The Gudhi library: Simplification of Simplicial Complexes
Gudhi workshop

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Introduction

- **Goal**: represent and simplify huge simplicial complexes
- **Problem**: in some cases, the number of simplices may be too large for full representation
Introduction

- Goal: represent and simplify huge simplicial complexes
- Problem: in some cases, the number of simplices may be too large for full representation
- *Implicit* representation of simplices
Introduction

Plan

- Skeleton-blocker data-structure
  [SoCG11, IJCGTA12 Attali Lieutier Salinas]
- Experiments
  - Memory size
  - Simplification time
Introduction
Storing implicitly the set of simplices

- Compact representation for Flag complexes (can be represented only with a graph)
- What about other complexes?

Flag-complex = every clique is a simplex
Introduction
Storing implicitly the set of simplices

- Compact representation for Flag complexes (can be represented only with a graph)
- What about other complexes?

Flag-complex nearly everywhere
Introduction
Storing implicitly the set of simplices

- Compact representation for Flag complexes (can be represented only with a graph)
- What about other complexes?
Skeleton-blockers data-structure

Definition of the data-structure

Simplicial complex

Encoding

Graph + Blockers

\( Sk(1)(K) = \{ab, ac, ad, bc, bd, dc, df, ec, fc, fe\} \)

\( \text{Blockers}(K) = \{bcd, cdf\} \)

The pair \([Sk(1)(K), \text{Blockers}(K)]\) is sufficient to encode entirely \(K_5/1\)
Skeleton-blockers data-structure

Definition of the data-structure

- $Sk^{(1)}(K) = \{ab, ac, ad, bc, bd, dc, df, ec, fc, fe\}$
- $Blockers(K) = \{bcd, cdf\}$

Skeleton-blockers data-structure [ALS 11]

- $Blockers(K) = \{\sigma \subseteq P \mid \sigma \not\in K \text{ and } \forall \tau \subsetneq \sigma, \tau \in K\}$
- The pair $[Sk^{(1)}(K), Blockers(K)]$ is sufficient to encode entirely $K$
Data structure size

- How many blockers in a simplicial complex?
- None for flag complexes such as the Rips complex
- How many in Delaunay, Tangential Delaunay, Cech complexes?
- How many in random 3-spheres?
  (3-dimensional manifold embedded in $\mathbb{R}^4$)
Data structure size

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  (3-dimensional manifold embedded in $\mathbb{R}^4$)

Random 2-sphere with 200 points
Data structure size

- How many blockers in a simplicial complex?
- None for flag complexes such as the Rips complex
- How many in Delaunay, Tangential Delaunay, Čech complexes?
- How many in random 3-spheres? (3-dimensional manifold embedded in $\mathbb{R}^4$)
Data structure size

Popable blockers

- Removing a blocker in the data-structure may change the homotopy type

Removing blocker $cdf$ ... Removing blocker $bcd$ ...

changes homotopy type preserves homotopy type

- Removing a popable blocker does not change the homotopy type
Data structure size

- How many blockers in a simplicial complex?
- How many non-popable blockers in random 3-spheres? (3-dimensional manifold embedded in $\mathbb{R}^4$)
Data structure size

- How many blockers in a simplicial complex?
- How many non-popable blockers in random 3-spheres? (3-dimensional manifold embedded in $\mathbb{R}^4$)
Edge contractions

Definition

To simplify a complex, perform iterative topology-preserving edge contraction:

- Contracting an edge = identify two vertices in the complex
- May change the homotopy type if link condition does not hold

Rips complex
6000 contractions
6700 contractions
6787 contractions

(≈ 70 \cdot 10^6 simplices) (≈ 100 simplices)

- \( ab \mapsto c \) preserves the homotopy type
- \( ab \mapsto c \) changes the homotopy type
**Edge contractions**

**Definition**

To simplify a complex, perform iterative topology-preserving edge contraction:

- Contracting an edge = identify two vertices in the complex
- May change the homotopy type if link condition does not hold
- Link condition iff no blocker through the edge

![Rips complex](image1.png) 

6000 contractions

6700 contractions

6787 contractions

\( \approx 70 \cdot 10^6 \) simplices

\( \approx 100 \) simplices
Edge contraction implementation efficiency
How much time to reduce a sphere?

- Draw a random sphere in $\mathbb{R}^3$ with $n$ points
- How much time to simplify to a tetrahedron with edge contraction?

How many time to simplify?
Edge contraction implementation efficiency
How much time to reduce a sphere?

- **CGAL**: Surface mesh simplification package (Polyhedron_3)
- ≈ 65% times faster
Skeleton-blockers data-structure
Implementation

- Template based, user can provide its vertices / edges classes
- Constructor from graph/blockers / list simplices / top faces
- STL iterators for vertices / edges / blockers / simplices
- Edge contraction, collapse, ...

Simplex iteration

typedef ... Complex;
Complex complex;

// build complex

for (auto s : complex.simplex_range())
    cout << s << endl;
Edge contraction implementation

Genericity

- Policy-based design (adapted from CGAL Surface Mesh Simplification package)
- The user can provide its own cost, placement, validity, and visitor policies
- Genericity: simplification of urban mesh (submitted to CGF)

- Homology computation

Input mesh

Mesh simplified with SB package and specific policies
Time for a demo
Conclusion

On a practical point of view:

- Very sparse representation
- Generic and efficient simplification

On a theoretical point of view:

- How many blockers and non popable blockers in a manifold?
- Persistence?