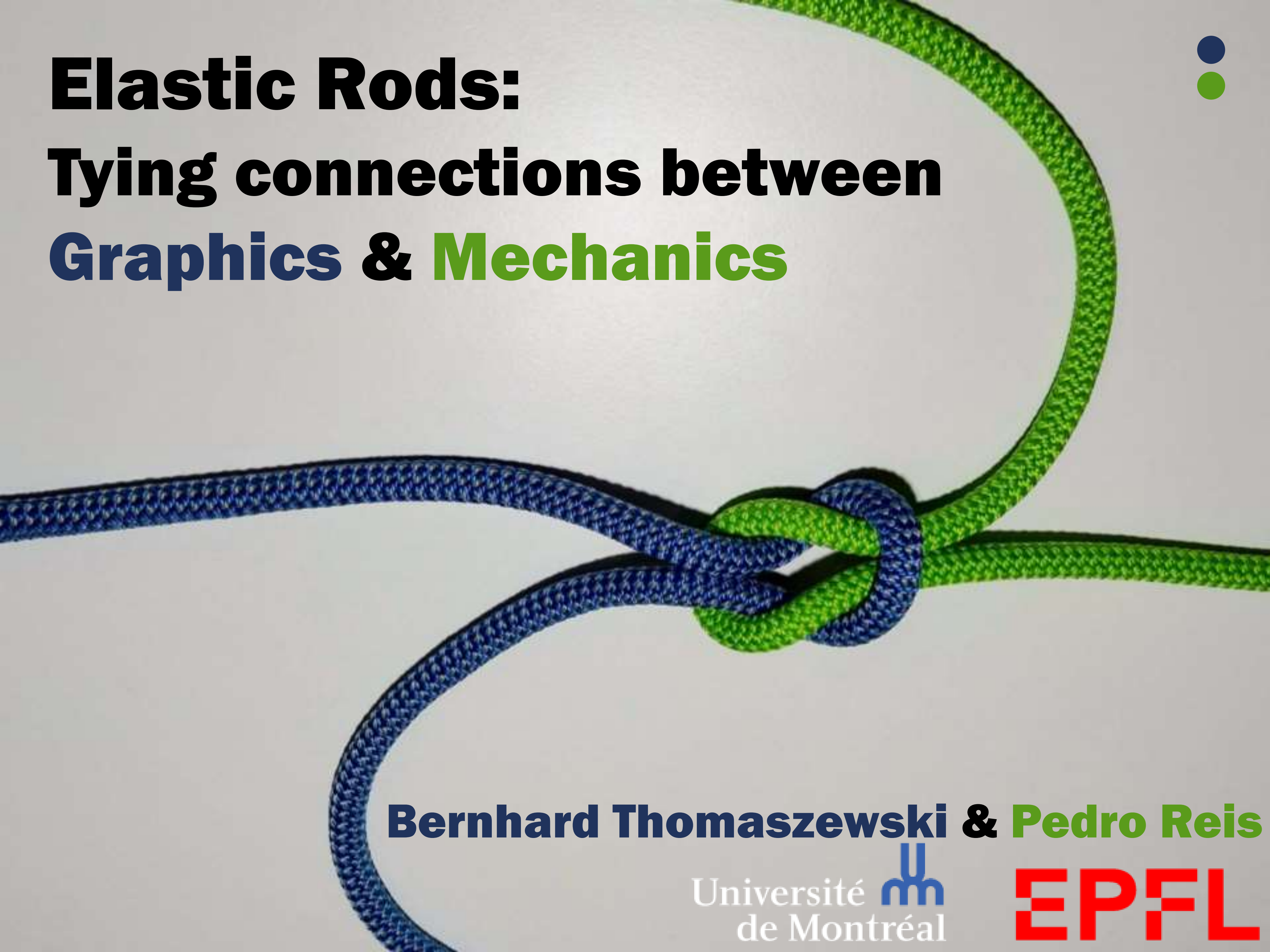
Elastic Rods: Tying connections between Graphics & Mechanics



Bernhard Thomaszewski & Pedro Reis

Université de Montréal

EPFL

FLEXLAB

FLEXIBLE STRUCTURES LABORATORY

http://flexlab.epfl.ch
: @flexlab_epfl

@EPFL since Nov. 2017

Mechanics of slender structures: '*Buckliphobia*' & '*Buckliphilia*':

Our goal is to **embrace mechanical instabilities of thin (soft) structures**, guided primarily through **precision model experiments**, towards understanding and exploiting **novel functional mechanisms** over a **wide range of length scales**.

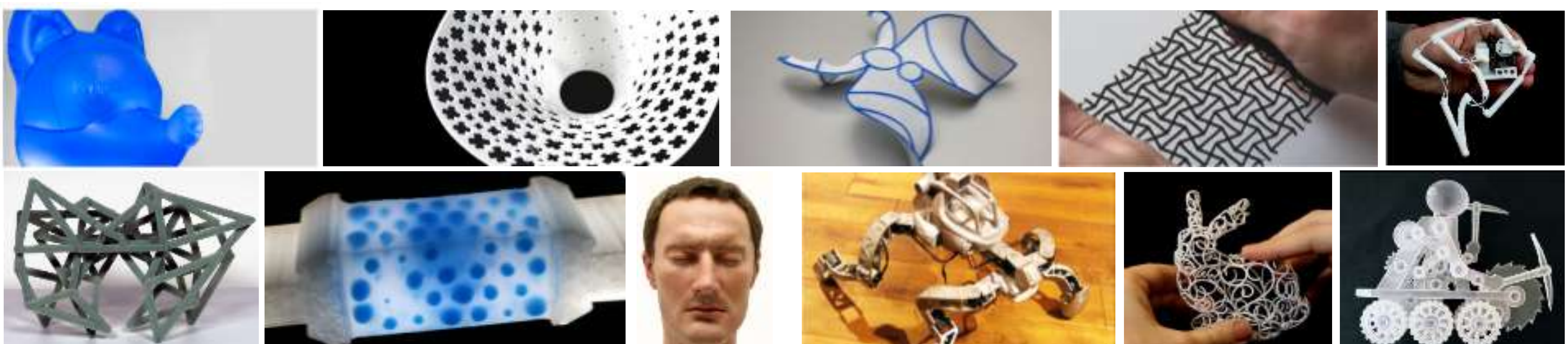
Computational Design Group

Simulation & Optimization-Driven Design

Université de Montréal

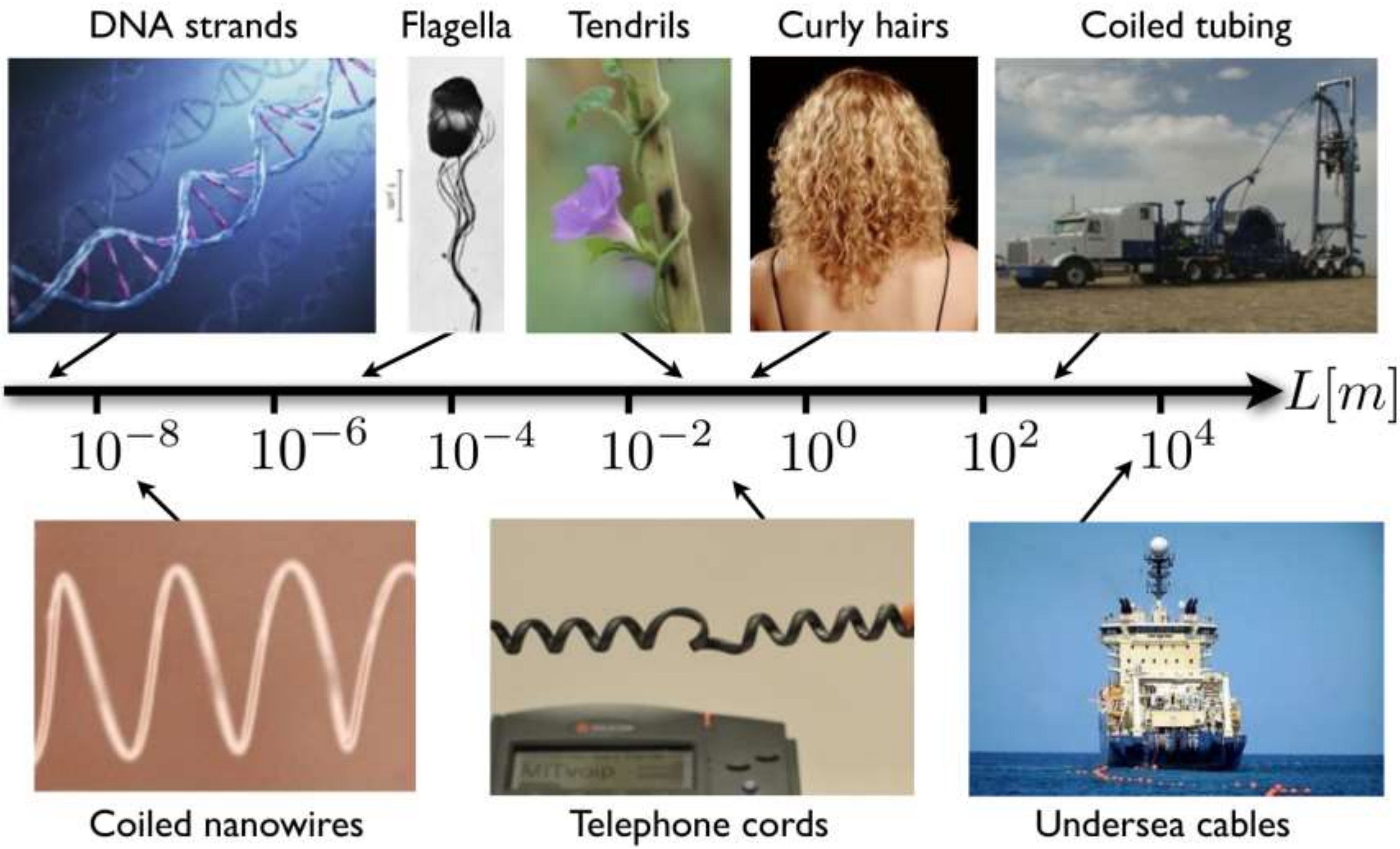
Research Themes

Physical Surface Structured Materials Mechanism Design Robotics Visual Simulation



Elastic Rods

Filamentary Structures...



... in nature and engineering.

Elastic Kirchhoff Rods

Gustav Kirchhoff

Alfred Clebsch

Augustus E. H. Love

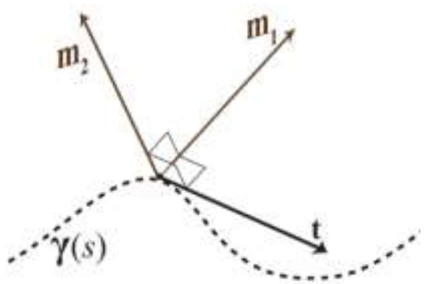
"… the classical theory of thin rods as developed by Kirchhoff [1859, 1876] and Clebsch [1862], and presented by Love [1892, 1906]."

E.H. Dill. *Kirchhoff's Theory of Rods*. Archive for History of Exact Sciences. Vol. 44, No. 1 (1992)

Elastic Kirchhoff Rods

Represent rod as an *adapted framed curve* $\Gamma(s) = \{\boldsymbol{\gamma}(s); F(\boldsymbol{s})\}$

- $\boldsymbol{\gamma}(s)$: $\mathbf{R} \to \mathbf{R}^3$ and arc-length parameterized curve describing the rod's *centerline*
- $F(s) = \{\boldsymbol{t}(s), \boldsymbol{m}_1(s), \boldsymbol{m}_2(s)\}$ an orthonormal *material frame* adapted to centerline, i.e., $\boldsymbol{t} = \boldsymbol{\gamma}'$



Bending:

$$b_1 = \mathbf{t}' \cdot \mathbf{m}_1 \quad b_2 = \mathbf{t}' \cdot \mathbf{m}_2$$

Twist:

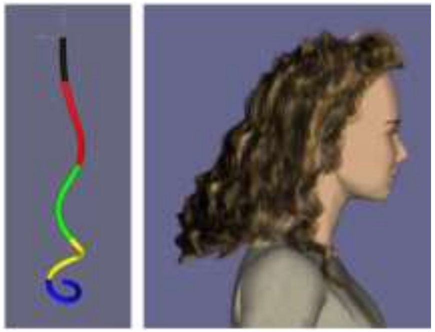
$$t = \mathbf{m}_1' \cdot \mathbf{m}_2$$

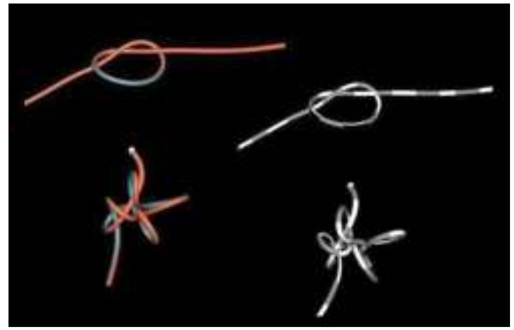
Stored energy*:

$$E(\Gamma) = \int_0^l k_b[b_1(s)^2 + b_2(s)^2] + k_t t(s)^2 \; ds$$

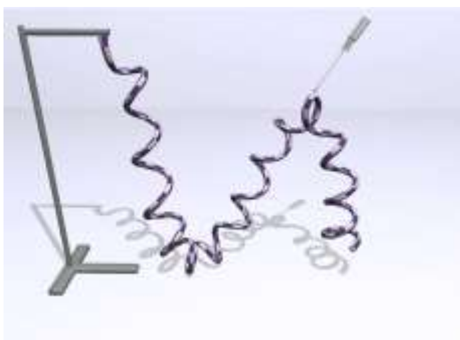
* for an initially-straight, isotropic rod

Rod Models from Graphics


[Bertails et al. '06]

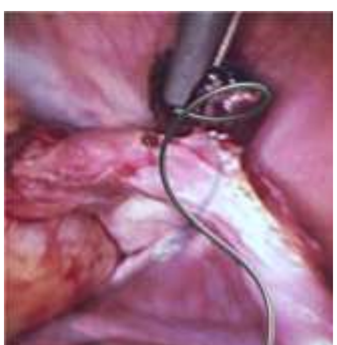

[Bertails '09]

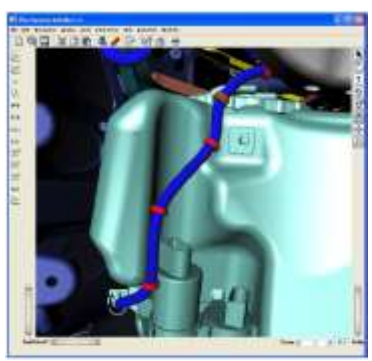

[Casati & Bertails-Descoubes '13]

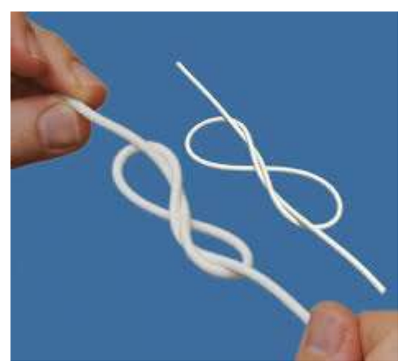

[Spillmann & Teschner '07]


[Spillmann & Teschner '08]


[Umetani et al. '13]


[Pai '02]


[Gregoire & Schoemer '07]


[Bergou et al. '08]

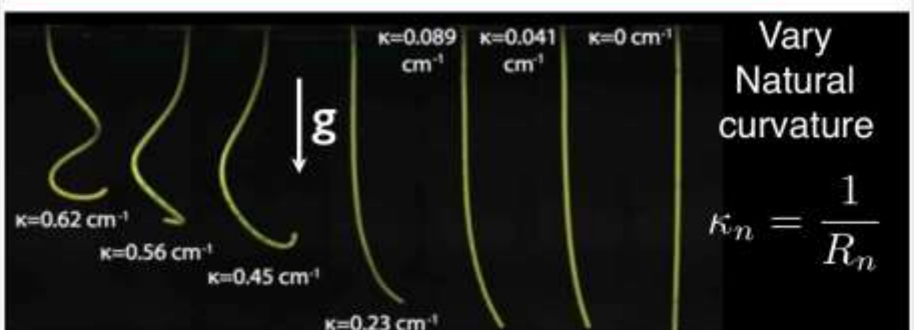

[Bergou et al. '10]

Thin Rods: Experiments

Fabrication of soft filaments

[A. Lazarus, J.T. Miller and P.M. Reis, *JMPS* 2013]
[A. Lazarus, J.T. Miller and P.M. Reis, *Soft Matter* 2013]

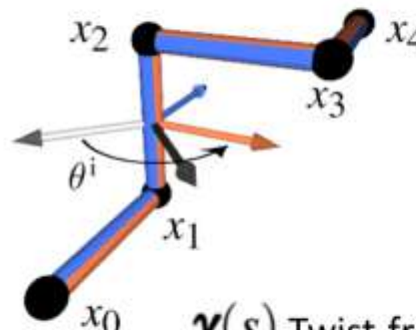
Suspended shapes of naturally curved rods



[A. Lazarus, J.T. Miller, B. Audoly and P.M. Reis, *PRL* 2014]

Thin Rods: Numerics

Discrete Elastic Rods



$\boldsymbol{\gamma}(s)$ Twist-free frame

The Hobbit, 2013

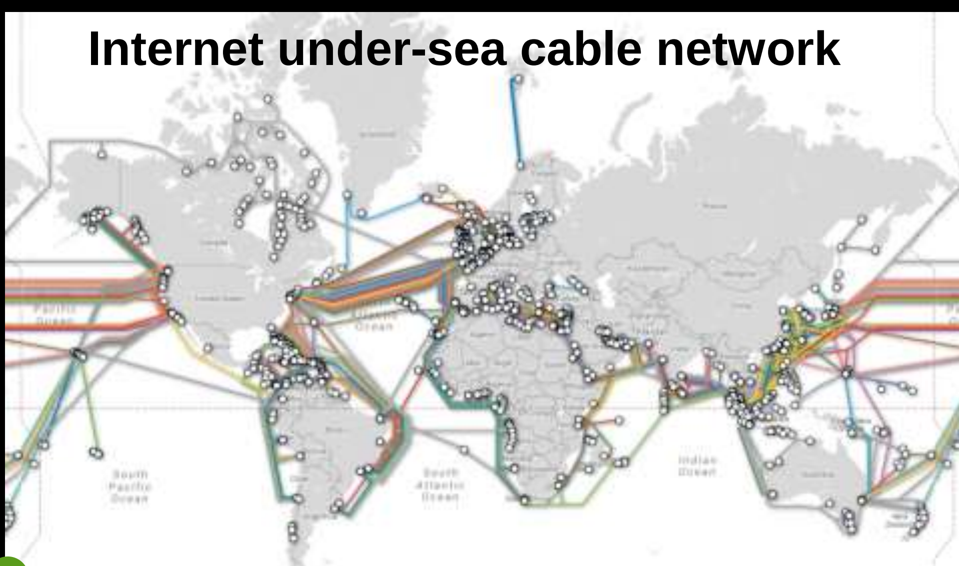
Simulation of ~100,000 hairs using DER

Example 1:

The elastic sewing machine & bacterial locomotion

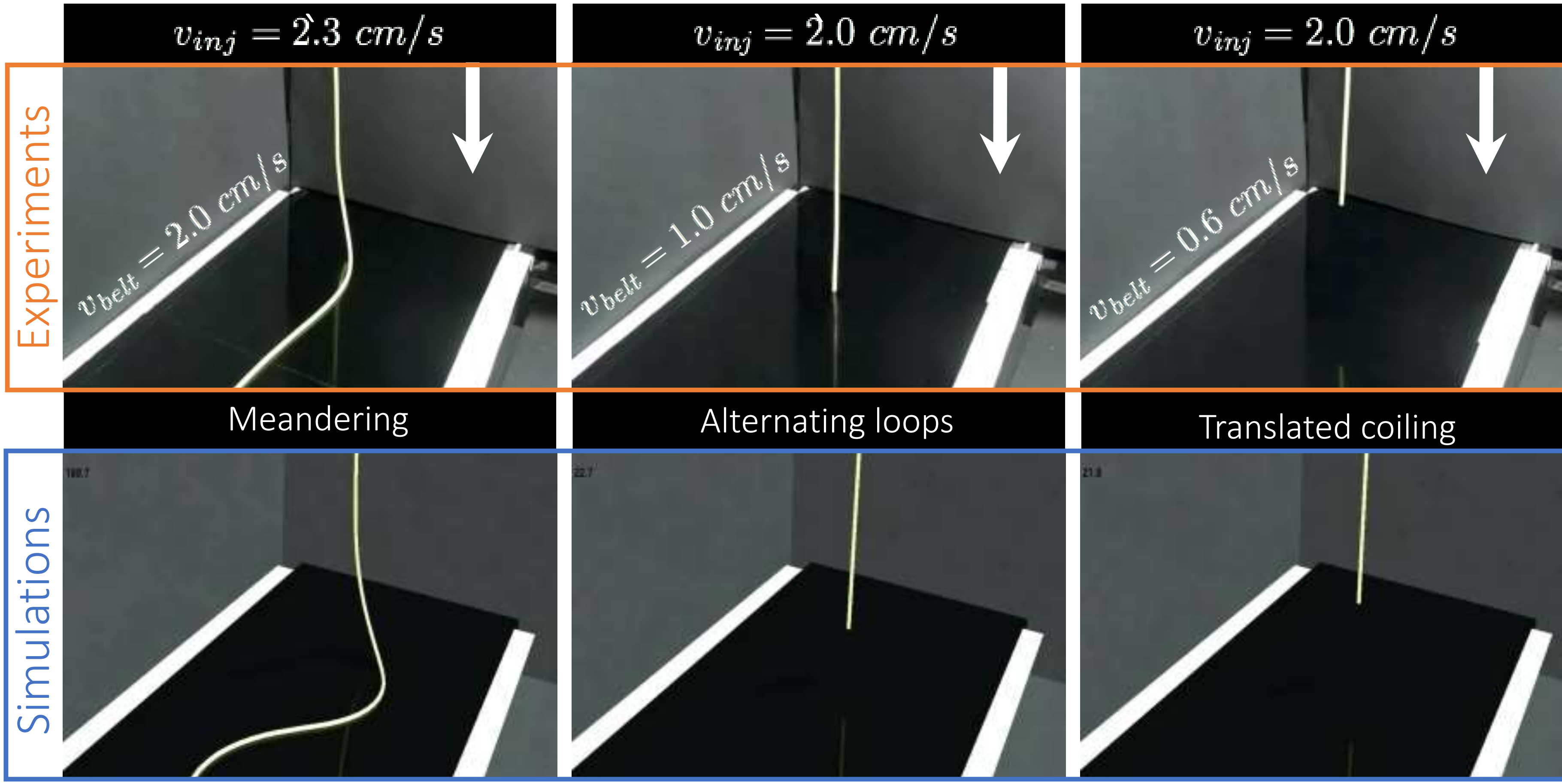
Deployment of pipelines/cables onto seabed

