Geometric Understanding in Higher Dimensions

What ~~is~~ will be new in GUDHI library version 1.4.0

Jean-Daniel Boissonnat, Paweł Dłotko, Marc Glisse, François Godi, Clément Jamin, Siargey Kachanovich, Clément Maria, Vincent Rouvreau and David Salinas

DataShape, Inria Saclay and Sophia-Antipolis

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Geometric Understanding in Higher Dimensions

GUDHI is a five years project supported by a Grant of the European Research Council and hosted by INRIA

- develop and understand geometrical data structures
- develop associated statistical, geometric and topological functions

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Geometric Understanding in Higher Dimensions

Topological data an...

https://en.wikipedia.org/wiki/Topological_data_analysis

Search

Workflow [edit]

The basic workflow in TDA is:[14]

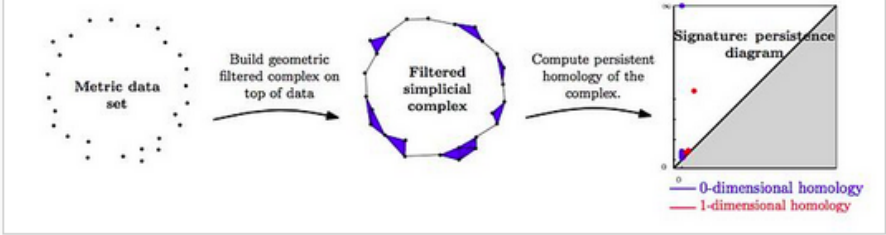
point cloud	→	nested complexes	→	persistence module	→	barcode or diagram

1. If X is a point cloud, replace X with a nested family of simplicial complexes X_r (such as the Čech or Vietoris-Rips complex). This process converts the point cloud into a filtration of simplicial complexes. Taking the homology of each complex in this filtration gives a persistence module

$$H_i(X_{r_0}) \to H_i(X_{r_1}) \to H_i(X_{r_2}) \to \cdots$$

2. Apply the structure theorem to provide a parameterized version of Betti number, **persistence diagram,** or equivalently, **barcode.**

Graphically speaking,



A usual use of persistence in TDA [15]

Wikipedia - TDA workflow

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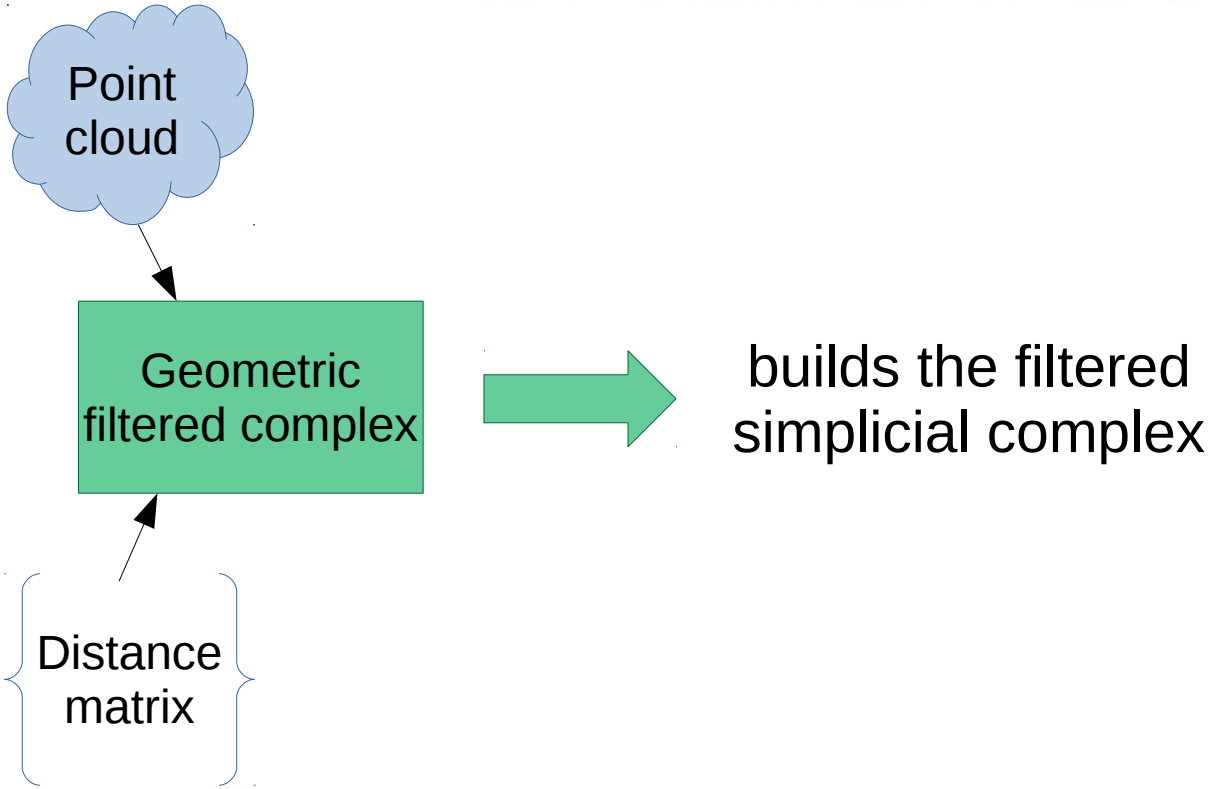