Pipe-
Lines
Under
The
Ocean



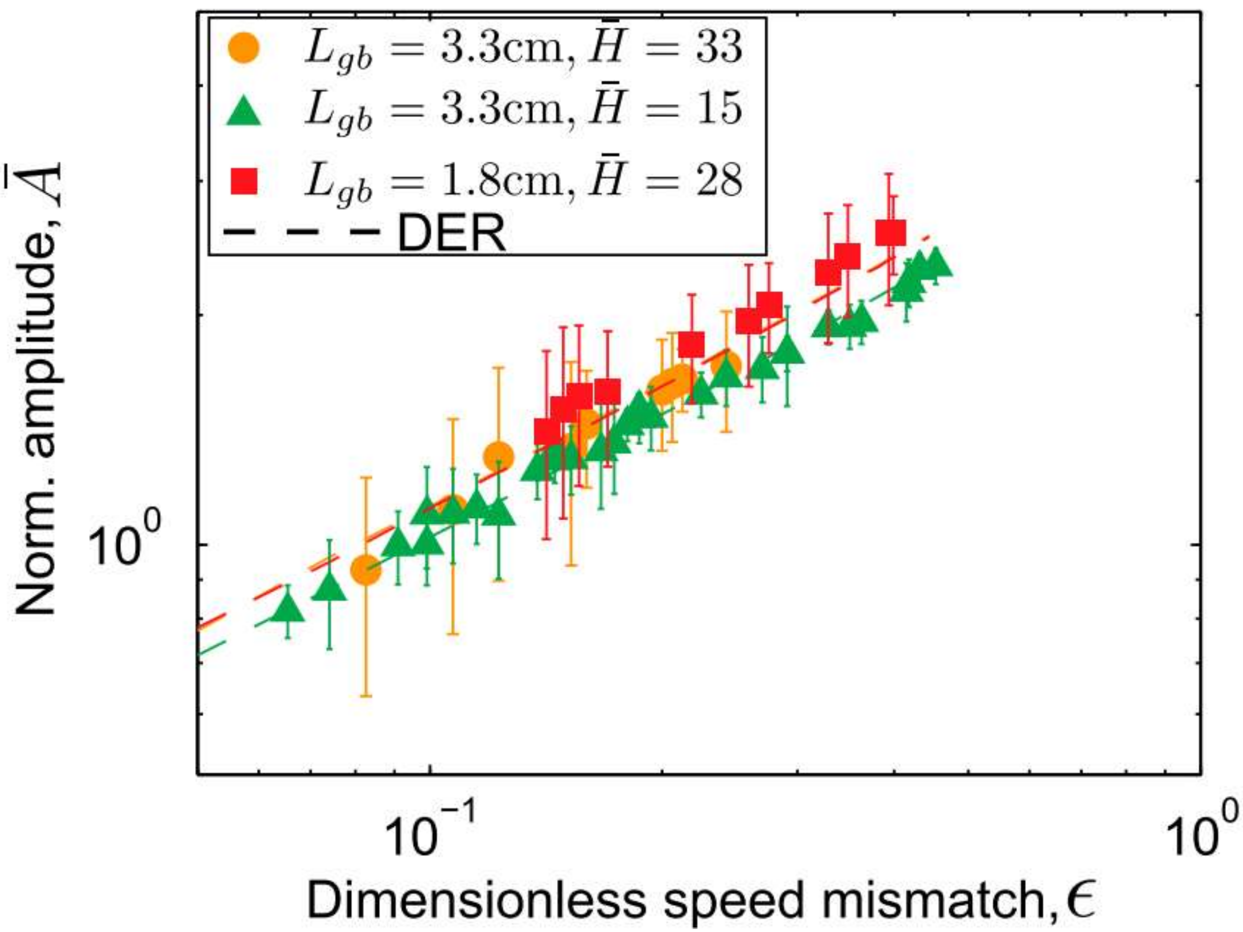
https://www.pri.org/stories/2015-04-20/what-links-global-internet-wires-inside-tubes-no-bigger-garden-hose

Deployment of an elastic rod onto a moving substrate



[PNAS 2014]
[Extreme Mech. Lett. 2015]
[J. App. Mech. 2015]

Excellent agreement between experiments (PDEs) and simulations (DER) with no fitting parameters!



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[Extreme Mech. Lett. 2015]
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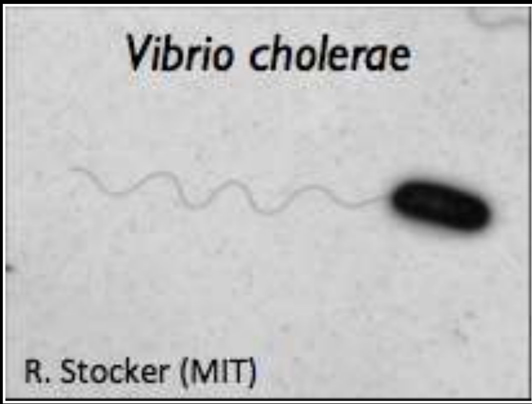
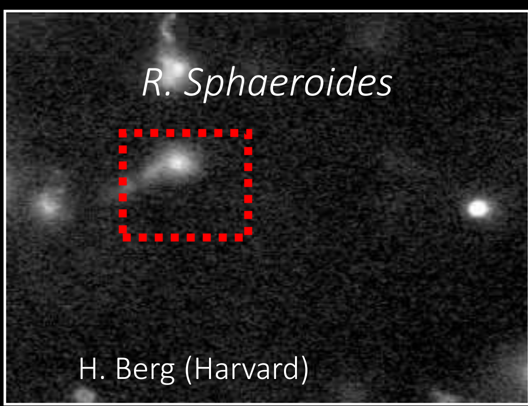
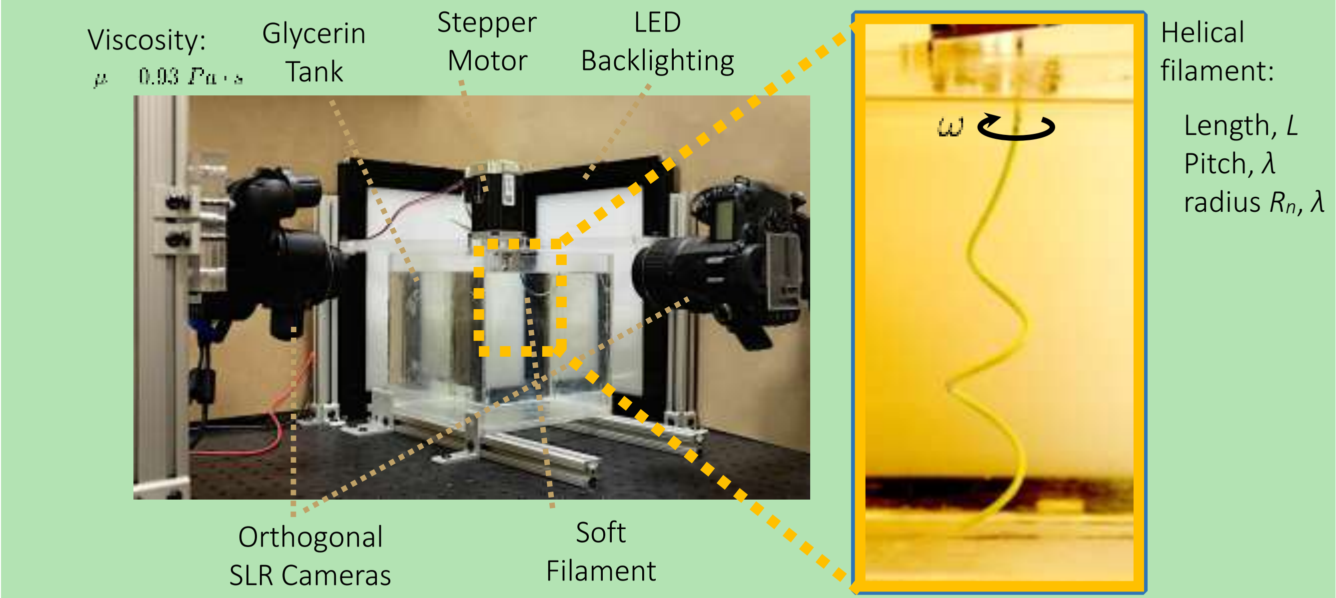




95% of bacteria in the ocean locomotive themselves by rotation of a single flexible helical flagellum



- **Couple DER code for rods with Lighthill's Slender Body Theory:**

  [J. Lighthill, "Flagellar hydrodynamics." SIAM Review (1976).]

- **Velocity $\mathbf{u}(s)$ at flagellum vs. force $\mathrm{f}(s)$ exerted by fluid.**

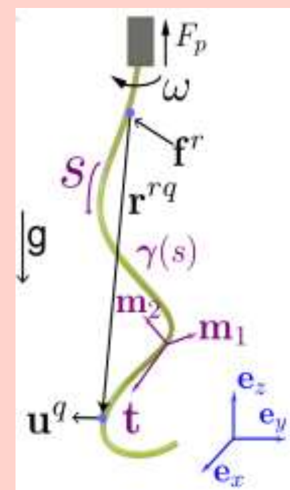
local non-local

$$\boxed{\mathbf{u}(s) = \frac{\mathbf{f}_\perp}{4\pi\mu} + \int_{r(s',s)>\delta} \mathbf{f}(s') \cdot \mathbf{J}(\mathbf{r}) ds'}$$

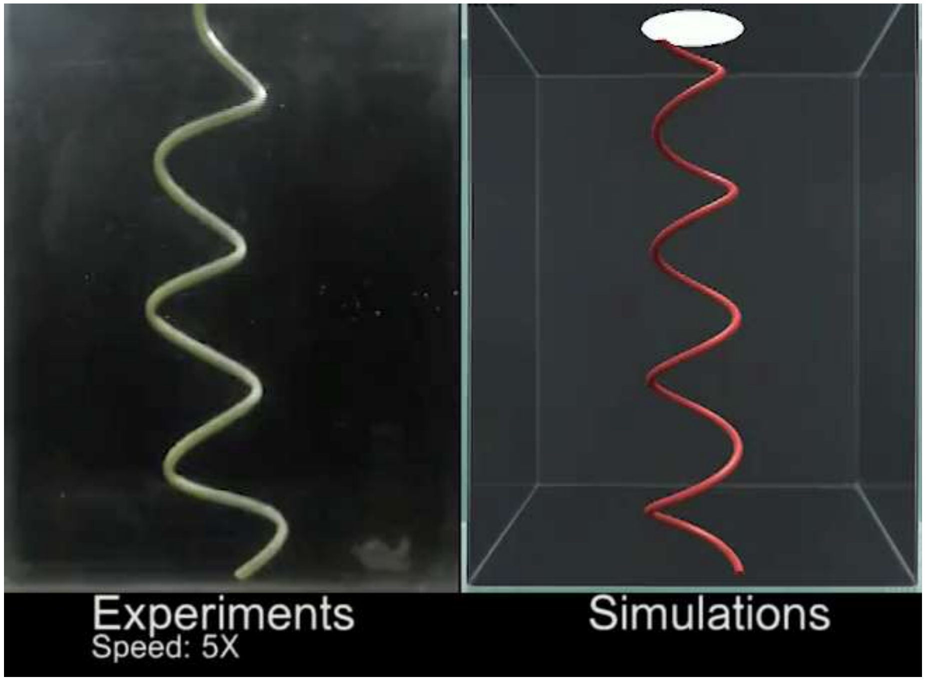
$$\mathbf{f}_\perp = \mathbf{f} \cdot \left(\mathbf{I} - \mathbf{t}\mathbf{t}^T\right) \quad \mathbf{J}(\mathbf{r}) = \frac{1}{8\pi\mu}\left(\frac{\mathbf{I}}{|\mathbf{r}|} + \frac{\mathbf{r}\mathbf{r}^T}{|\mathbf{r}|^3}\right)$$

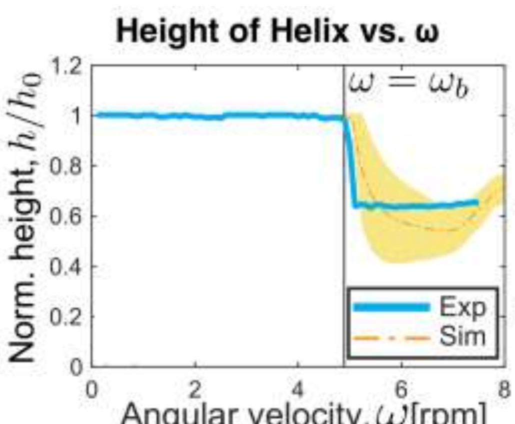
Cutoff

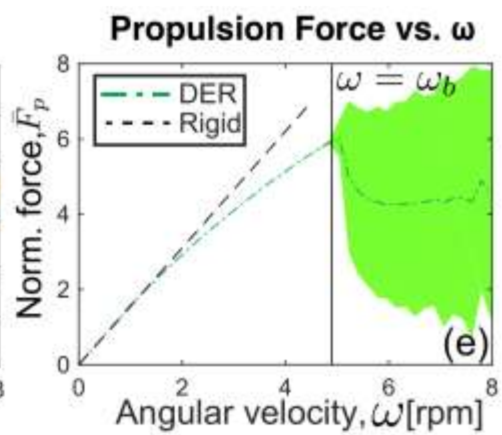
$$\delta = \frac{1}{2} r_o \sqrt{e}$$

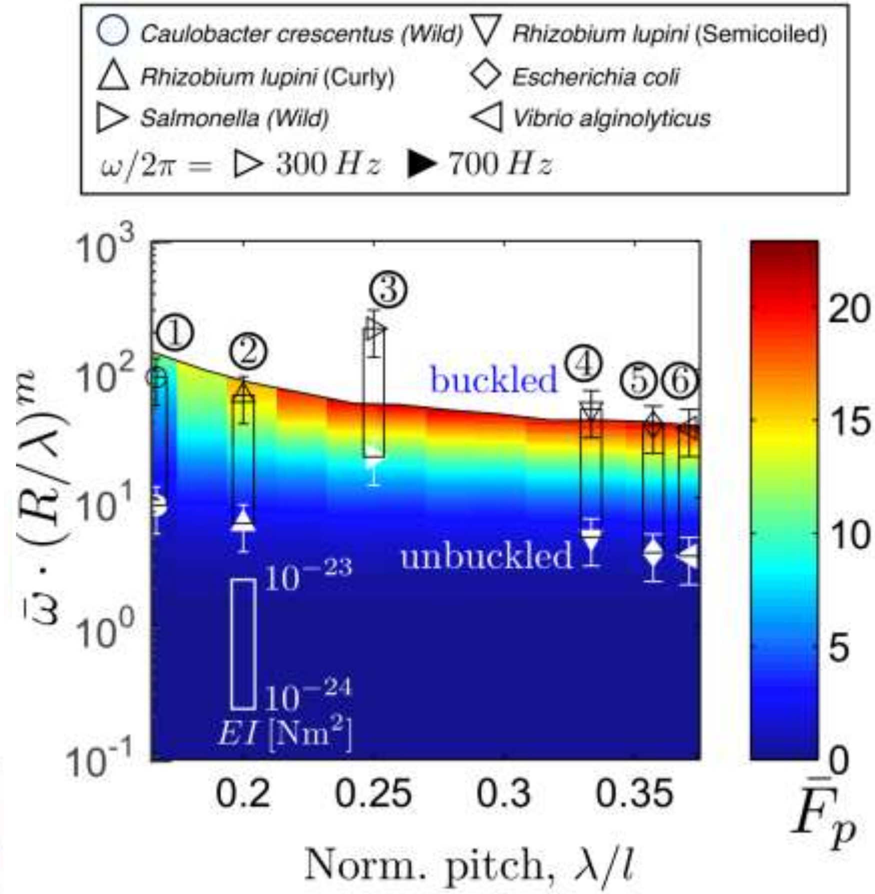


Dynamics of rotating helical rods in viscous fluids | Bacterial locomotion







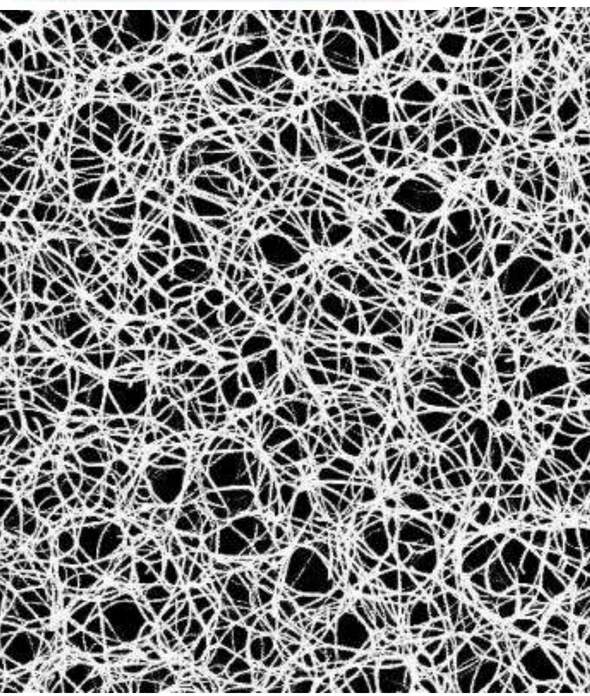


Could some bacteria be exploiting this buckling mechanism for turning?