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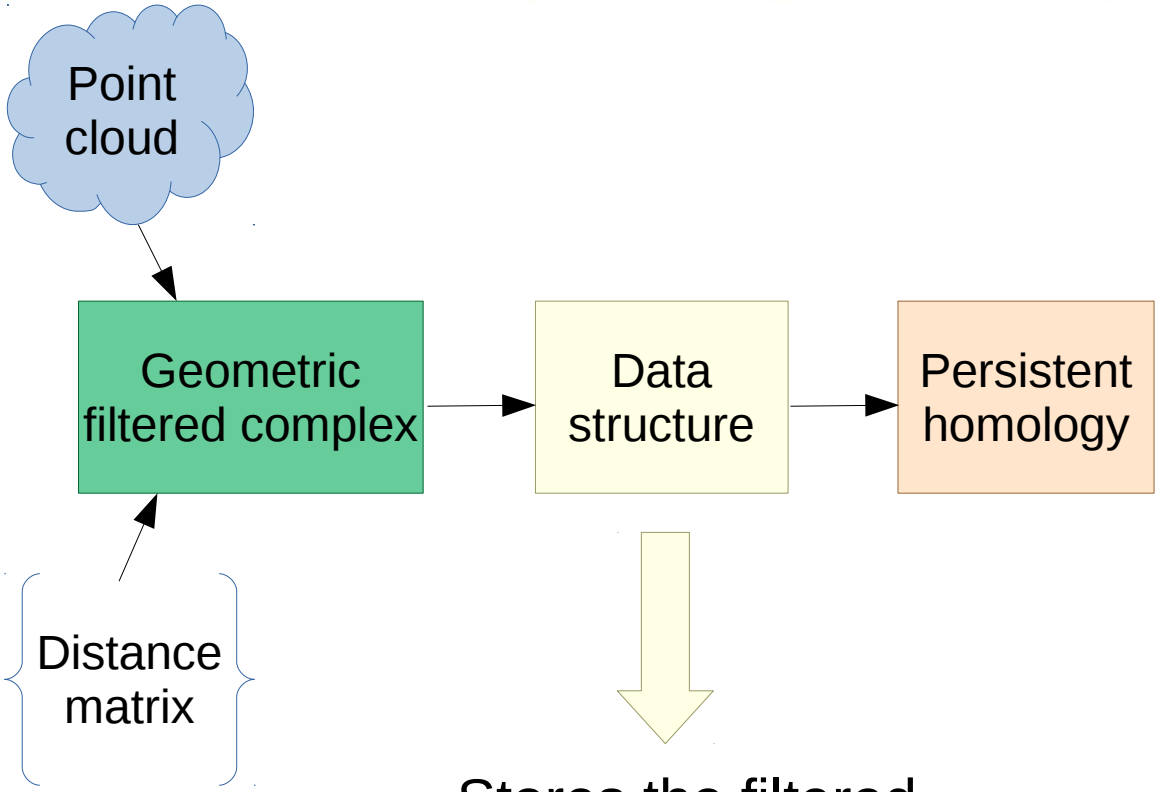

- Stores the filtered simplicial complex
- Interfaces with persistence modules

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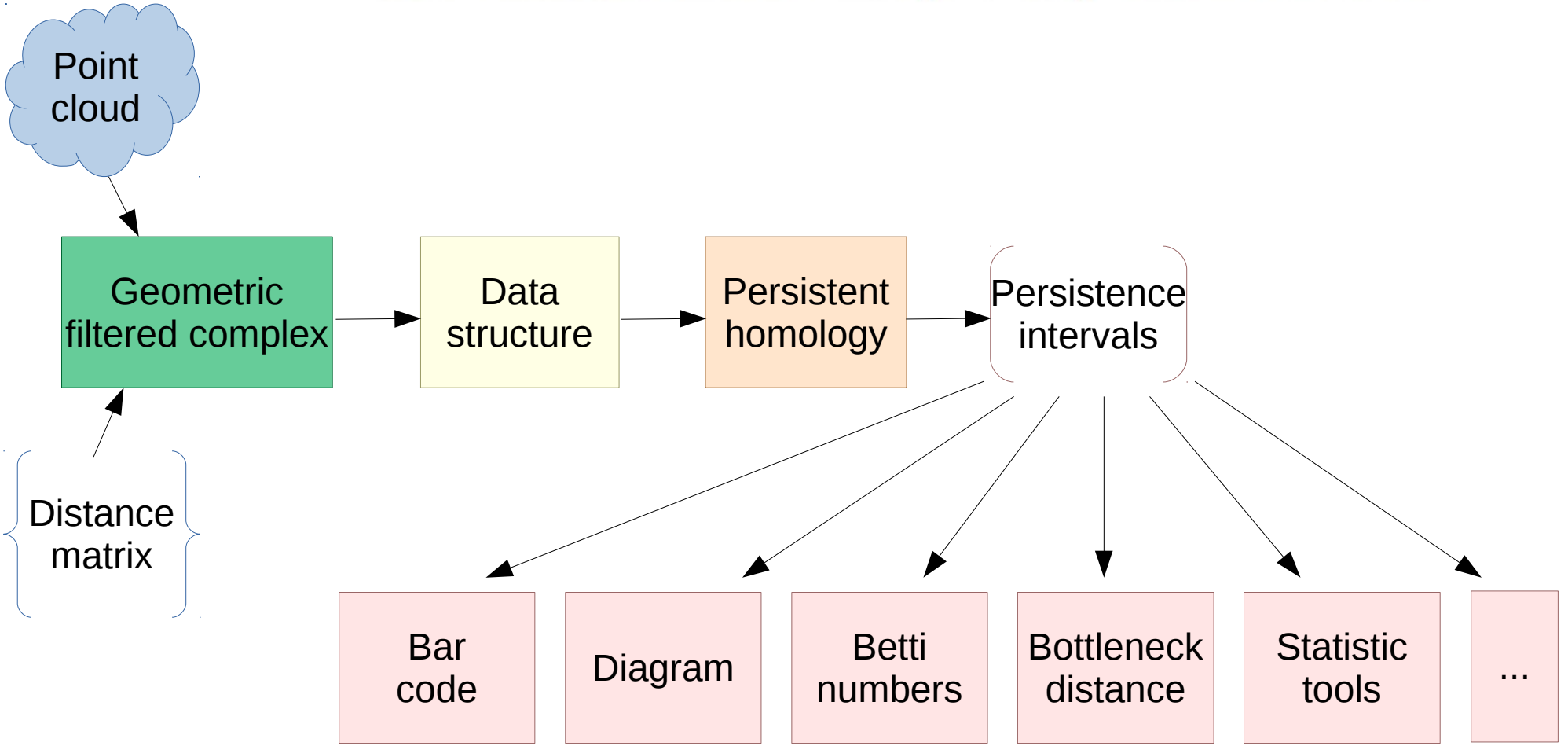