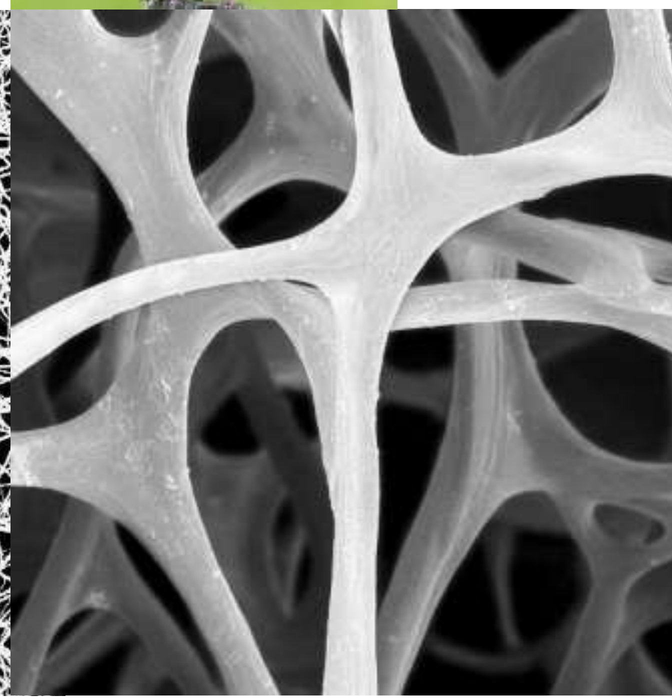
Example 2:

Structured Sheet Materials

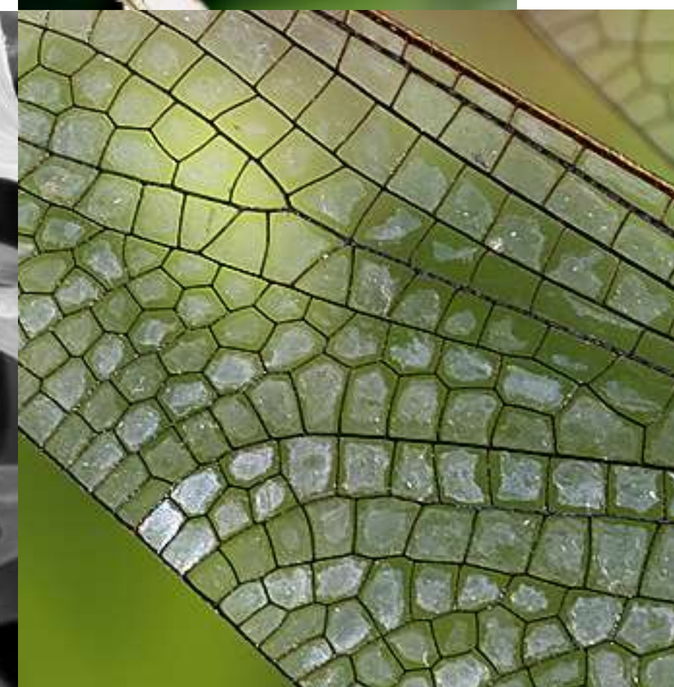
Natural Network Materials



Muscle tissue (actin filaments)



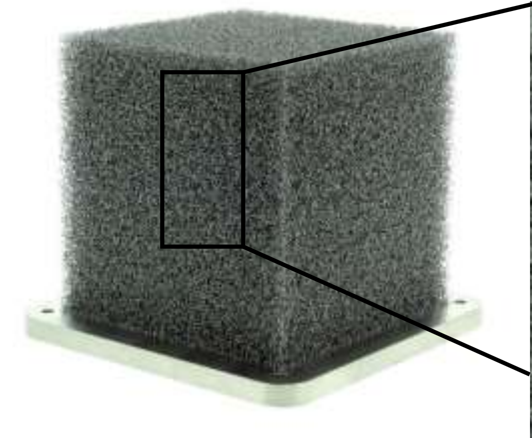
Cancellous bone (trabeculae)



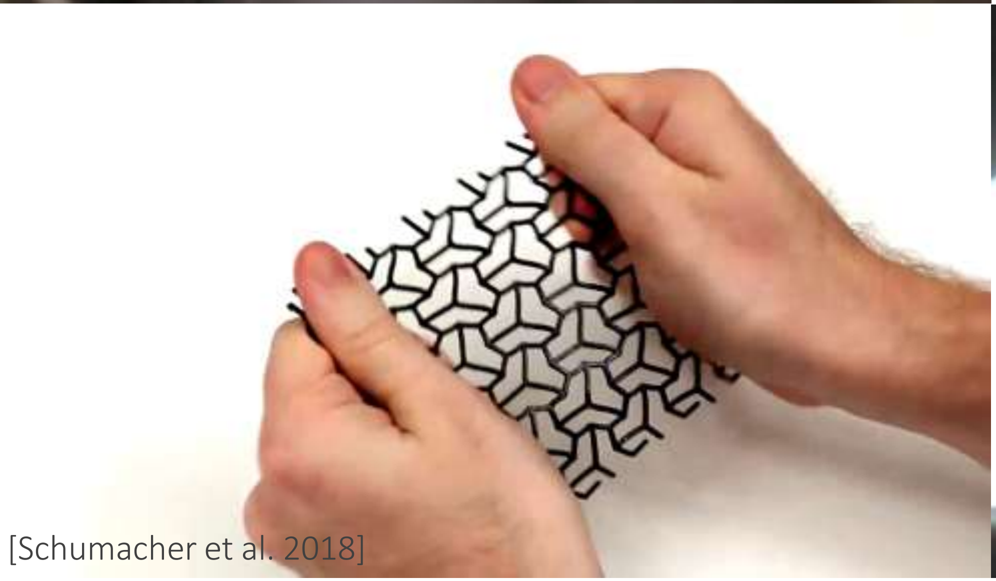
Vein structure (chitin, resilin)

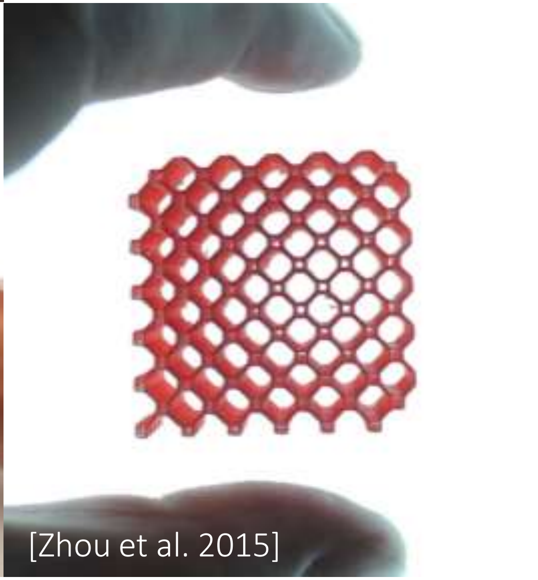
Digital Network Materials


[Carbon]

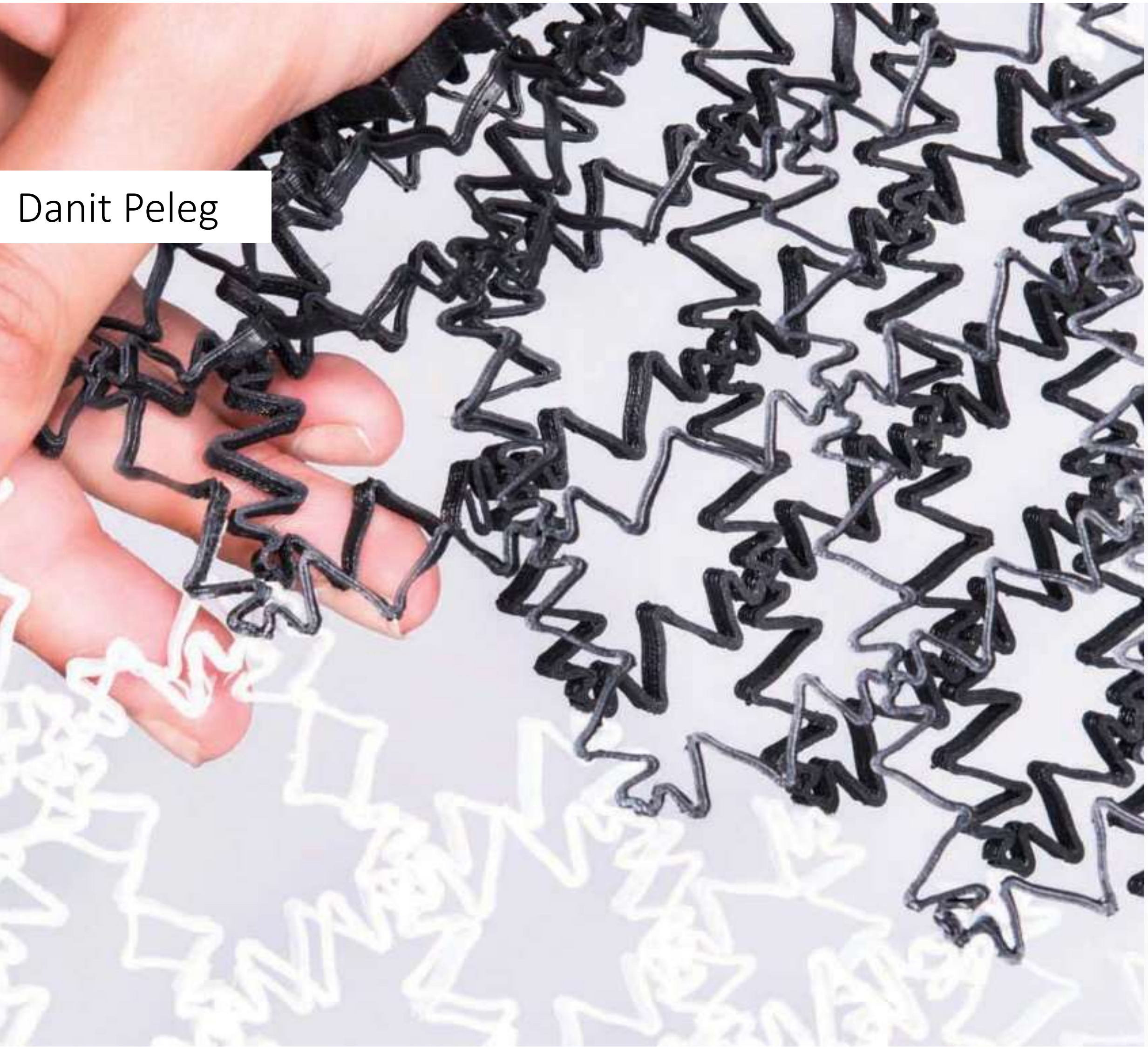


[Renishaw]


[Schumacher et al. 2018]

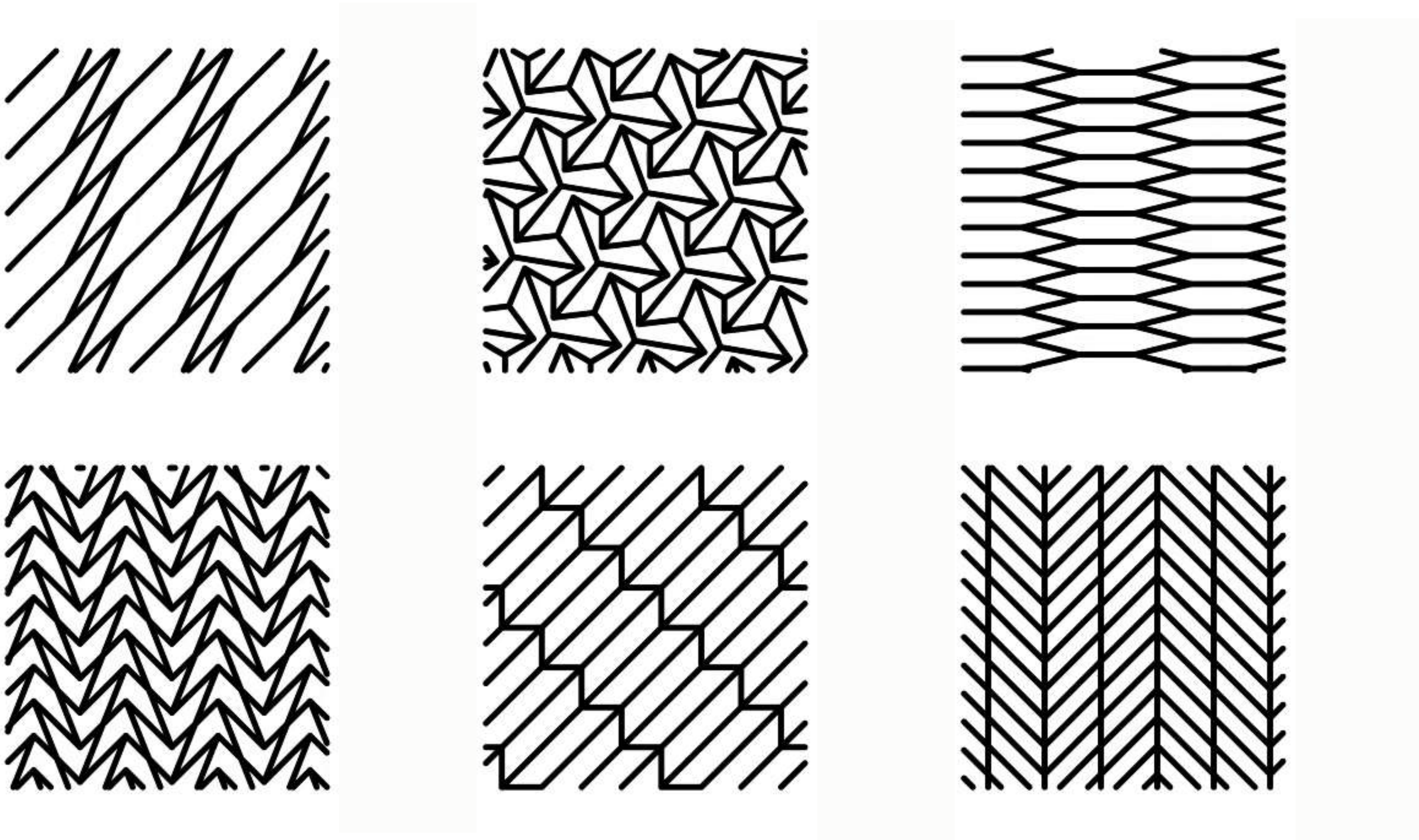

[Zhou et al. 2015]

3D-Printed Fabric



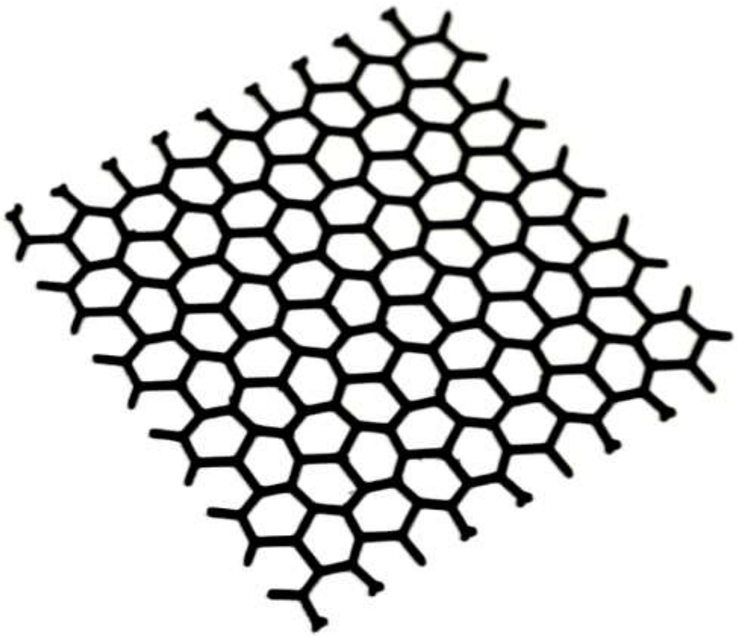
Danit Peleg

Elastic Isohedral Tilings

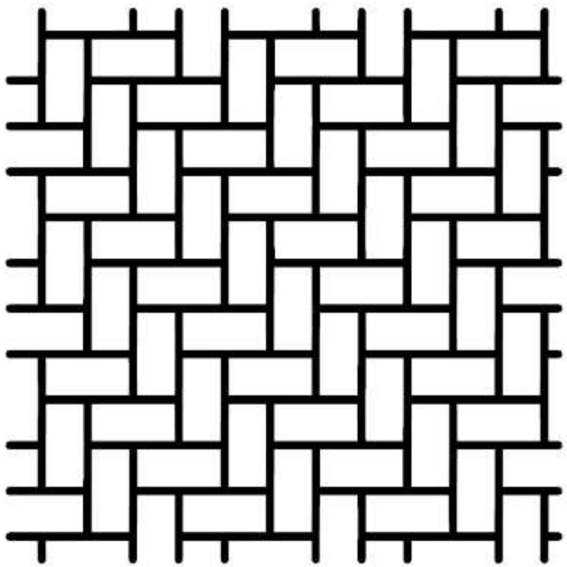


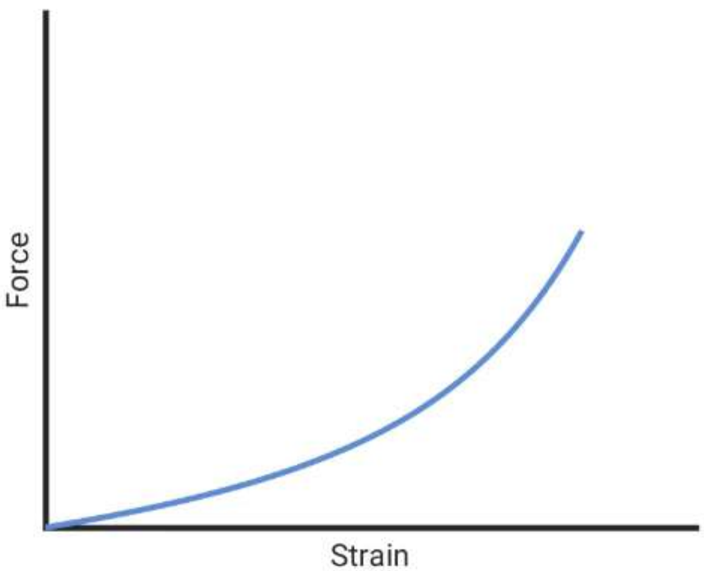
Schumacher, Marschner, Gross, Thomaszewski. *Structured Sheet Materials*. **SIGGRAPH '18.**