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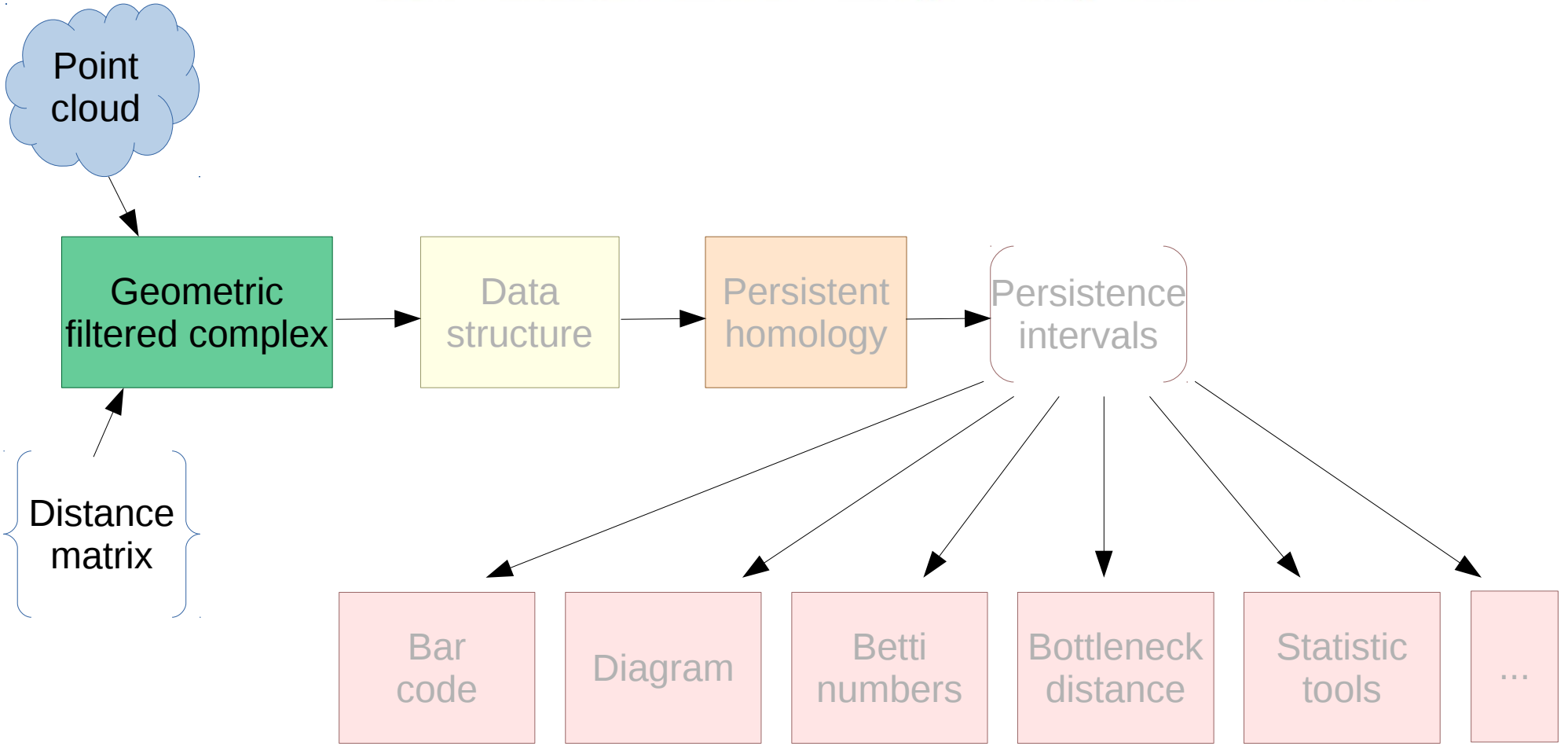