3D-Printed Tilings

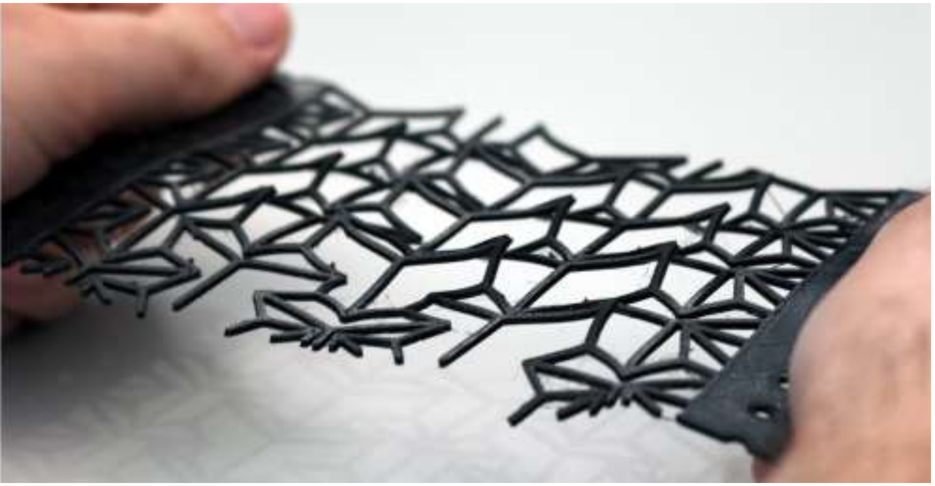


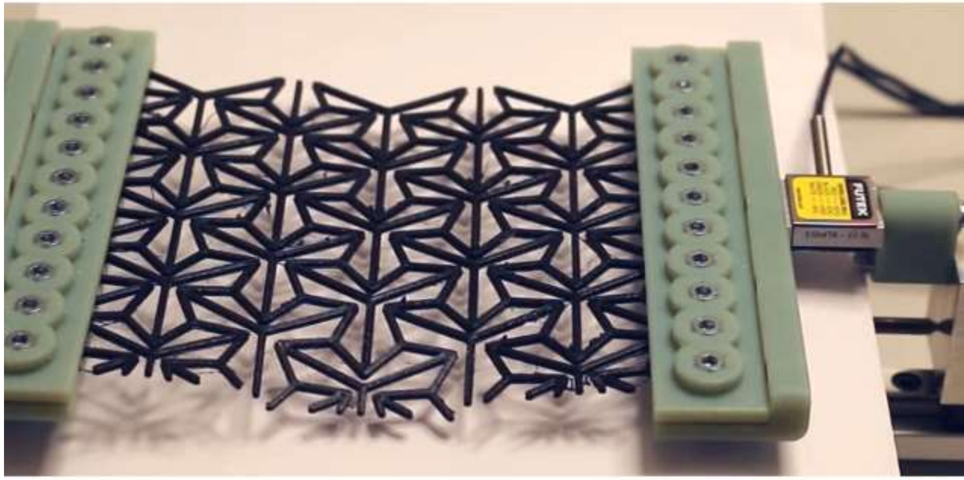
Rod Network Mechanics

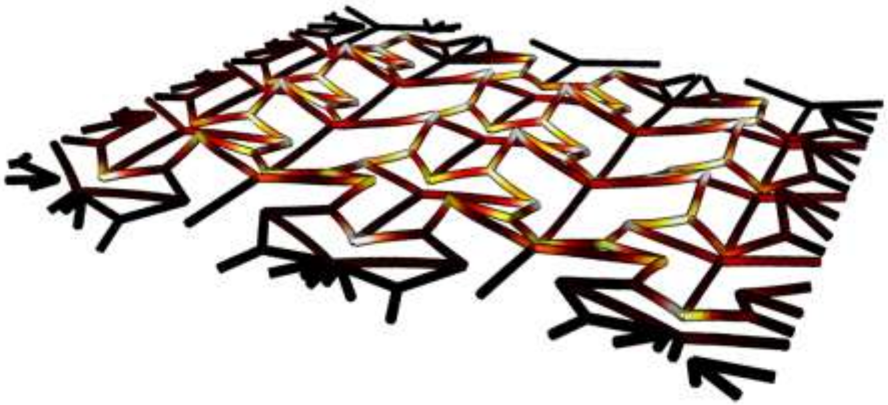


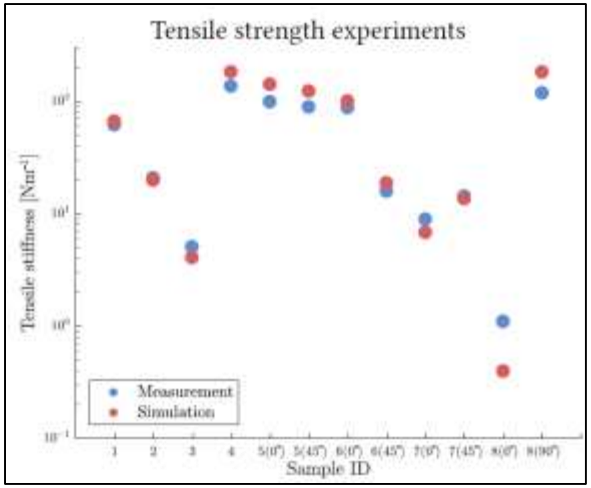


Simulation



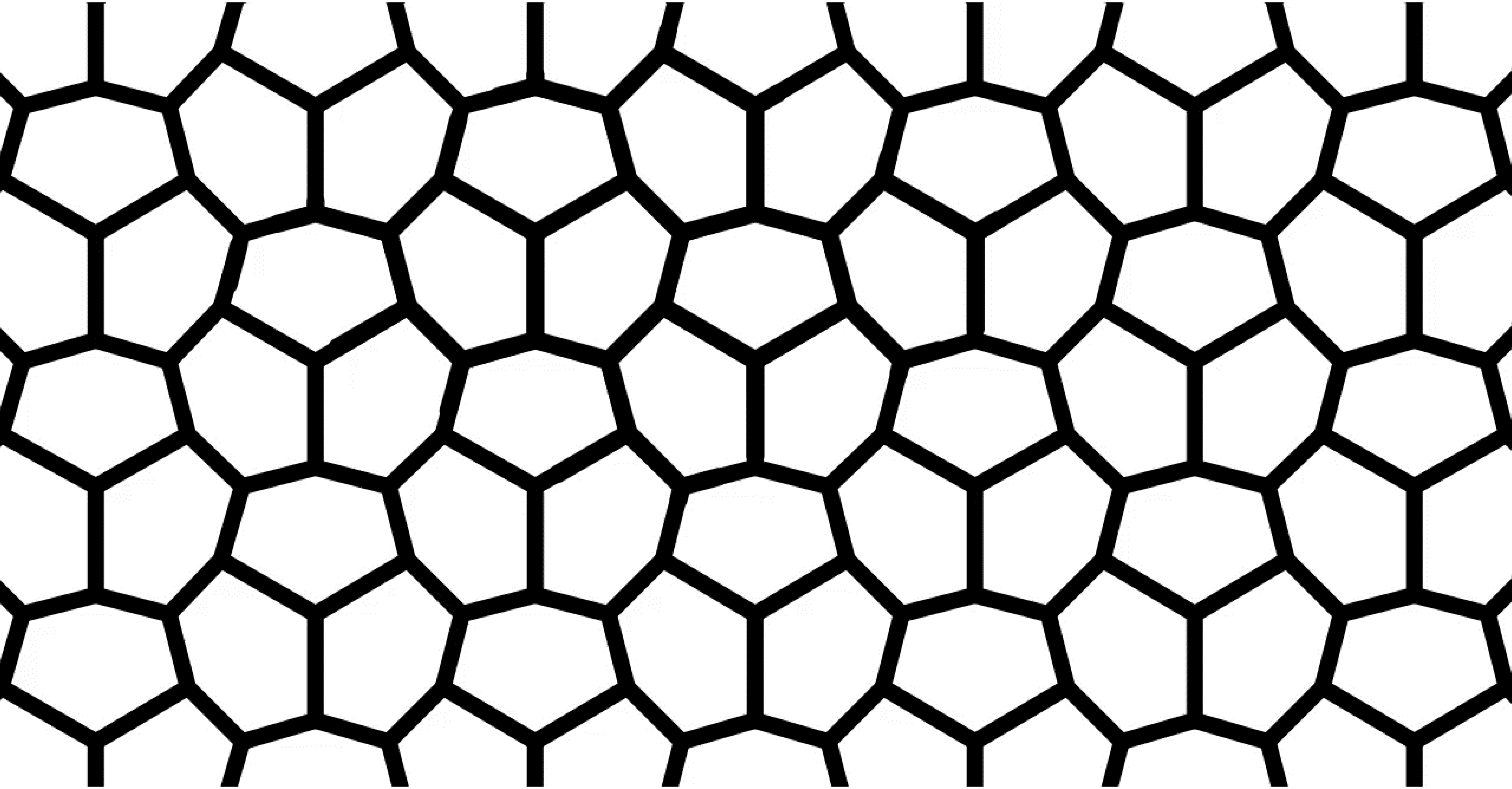




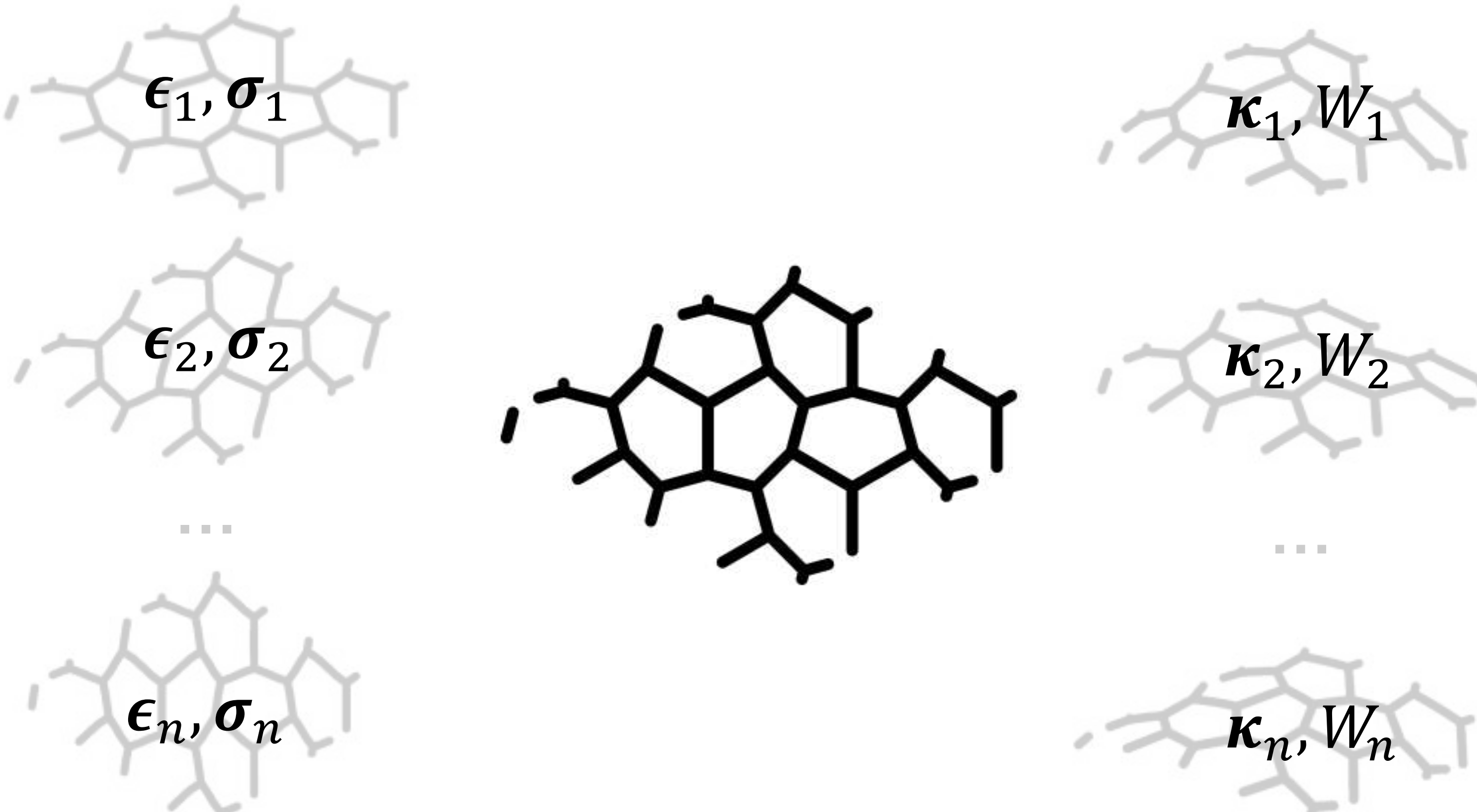


Discrete Elastic Rods ([Bergou '08,'10])
+ Extension to Networks ([Perez '15], [Zehnder '16])

Mechanical Characterization



Mechanical Characterization



Macromechanical Model

$$W = \epsilon : \mathbb{C} : \epsilon + \kappa : \mathbb{B} : \kappa$$

Membrane Bending

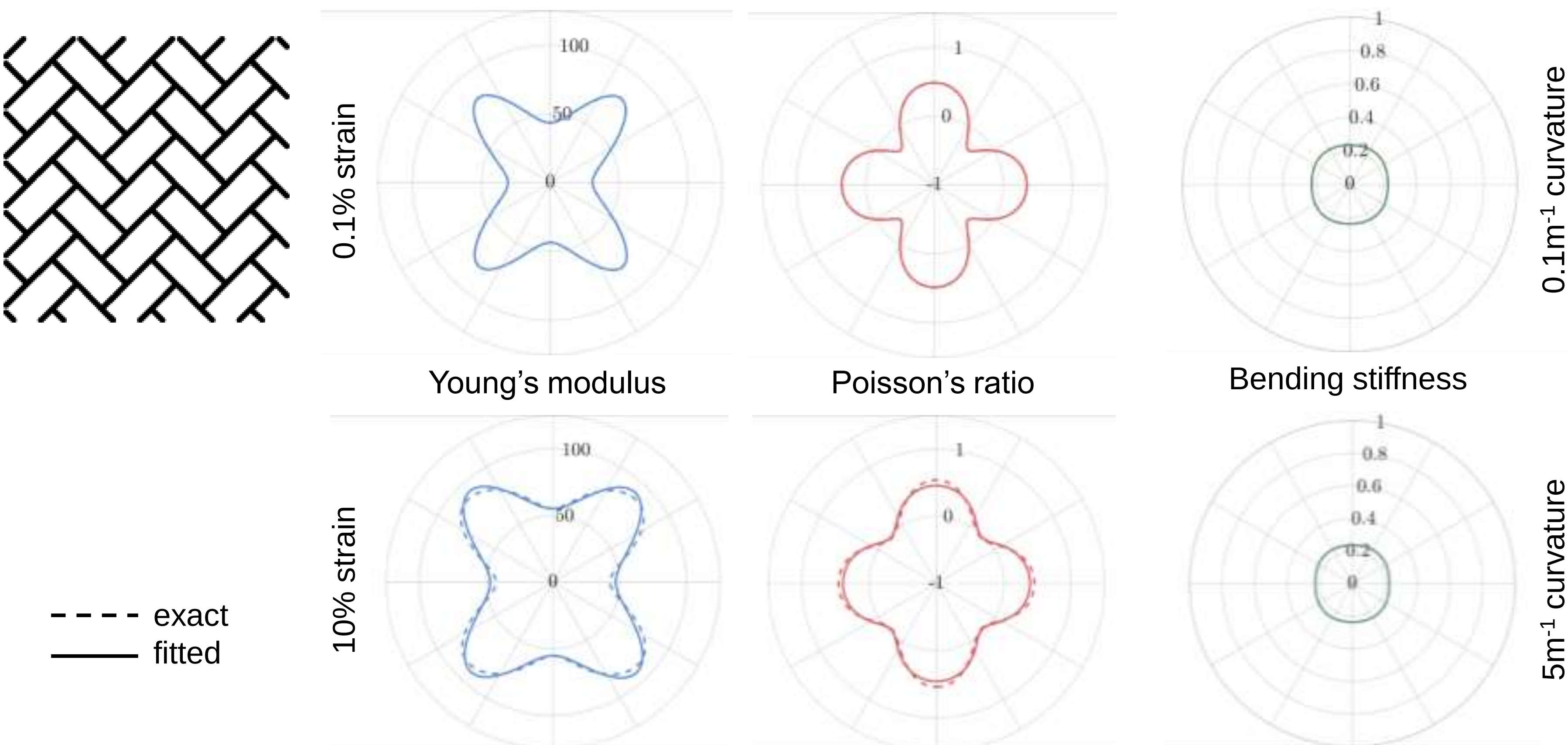
Membrane

$$\mathbb{C}(\boldsymbol{\epsilon}_1, \boldsymbol{\sigma}_1, \boldsymbol{\epsilon}_2, \boldsymbol{\sigma}_2, \ldots, \boldsymbol{\epsilon}_n, \boldsymbol{\sigma}_n)$$

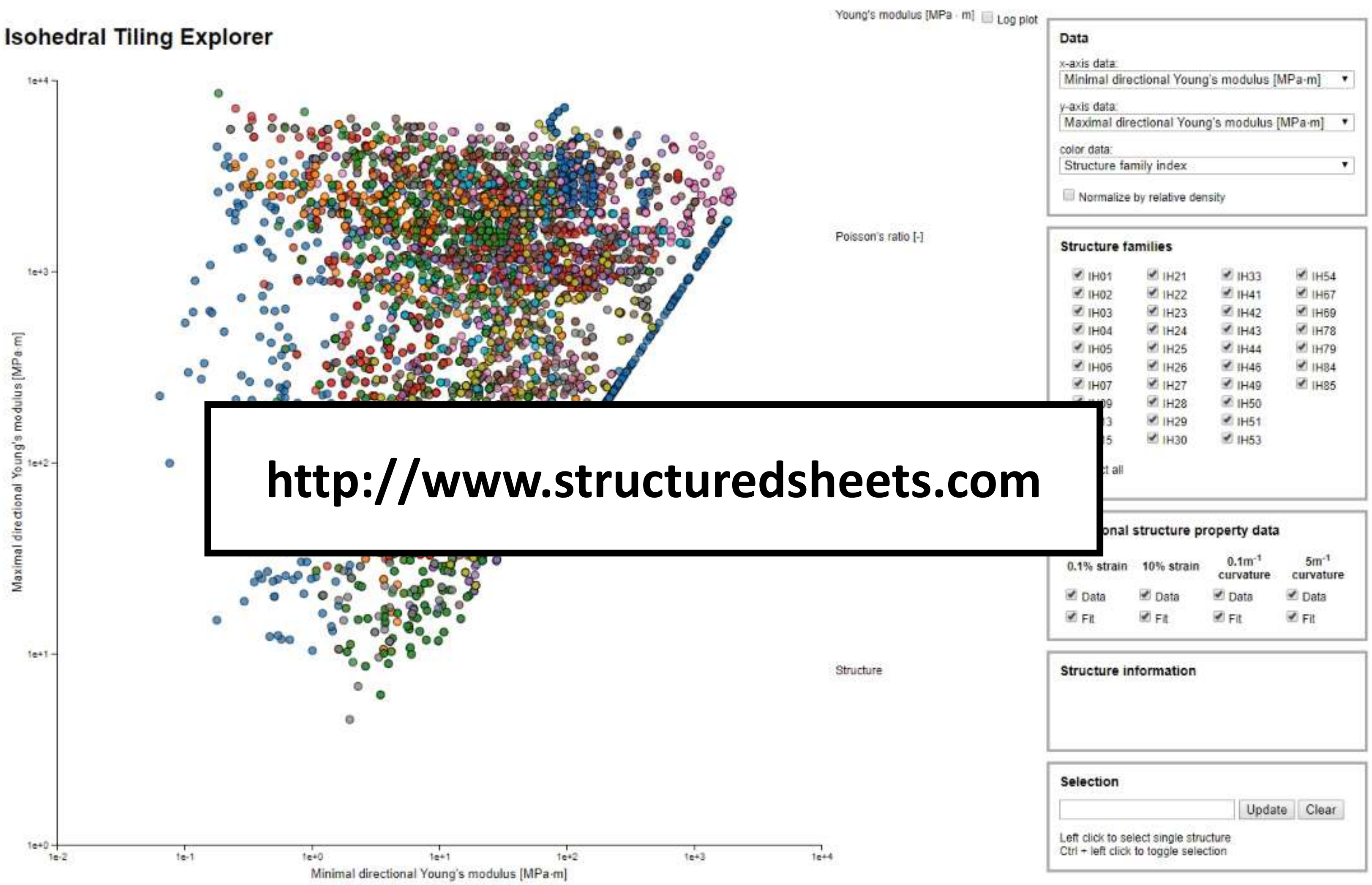
Bending

$$\mathbb{B}(\boldsymbol{\kappa}_1, W_1, \boldsymbol{\kappa}_2, W_2, \ldots, \boldsymbol{\kappa}_n, W_n)$$

Macromechanical Representation

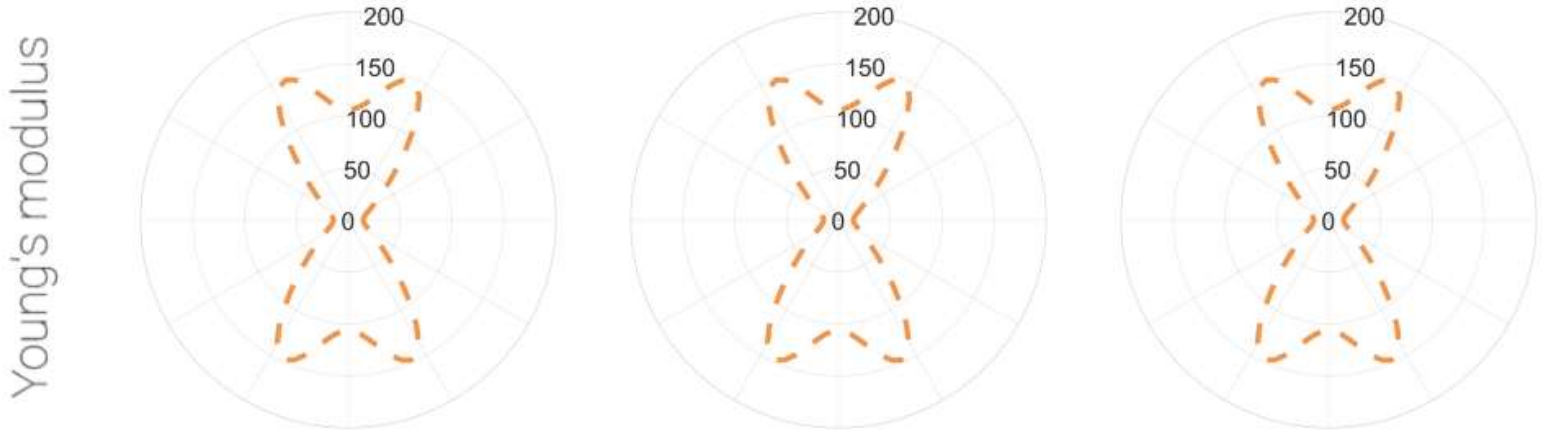


Exploration



http://www.structuredsheets.com

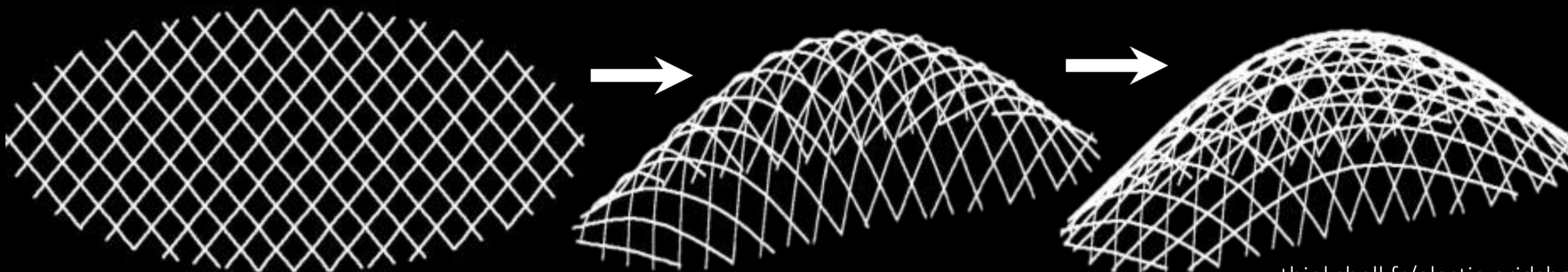
Inverse Design



Example 3:

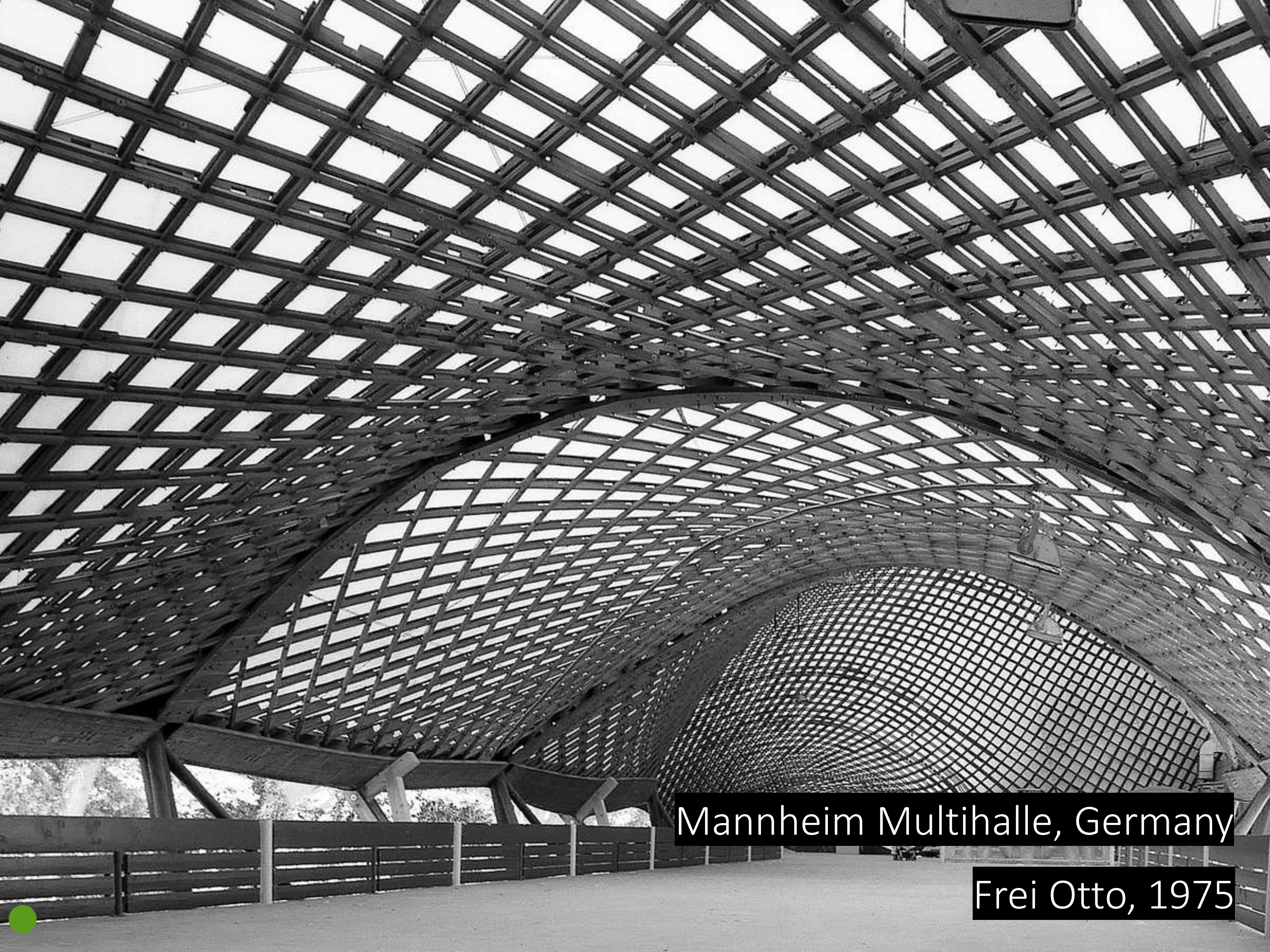
Elastic gridshells

Shaping through buckling in elastic gridshells



thinkshell.fr/elastic-gridshell/

Forum Café Gridshell for Solidays Festival, Paris (2011)