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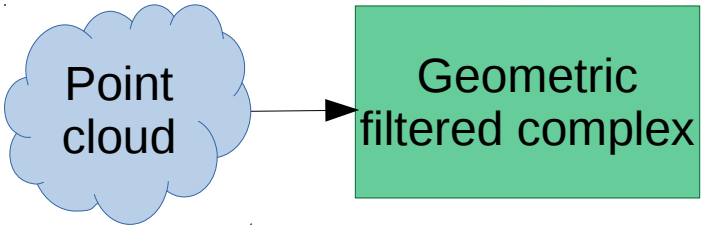
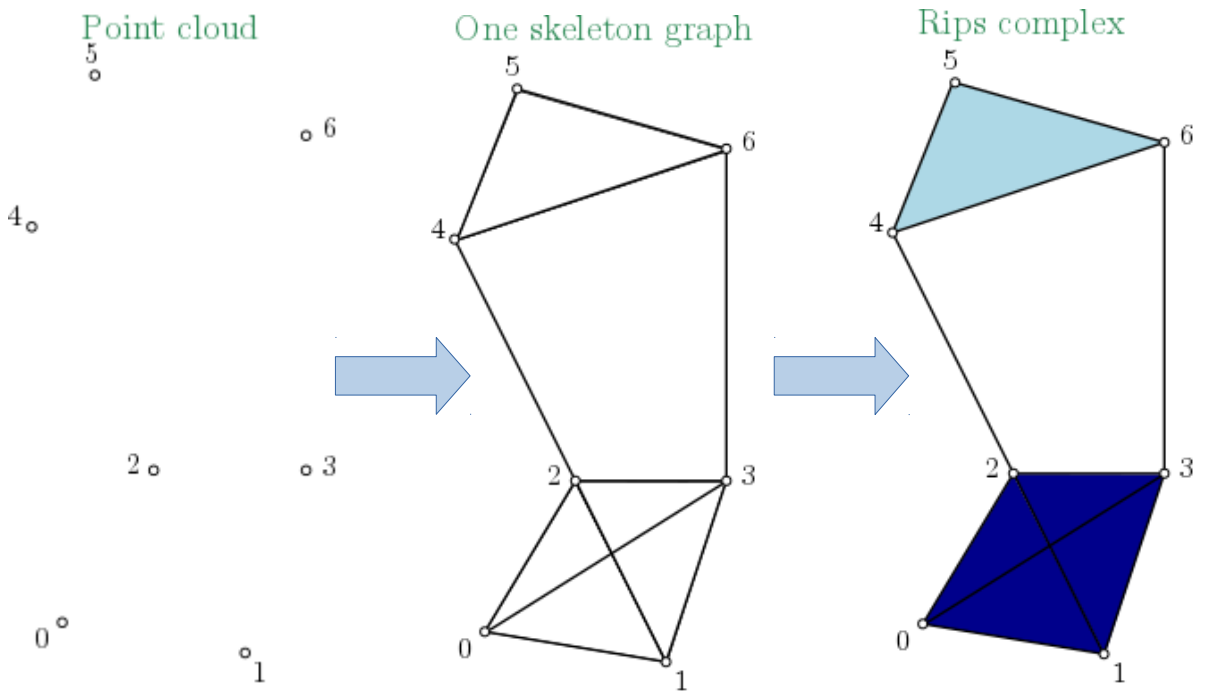


Geometric filtered complex – Rips from a point cloud



by Clément Maria

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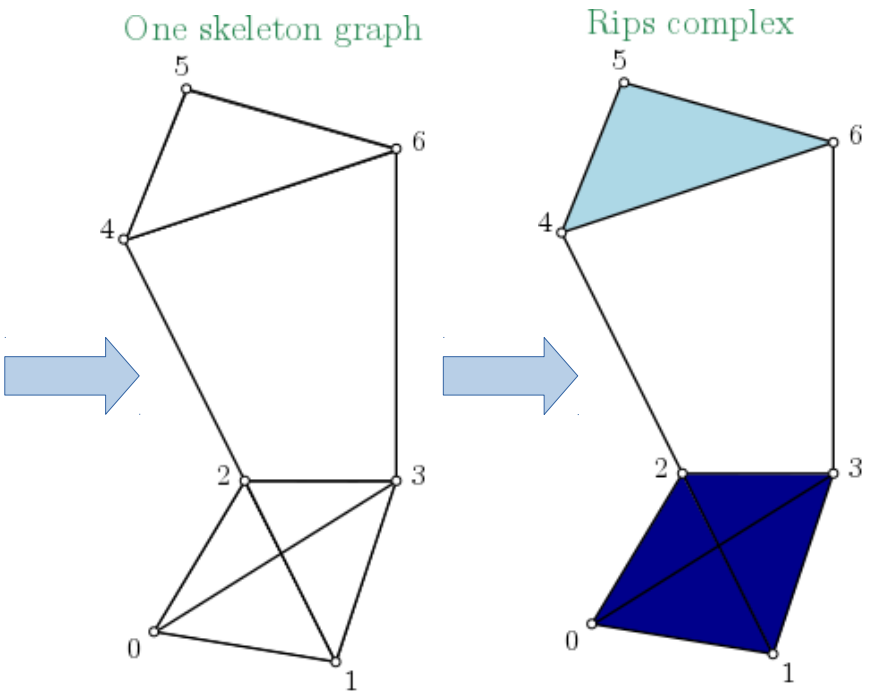
Geometric Understanding in Higher Dimensions

Distance matrix → Geometric filtered complex

Geometric filtered complex – Rips from a distance matrix

0	6.08	5.83	9.43	13.04	18.03	17.89
6.08	**0**	6.71	6.32	15.65	19.65	17.12
5.83	6.71	**0**	5	8.94	13.15	12.08
9.43	6.32	5	**0**	12.04	14.76	11
13.04	15.65	8.94	12.04	**0**	14.76	9.49
18.03	19.65	13.15	14.76	5.38	**0**	7.28
17.89	17.12	12.08	11	9.49	7.28	**0**



by Paweł Dłotko

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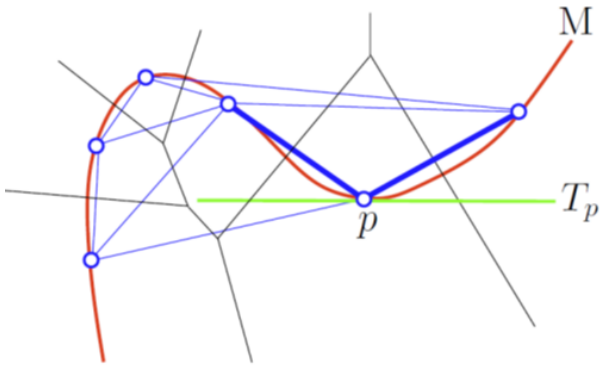
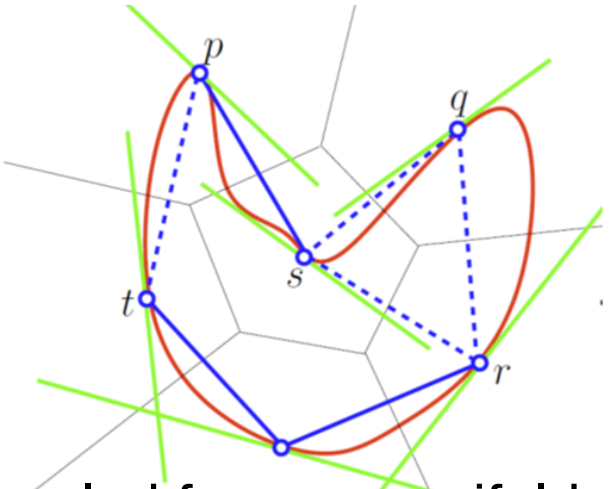
Distance matrix → Geometric filtered complex

CGAL

Geometric filtered complex – Tangential complex





- Suppose we have a set of points sampled from a manifold.
- For every point construct tangent space at that every p ∈ L.
- For every p ∈ L, construct its star and glue the stars of neighbouring points if they agree.
- Based on *Jean-Daniel Boissonnat and Arijit Ghosh Manifold reconstruction using Tangential Delaunay Complexes*.

by Clément Jamin

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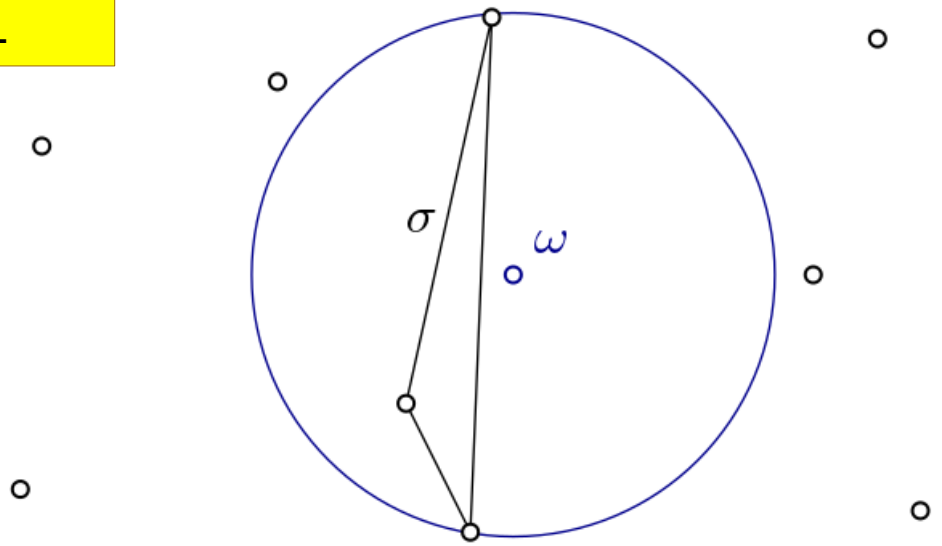
Distance matrix → Geometric filtered complex