Mannheim Multihalle, Germany

Frei Otto, 1975

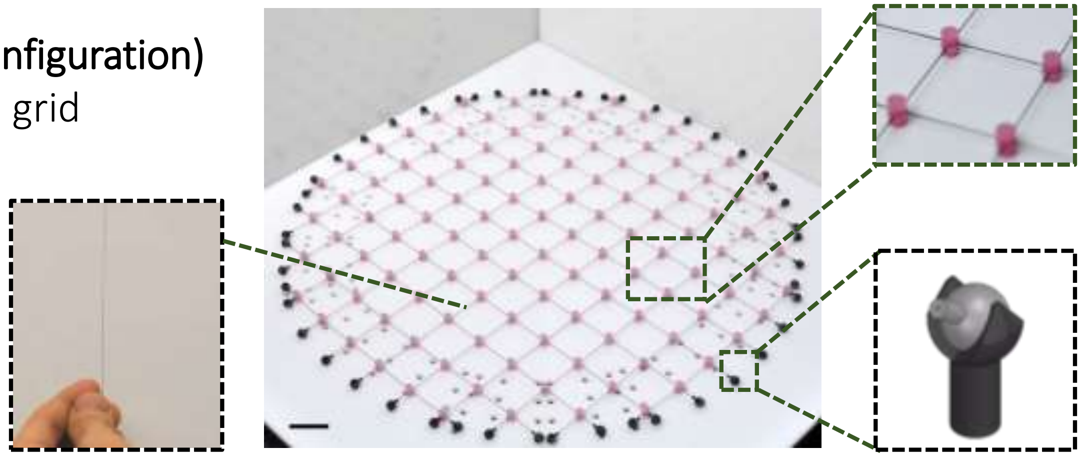
Anatomy and Actuation of an Elastic Gridshell

Footprint (rest configuration)

* Quadrilateral grid

Rods:

* Nitinol
* $E = 83\,\mathrm{GPa}$
* $d = 254\,\mu\mathrm{m}$

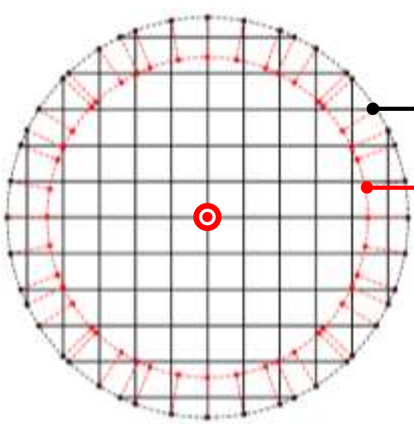


Joints:

* VPS $E = 1.3\,\mathrm{MPa}$
* diameter $d = 3\,\mathrm{mm}$
* height $h = 5\,\mathrm{mm}$

Boundary points:

* 3D printed ball joints
* Pinned B.C.s



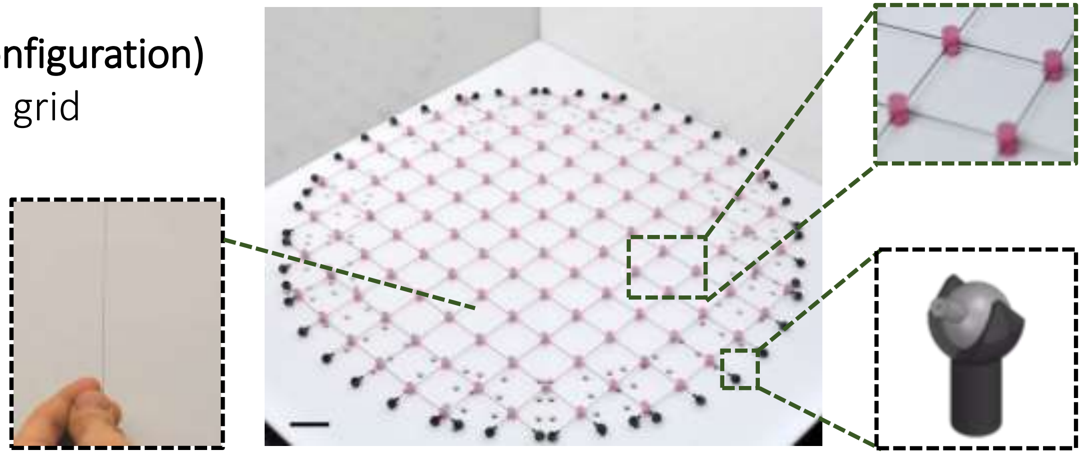
Original boundary of footprint

Actuated boundary

Anatomy and Actuation of an Elastic Gridshell

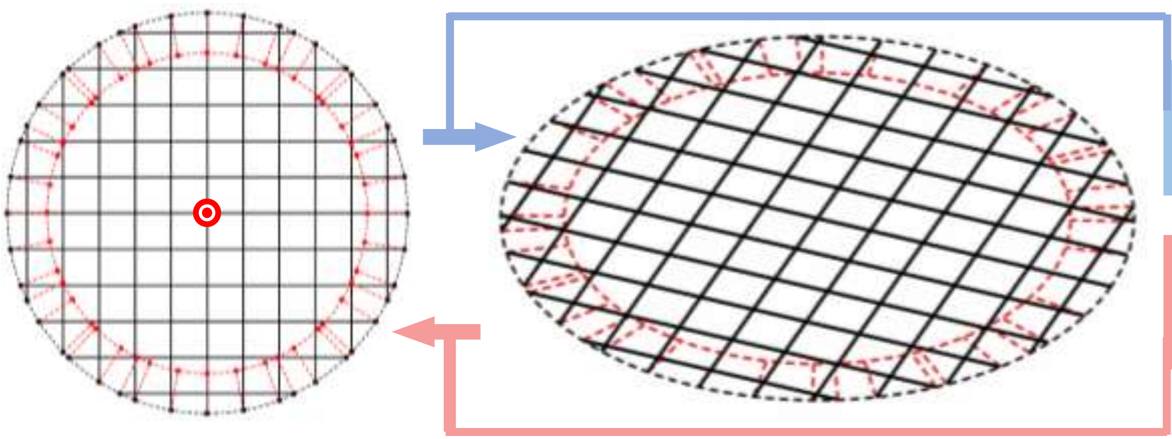
Footprint (rest configuration)
- * Quadrilateral grid

Rods:
- * Nitinol
- * $E = 83\,\mathrm{GPa}$
- * $d = 254\,\mu\mathrm{m}$



Joints:
- * VPS $E = 0.23, \mathrm{MPa}$
- * diameter $d = 3\,\mathrm{mm}$
- * height $h = 5\,\mathrm{mm}$

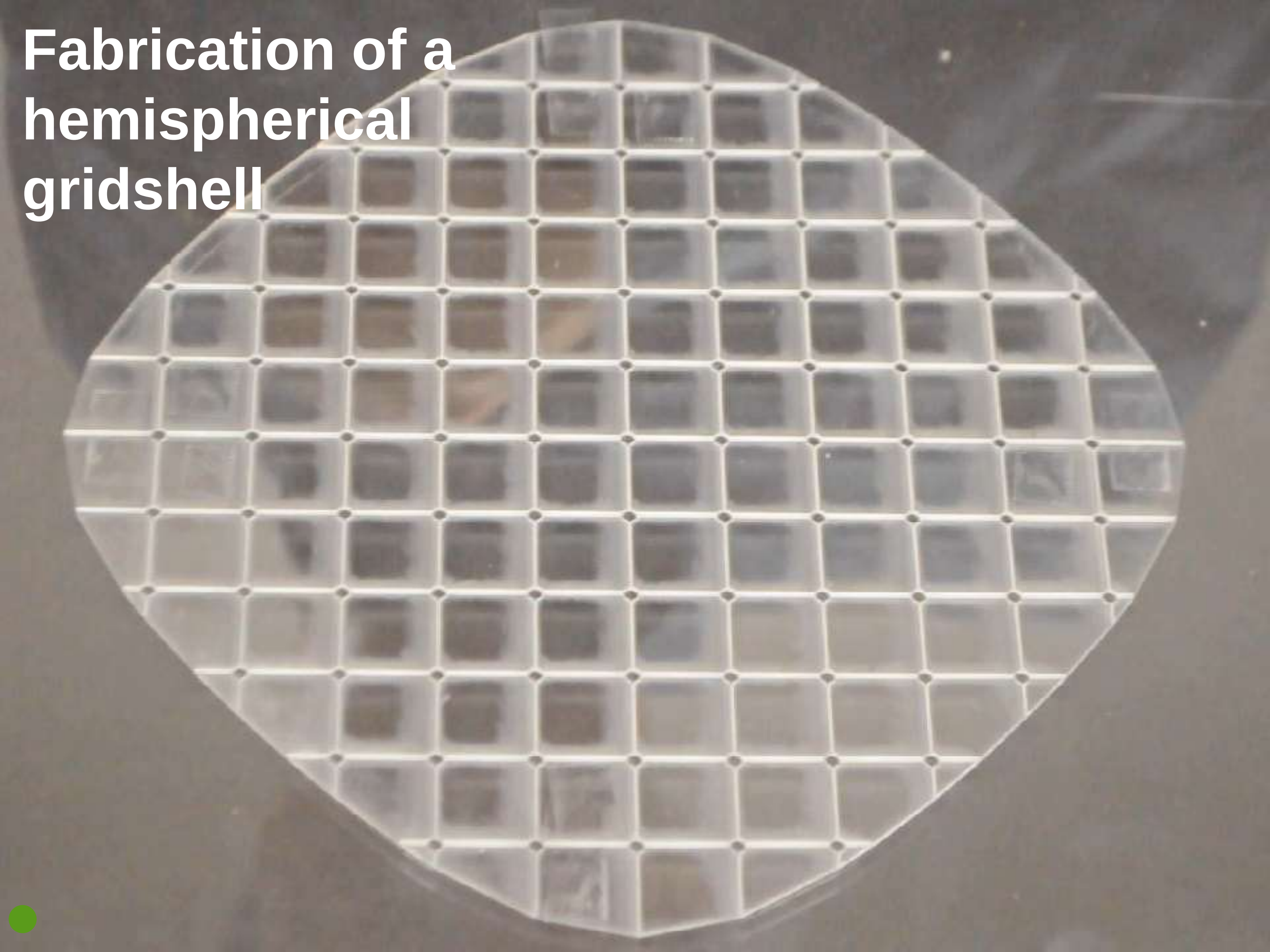
Boundary points:
- * 3D printed ball joints
- * Pinned B.C.s



Given a footprint, what is the elastic gridshell generated?

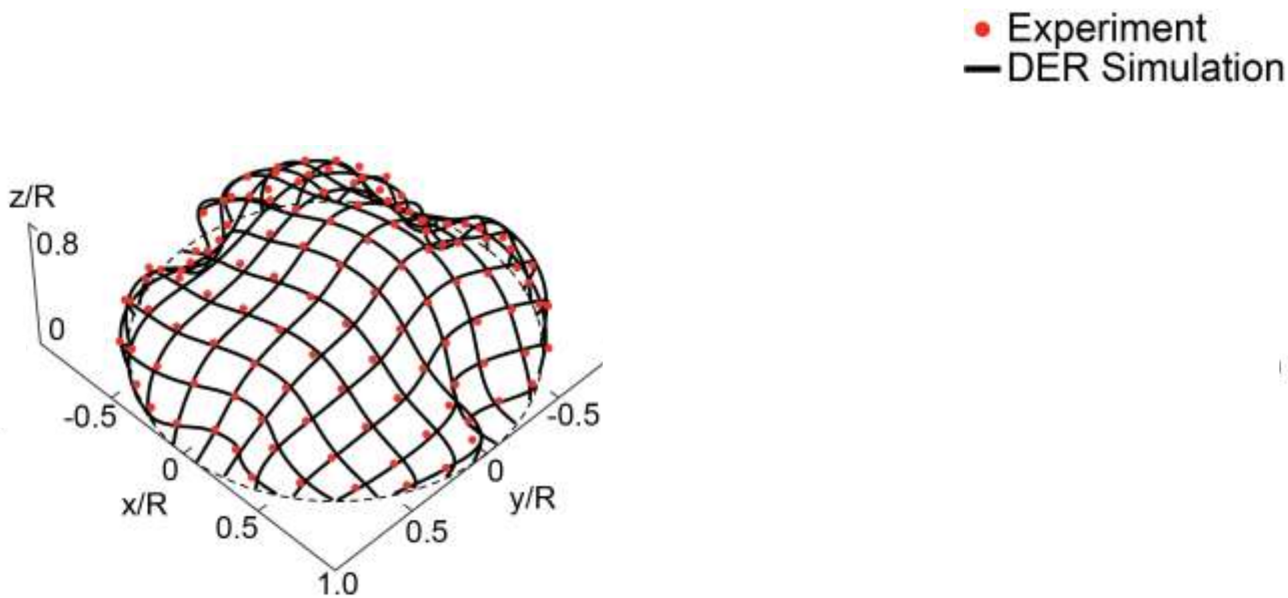
Given a target shape, what should be the footprint (**inverse design**)?

Fabrication of a hemispherical gridshell



Physical Experiments v. DER

3D Digital Scanning (NextEngine)

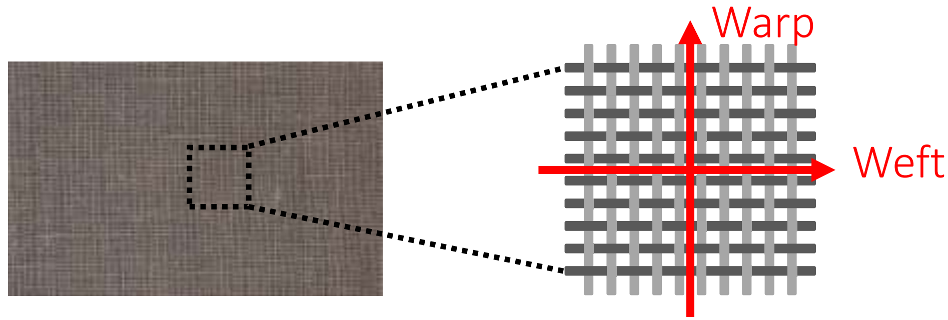


- Excellent quantitative agreement between exps. & DER.
- Multiplicity of states for same input parameters.

Theory of Chebyshev Nets

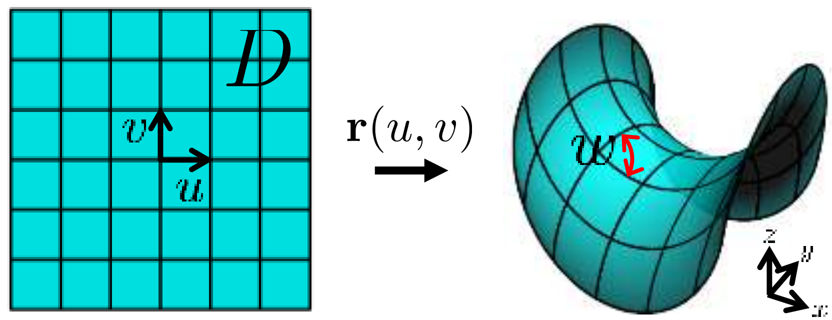
***Pafnuty Chebyshev* (1821-94)**

- ~~Probability, Chebyshev polynomials, number theory~~
- How do textiles drape?



Chebyshev Net (1878):

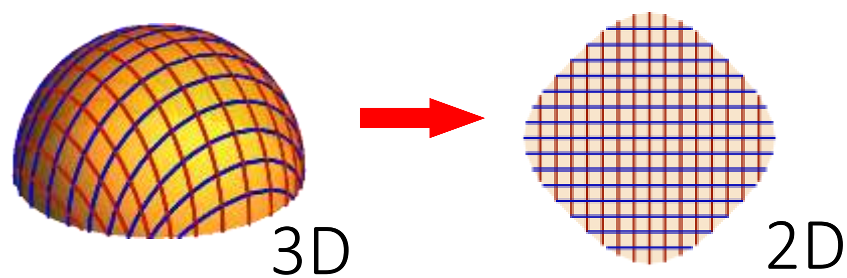
- Maps 2D-to-3D:
  $r : D \subset \mathbb{R}^2 \rightarrow \mathbb{R}^3$
- Inextensible rods in u, v directions:
  $|r_u| = |r_v| = 1$
- (Shearing) angle changes of tangent vectors at a point deform the metric:
  $\omega(u,v) = \angle(r_u, r_v)$



Gauss equation for Chebyshev nets:

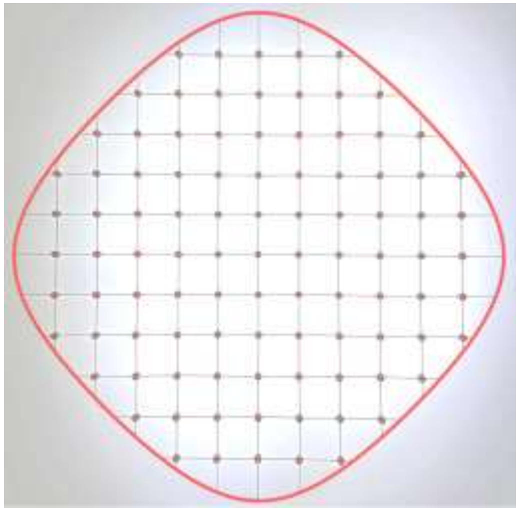
$$-K(u,v)\sin\omega(u,v) = \omega_{uv}(u,v)$$