CGAL

Geometric filtered complex – Witness complex



- For large point clouds, select small, representative collection of points L called landmarks.
- Build a complex on landmark points. Add a simplex if a witness exists.
- Version with and without filtration.

by Siargey Kachanovich

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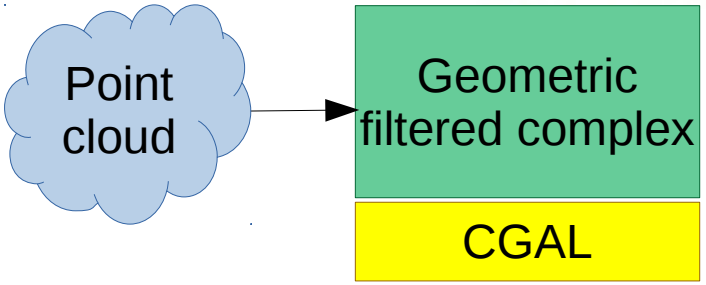
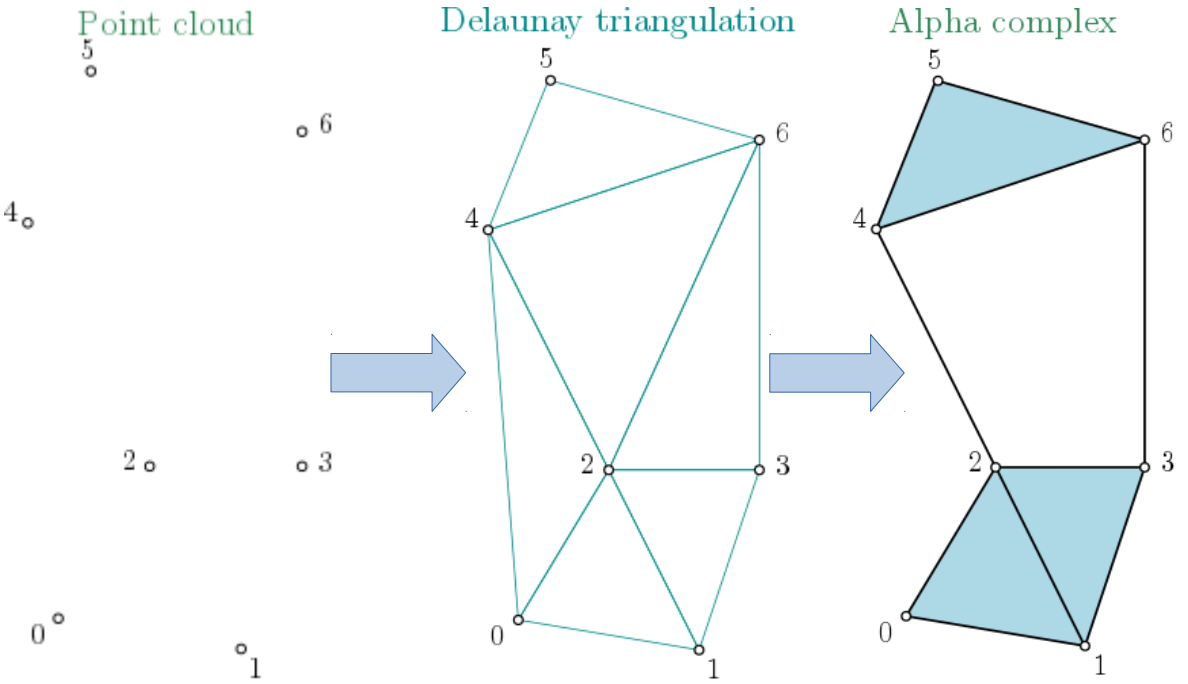