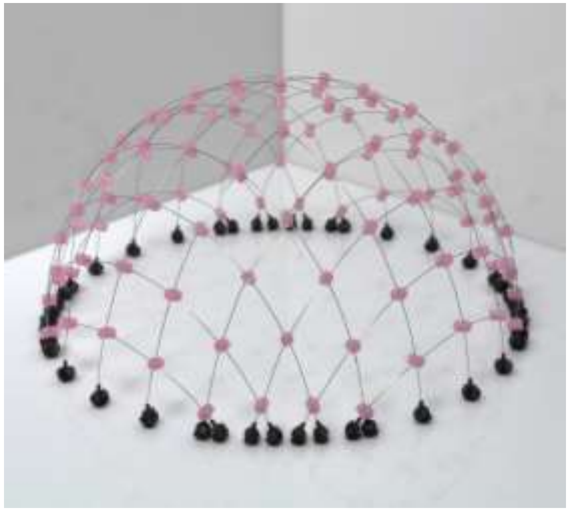
For $K = 1$ **[Chebyshev, 1878]**



Spherical Elastic Gridshells

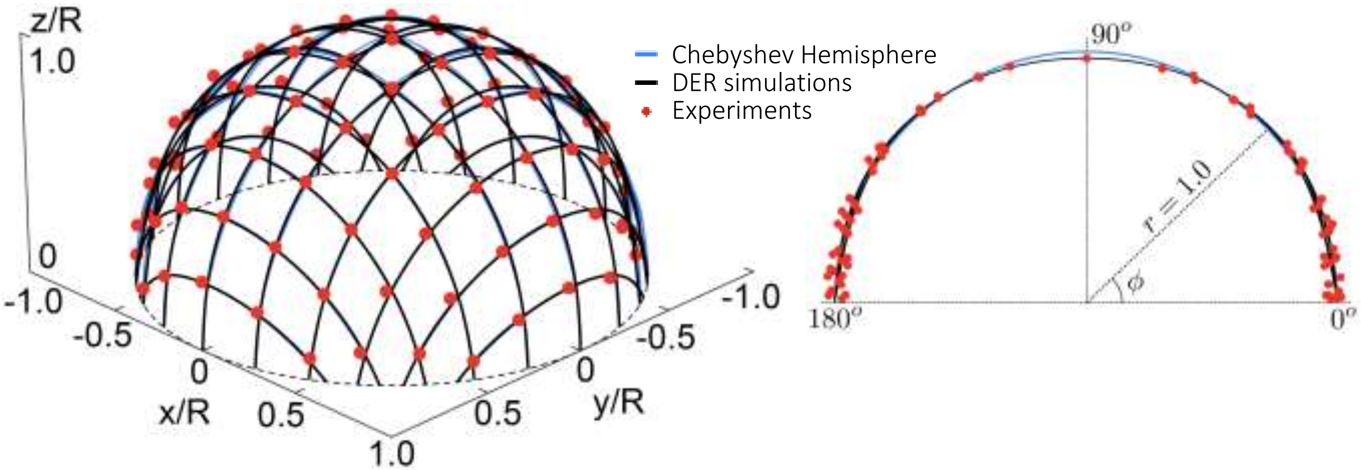
2D Footprint





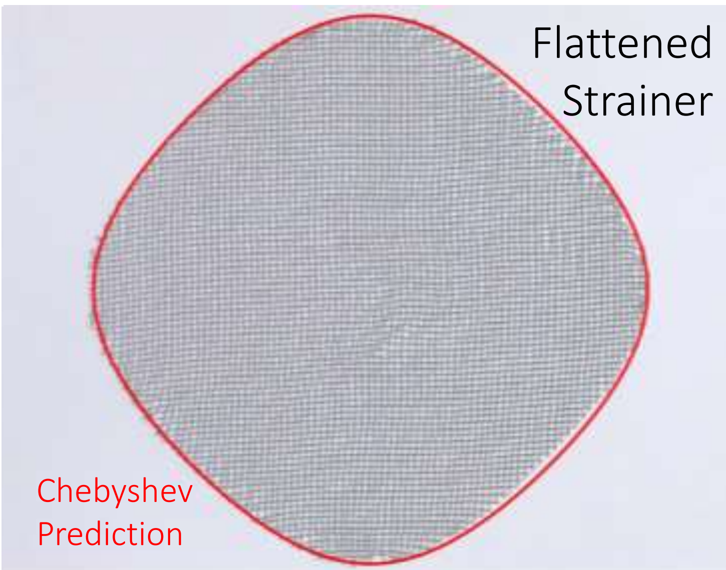
Actuated hemispherical gridshell

3D Digital Scanning



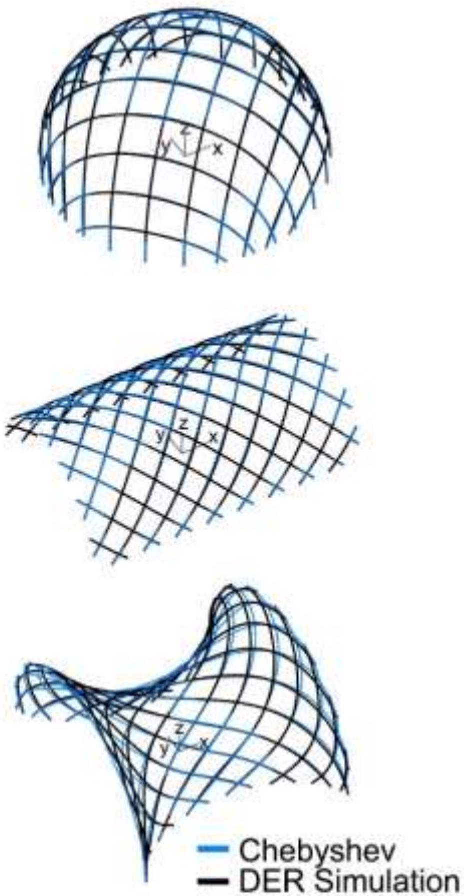
Excellent quantitative agreement!
Max. deviation ~2%

Let’s cut a pasta strainer

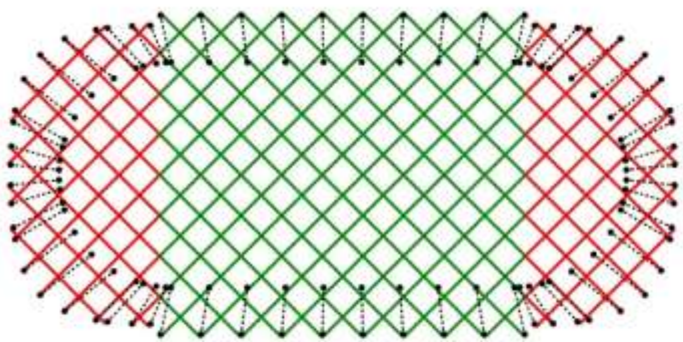


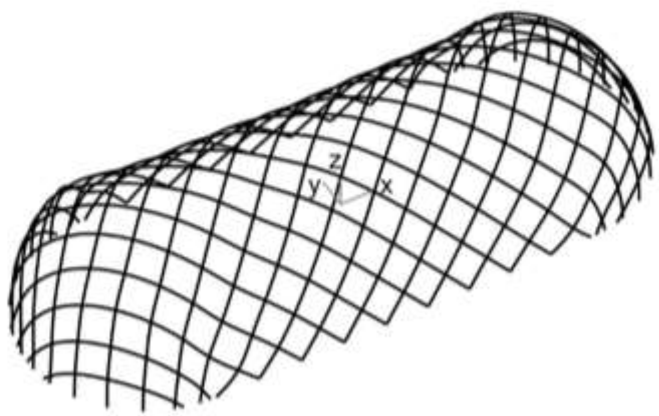
Cut

Building with blocks: more complex gridshells

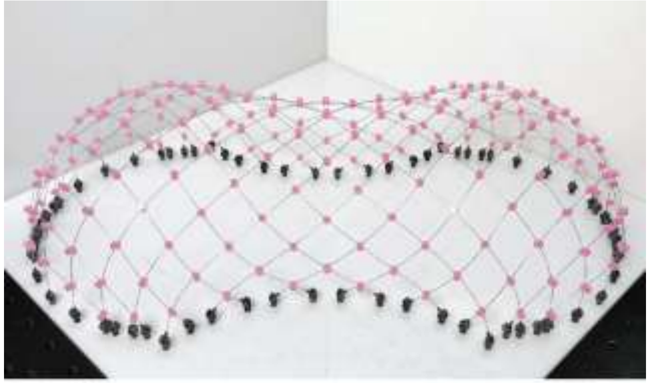


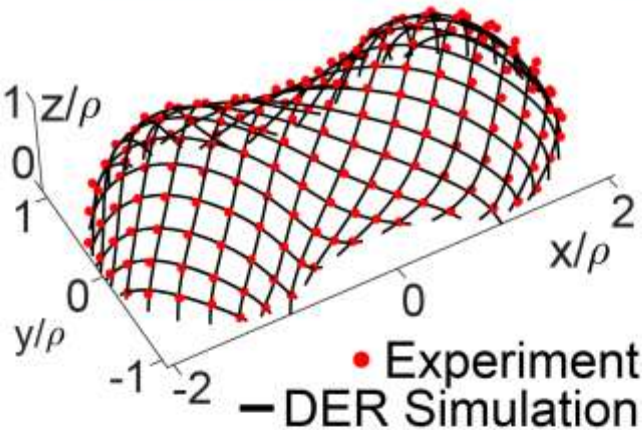
Quarter-sphere + Cylinder





Quarter-sphere + Saddle





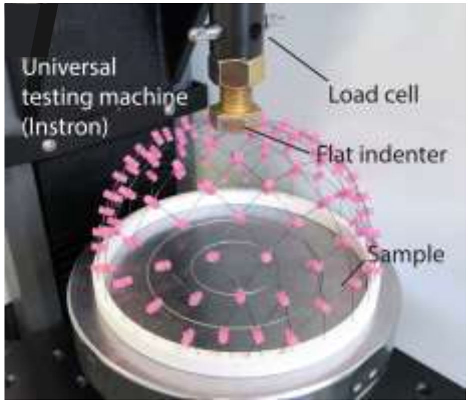
Rigidity for a gridshell under point indentation?

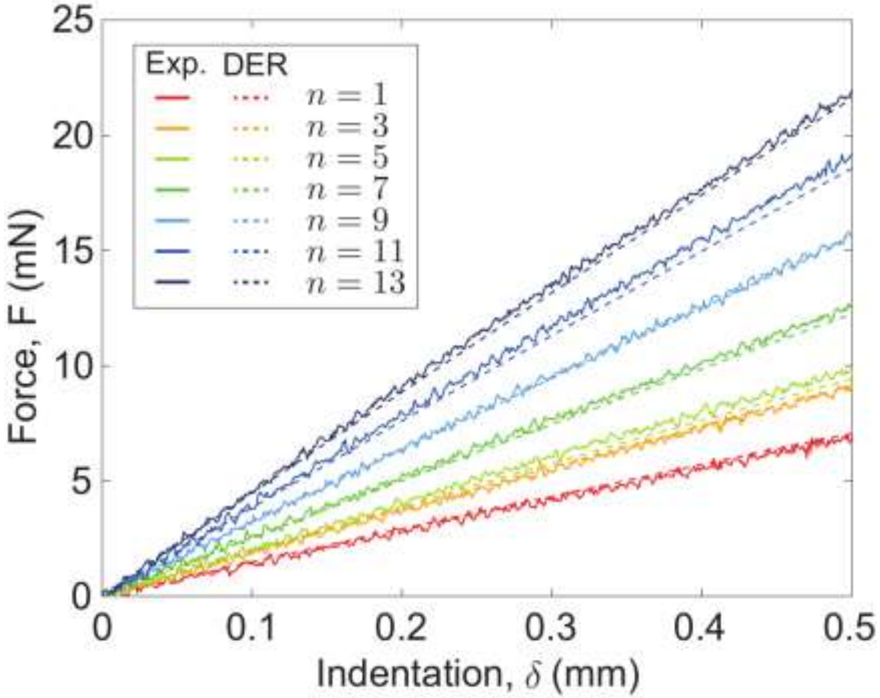
For Hemispherical shell: $$\frac{K^{o}}{Et} = \frac{4}{\sqrt{3(1-\nu^2)}}\left(\frac{t}{R}\right)$$

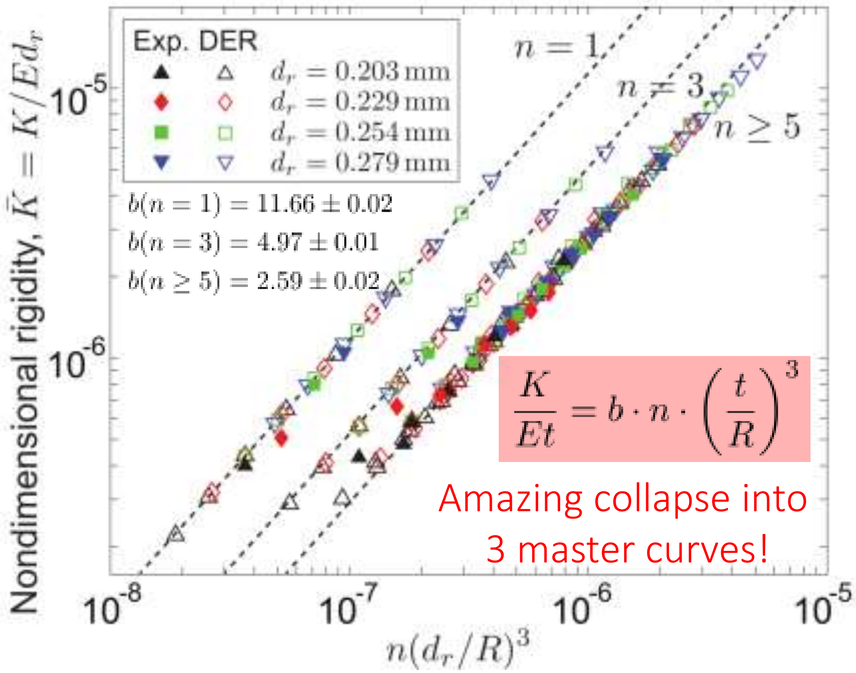
[Reissner, 1946]

For Euler-Bernoulli beam: $$\frac{K^{l}}{Et} \sim \left(\frac{t}{R}\right)^3$$

Q: What is K for hemispherical gridshell?







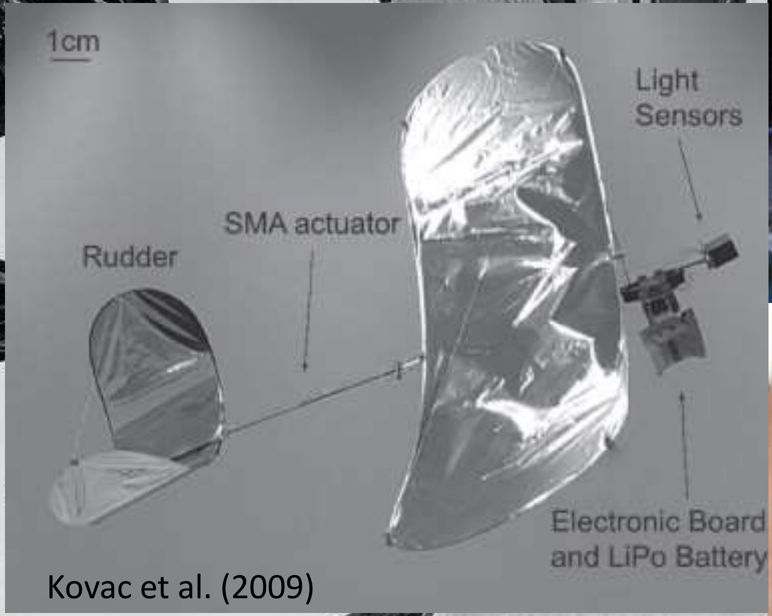
$$\frac{K}{Et} = b \cdot n \cdot \left(\frac{t}{R}\right)^3$$

Amazing collapse into 3 master curves!

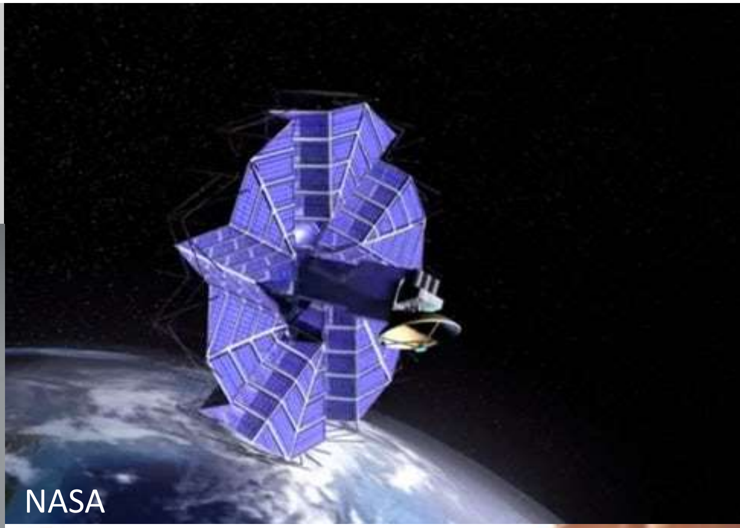
Example 4:

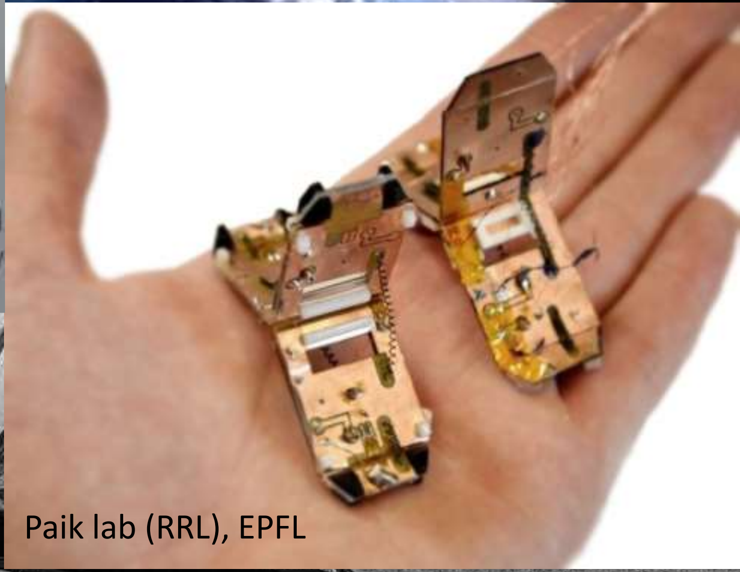
Self-deploying Surfaces

Self-Deploying Surfaces



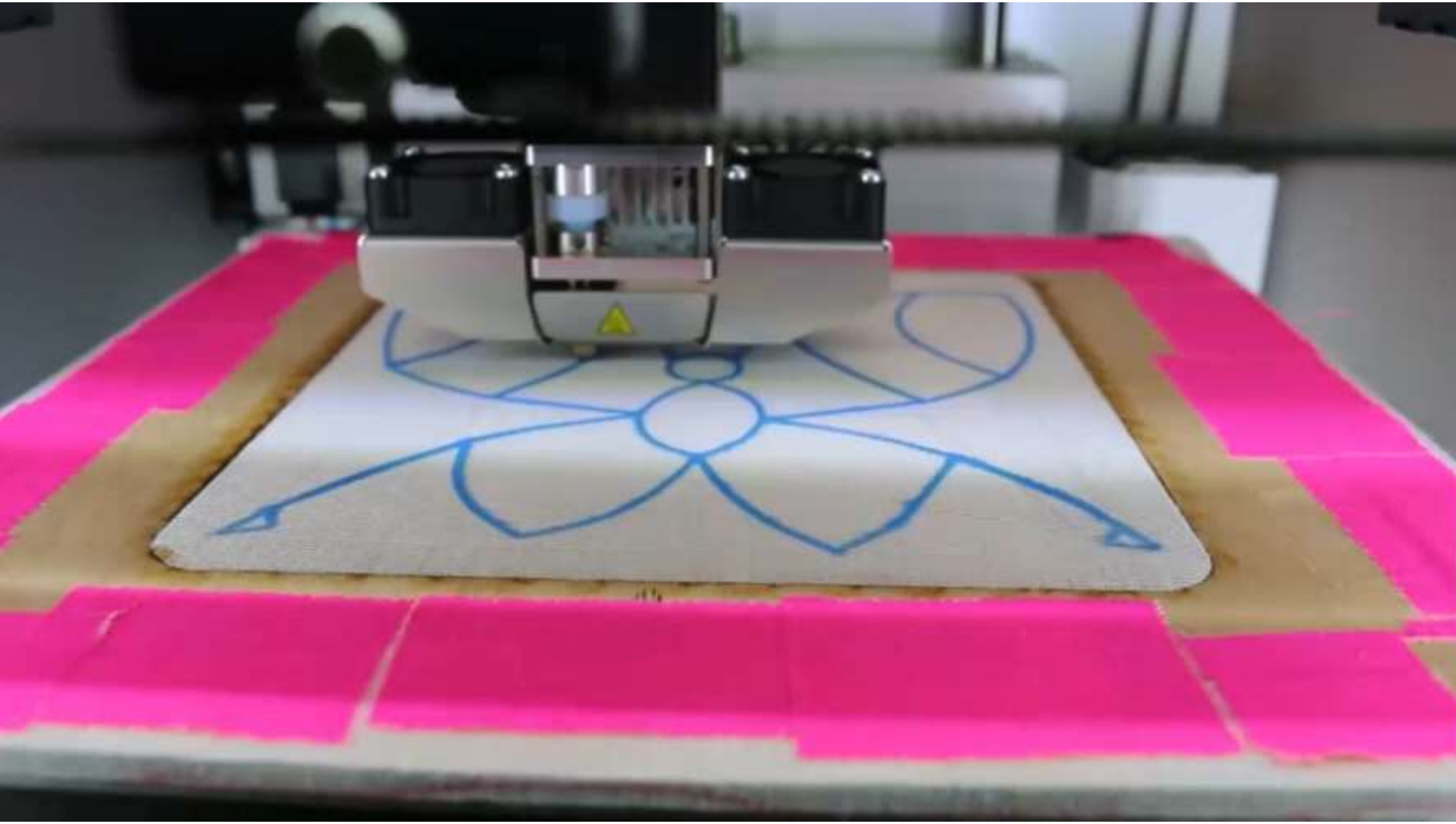
Kovac et al. (2009)


NASA

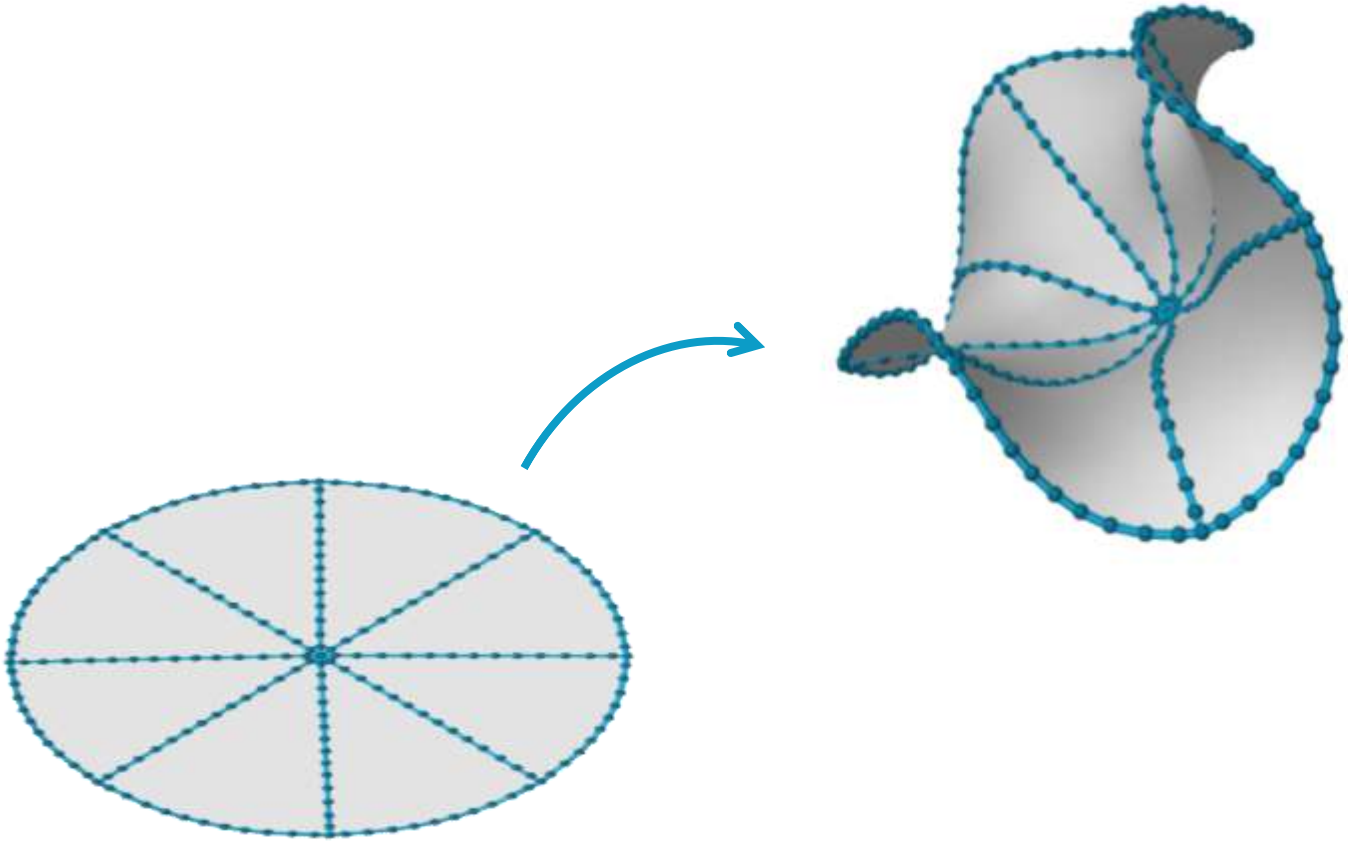

Paik lab (RRL), EPFL

Self-Deploying Surfaces

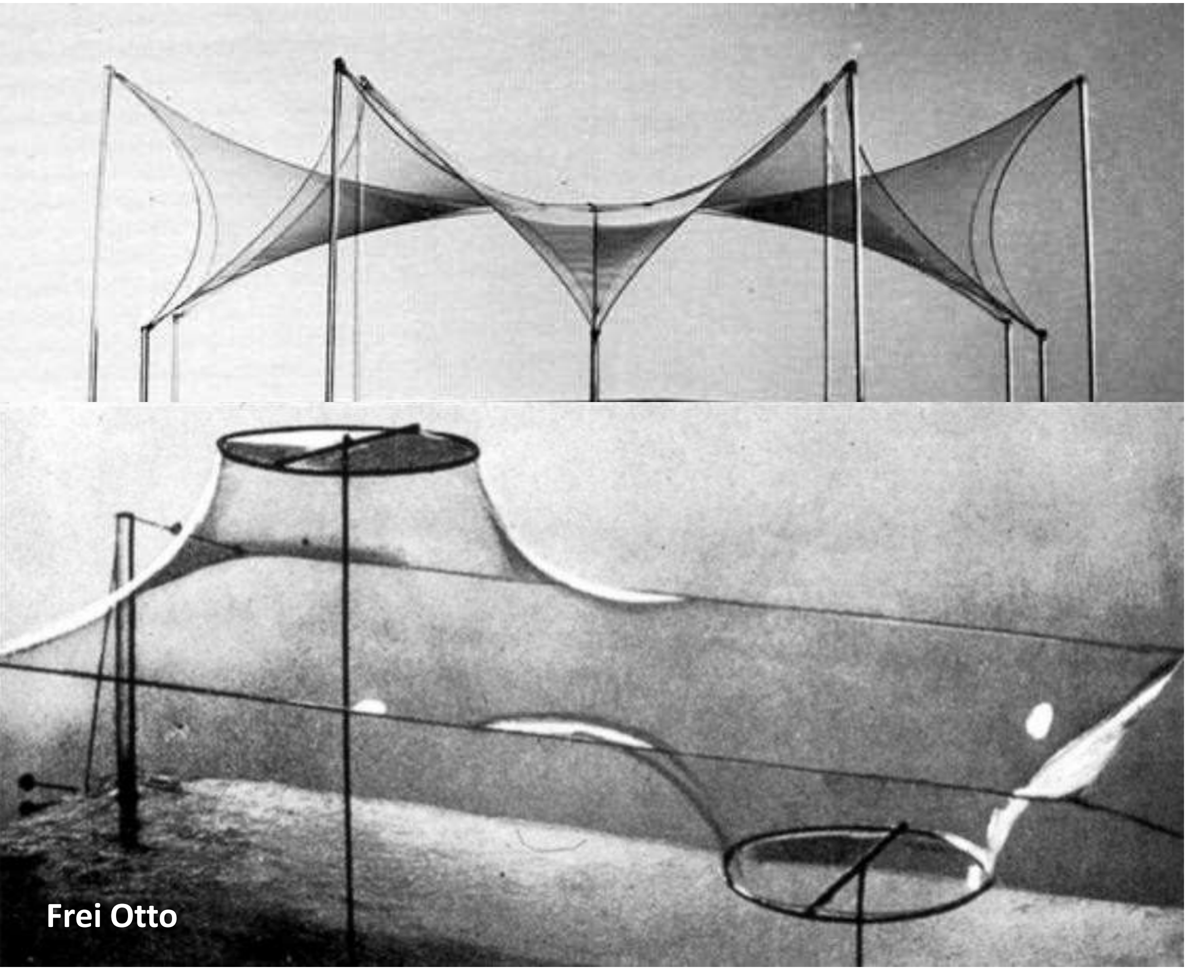
Perez, Otaduy, Thomaszewski. *Kirchhoff Plateau Surfaces.* SIGGRAPH '17.



Nonlinear Mechanics



Constrained Design Space



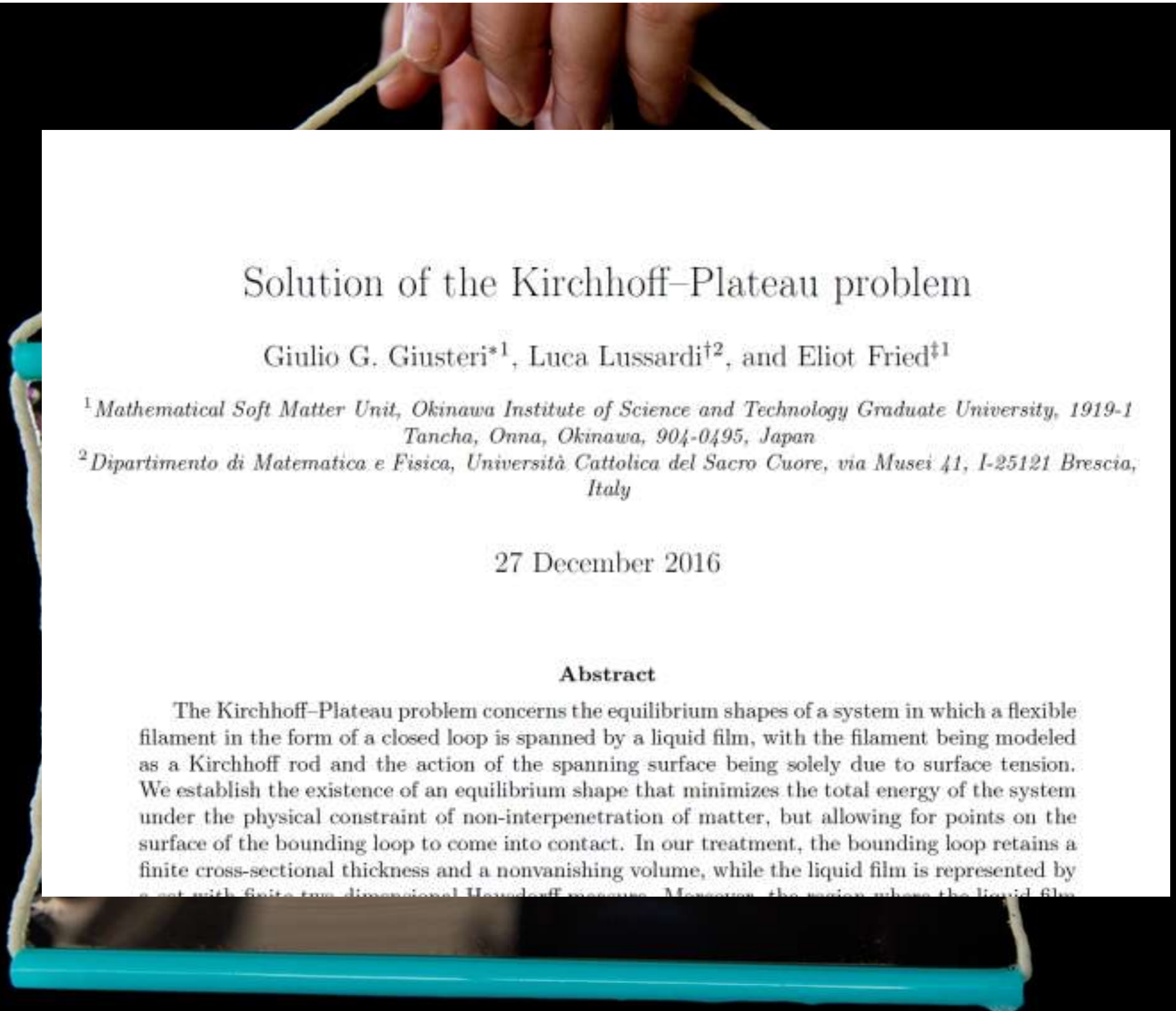
Minimal surfaces have zero mean curvature throughout,

$$\frac{\kappa_1 + \kappa_2}{2} = 0$$

Minimal surfaces are either

- locally flat
  $$\kappa_1 = \kappa_2 = 0$$
- or saddle-shaped
  $$\kappa_1 = -\kappa_2 \neq 0$$

Kirchhoff Plateau Problem



Solution of the Kirchhoff–Plateau problem

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27 December 2016

Abstract

The Kirchhoff–Plateau problem concerns the equilibrium shapes of a system in which a flexible filament in the form of a closed loop is spanned by a liquid film, with the filament being modeled as a Kirchhoff rod and the action of the spanning surface being solely due to surface tension. We establish the existence of an equilibrium shape that minimizes the total energy of the system under the physical constraint of non-interpenetration of matter, but allowing for points on the surface of the bounding loop to come into contact. In our treatment, the bounding loop retains a finite cross-sectional thickness and a nonvanishing volume, while the liquid film is represented by

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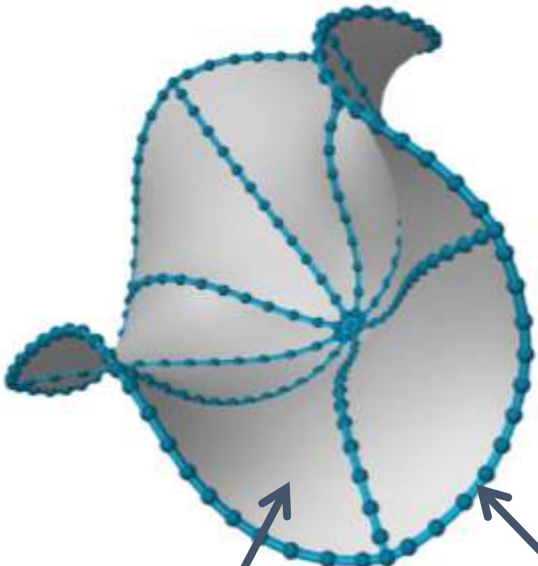
Computational Model

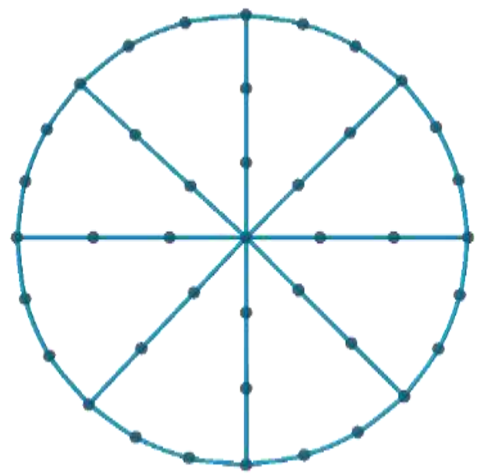
Nonlinear Mechanics



CST finite elements with orthotropic material

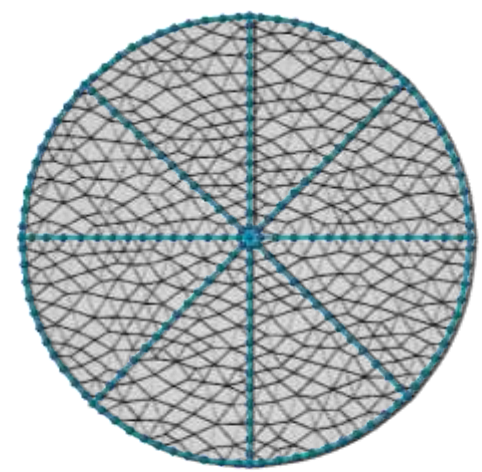
Discrete Elastic Rods

Parameters



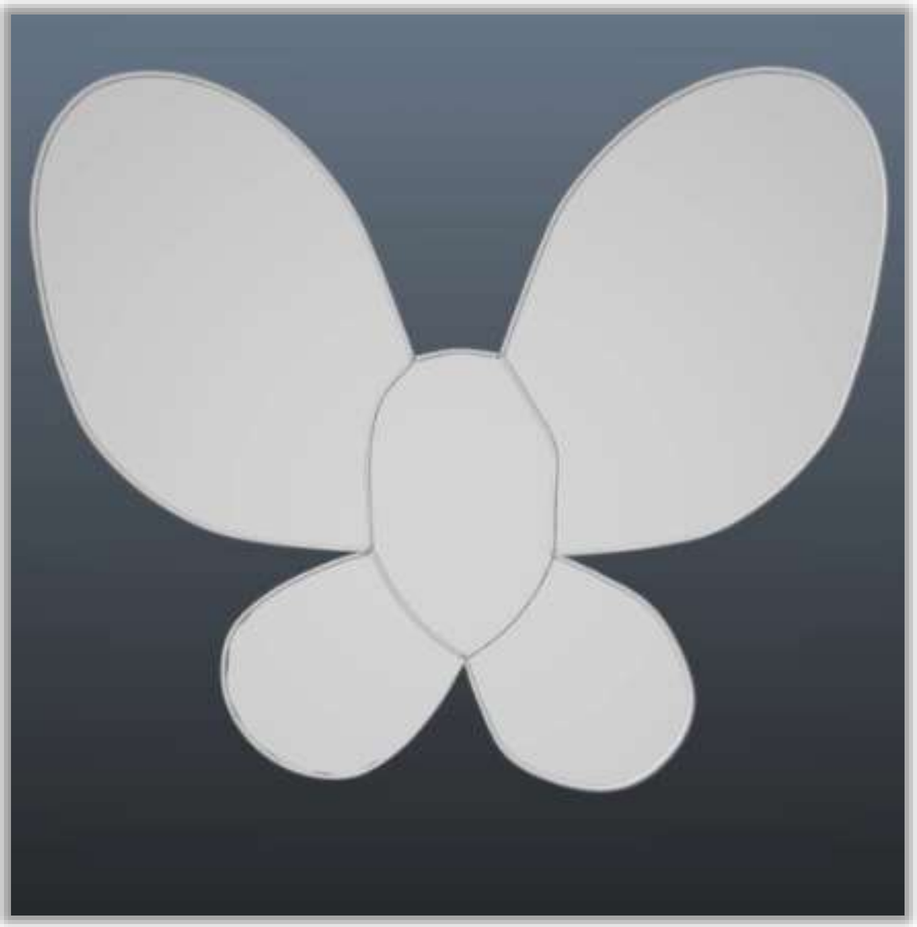
Rod control points and cross sections

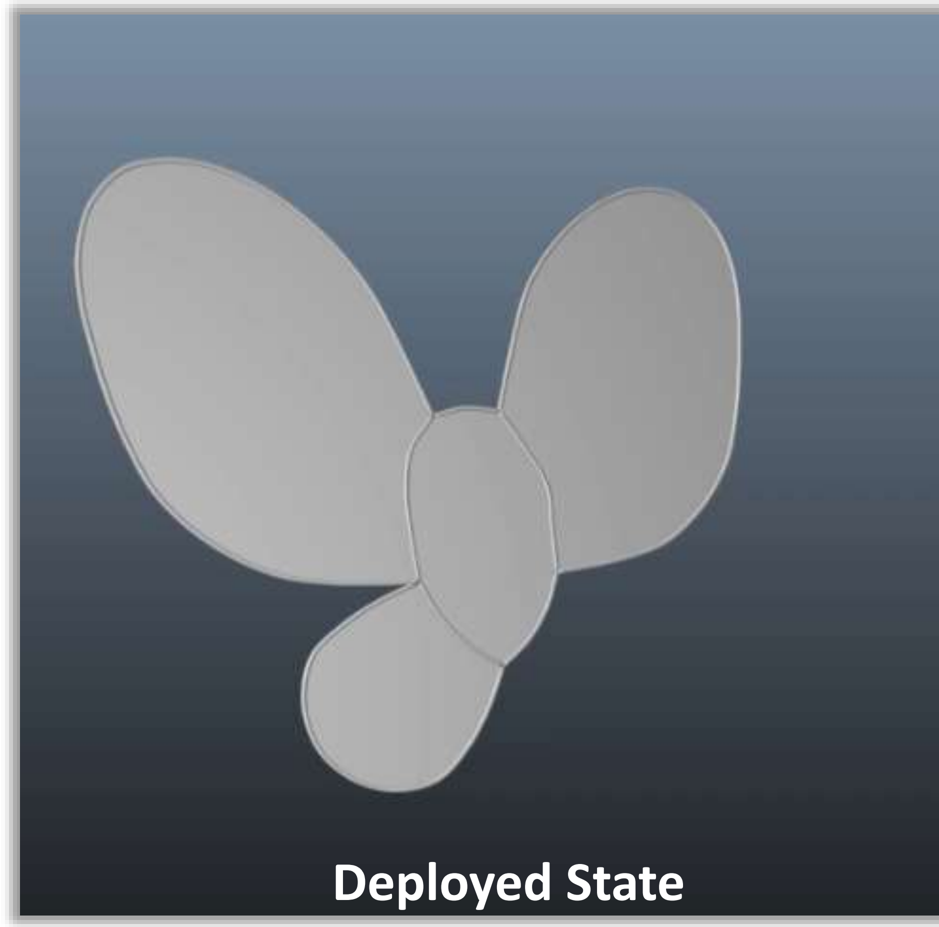
Simulation Mesh



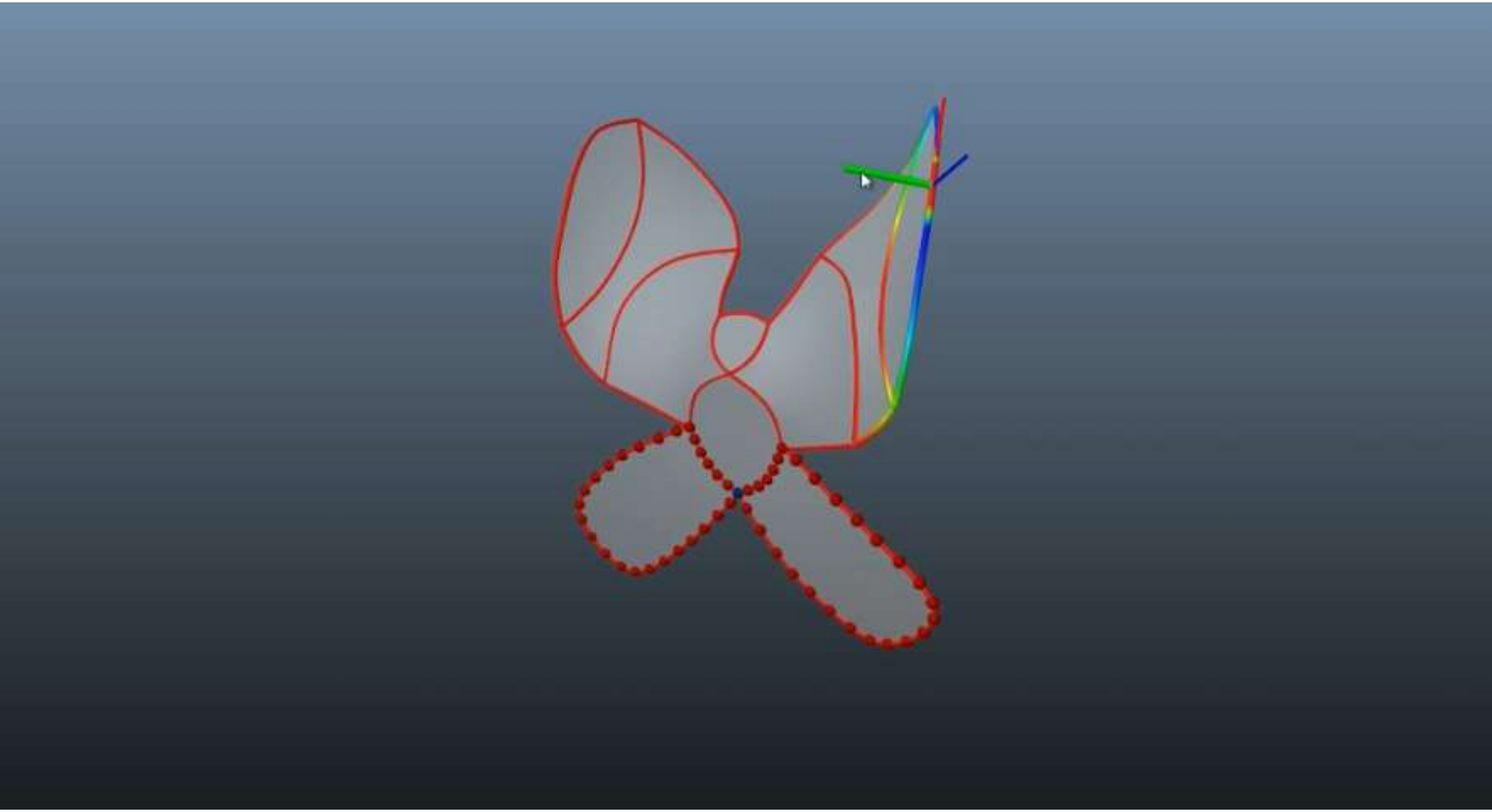
Coupled membrane and rod elements (collocation)

Forward Design

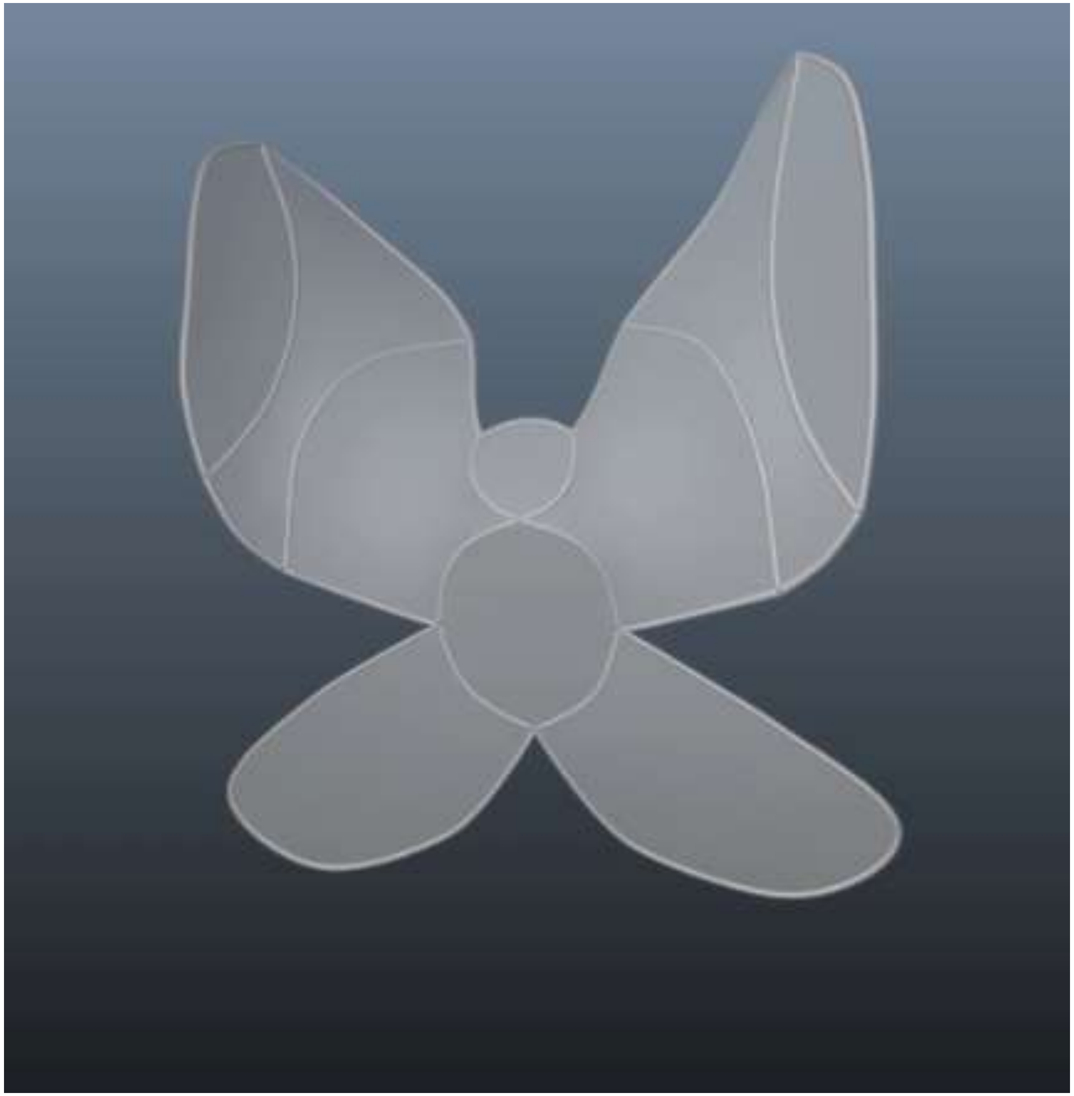


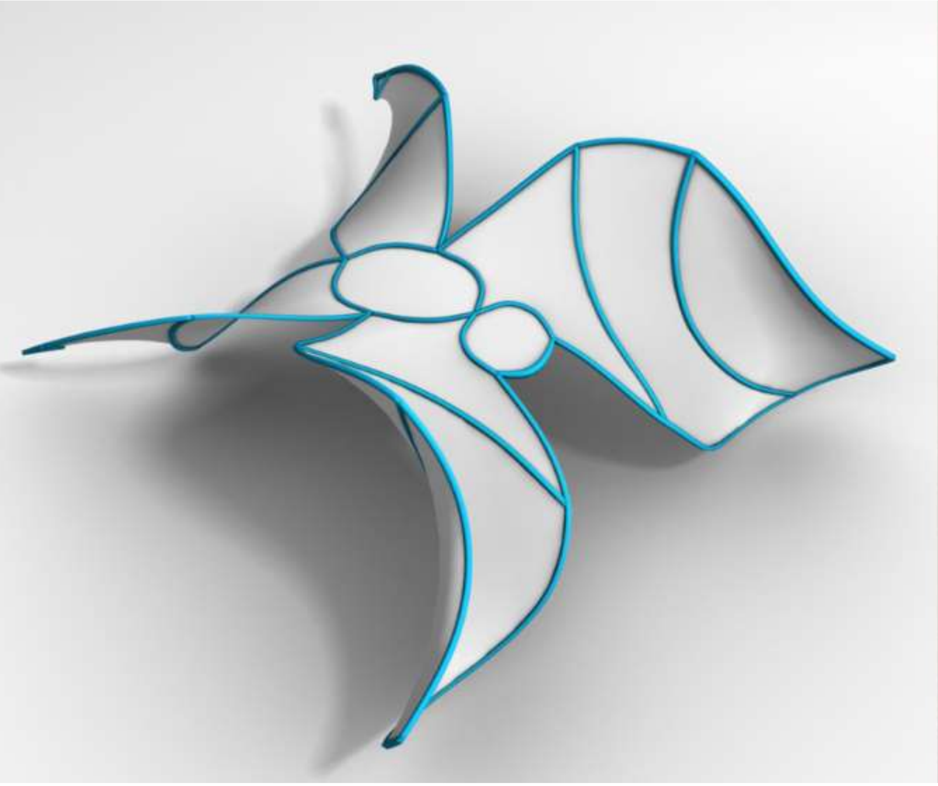
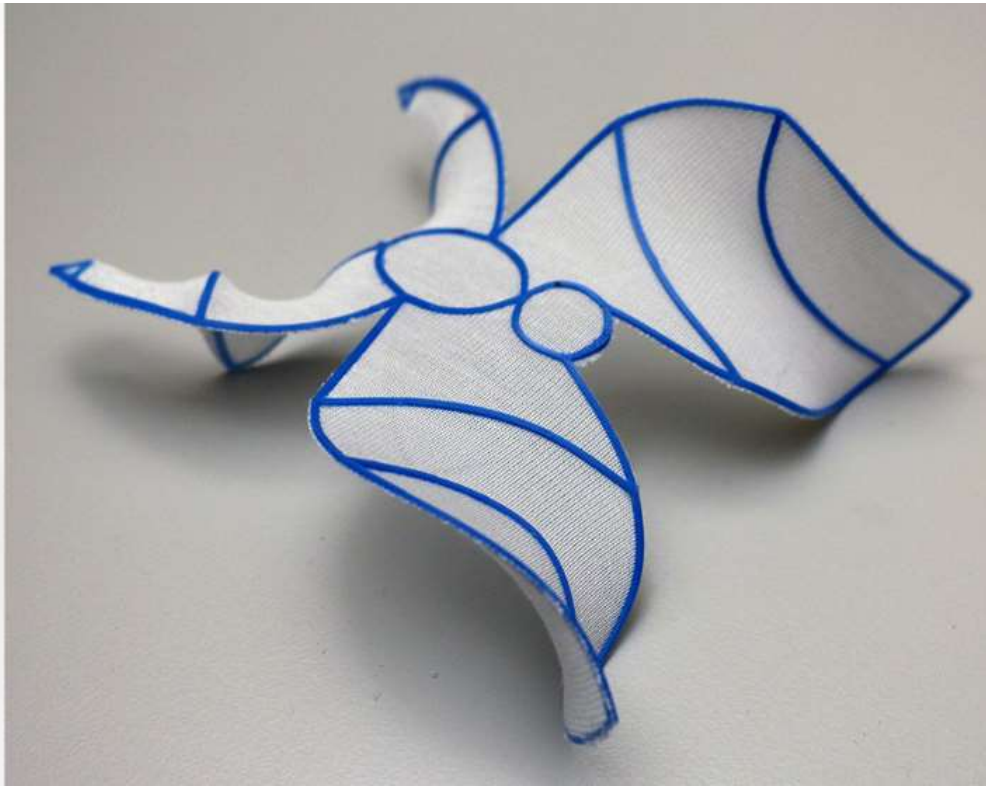


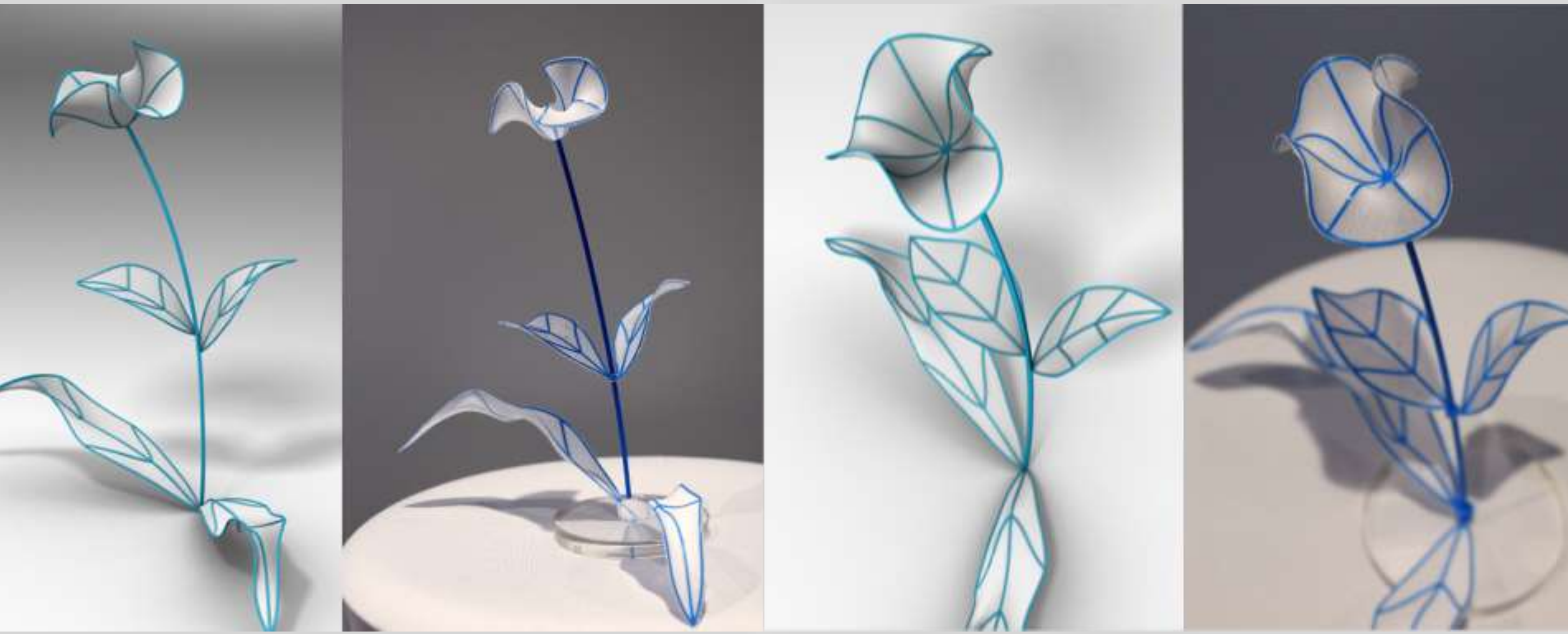
Inverse Design

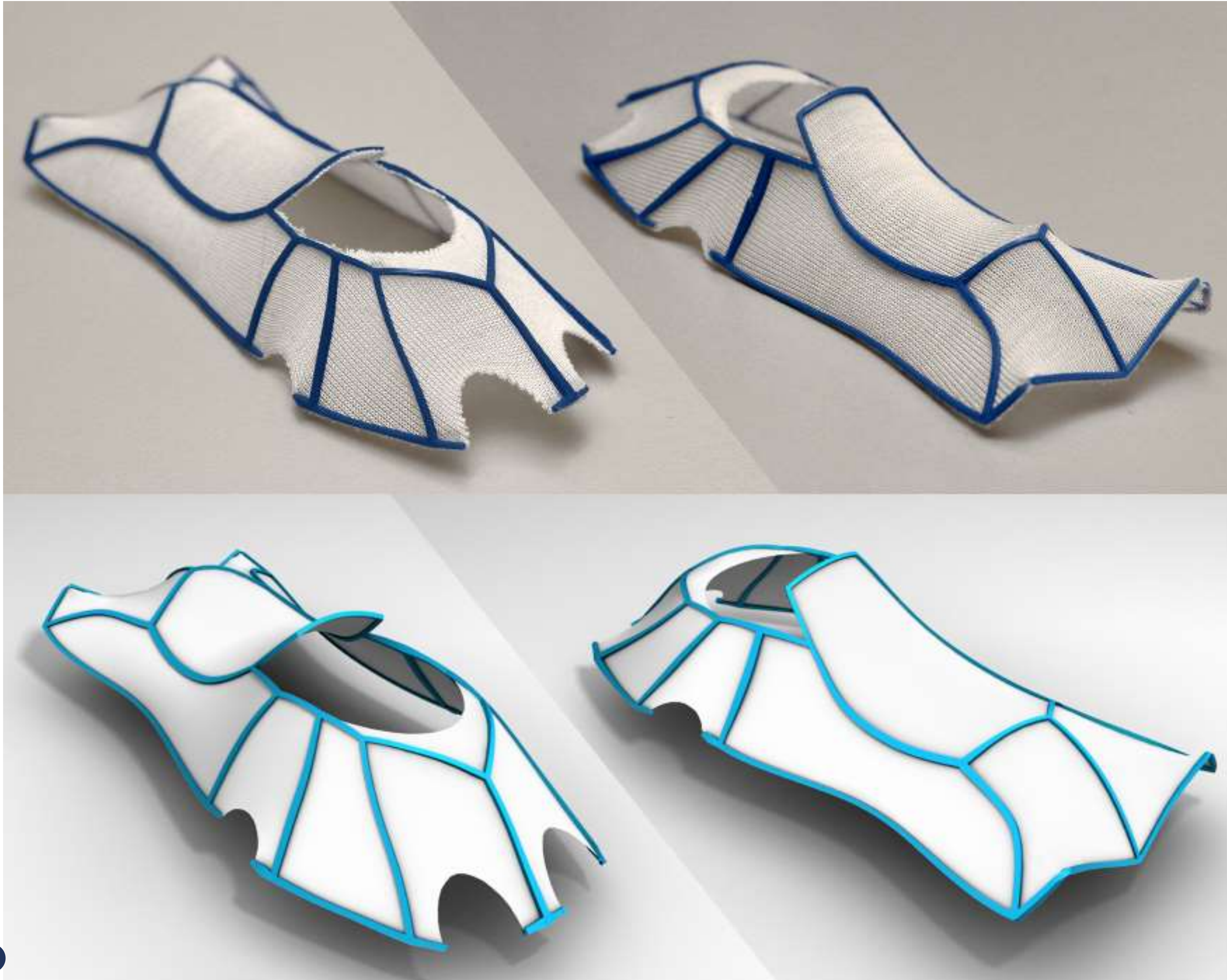


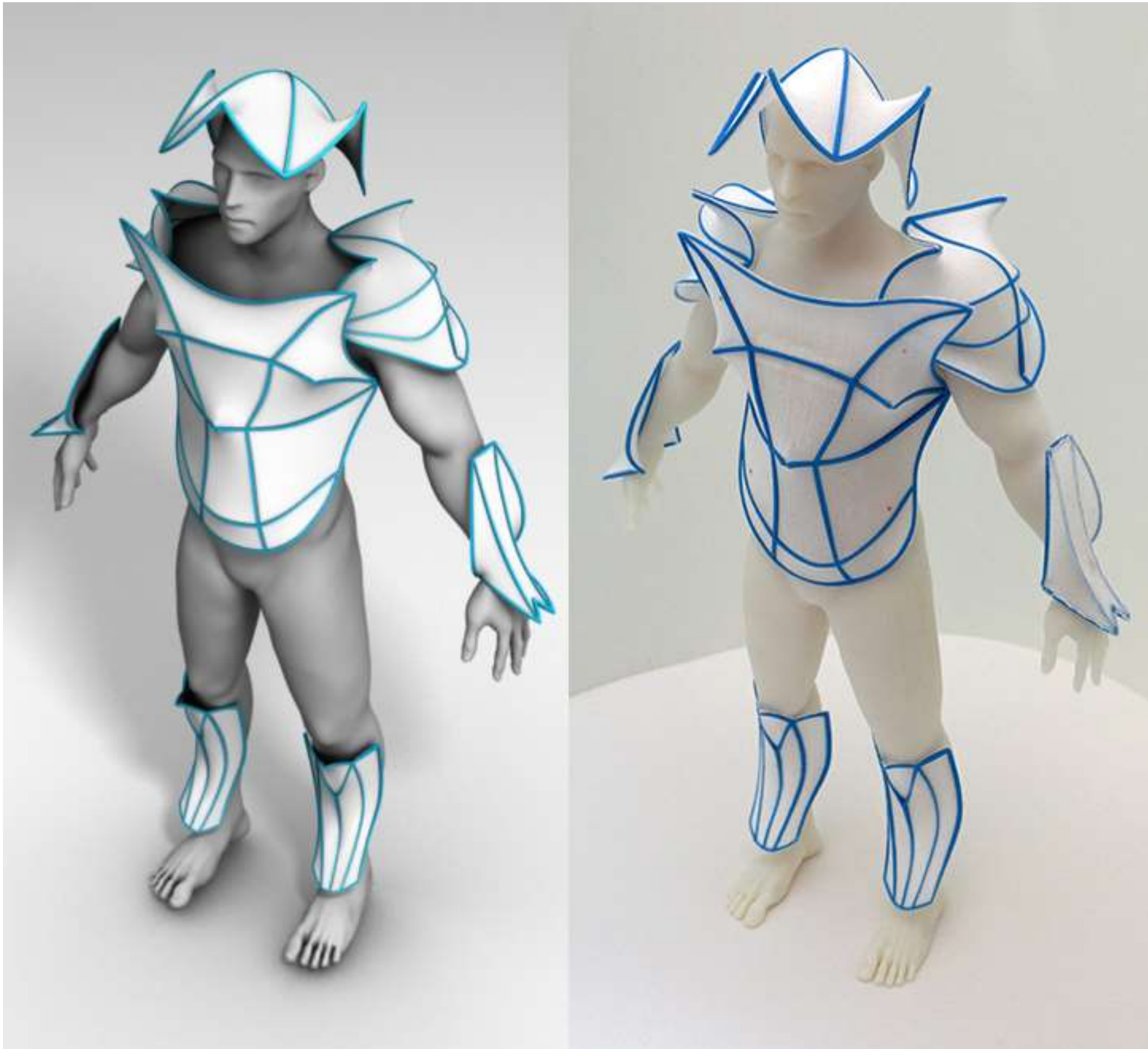
Exploring Design Variations









Concluding thoughts...

- **Computer graphics** tools and **physically-based simulation** offer unprecedented opportunities as quantitatively predictive tools in **experimental mechanics**.
- Timely **engineering applications & physical scenarios** and novel **experimental mechanics** tools can challenge and push **physically-based simulation** into new grounds.

Let's
Keep
Mixing

Bernhard Thomaszewski
Pedro Reis
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