Geometric filtered complex – Alpha from a point cloud



+ Periodic alpha complexes in dimension 3.

by Marc Glisse & Vincent Rouvreau

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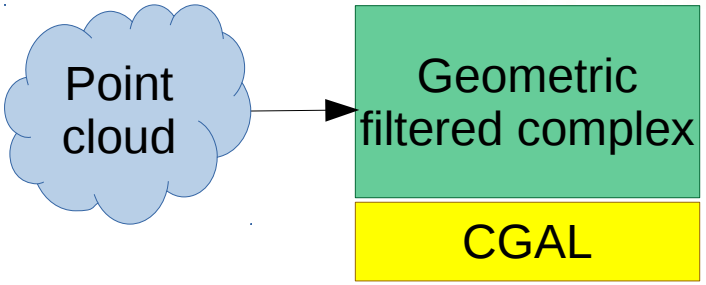
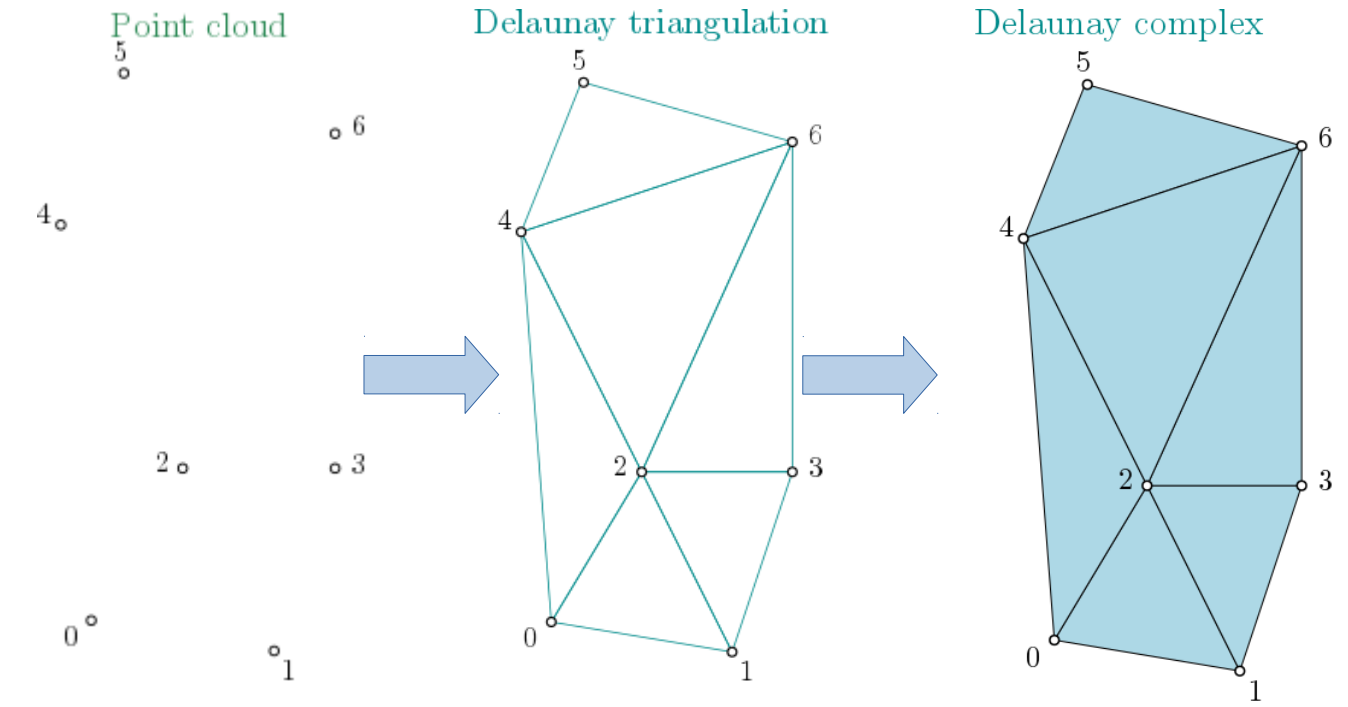


Geometric filtered complex - Delaunay from a point cloud



by Marc Glisse & Vincent Rouvreau

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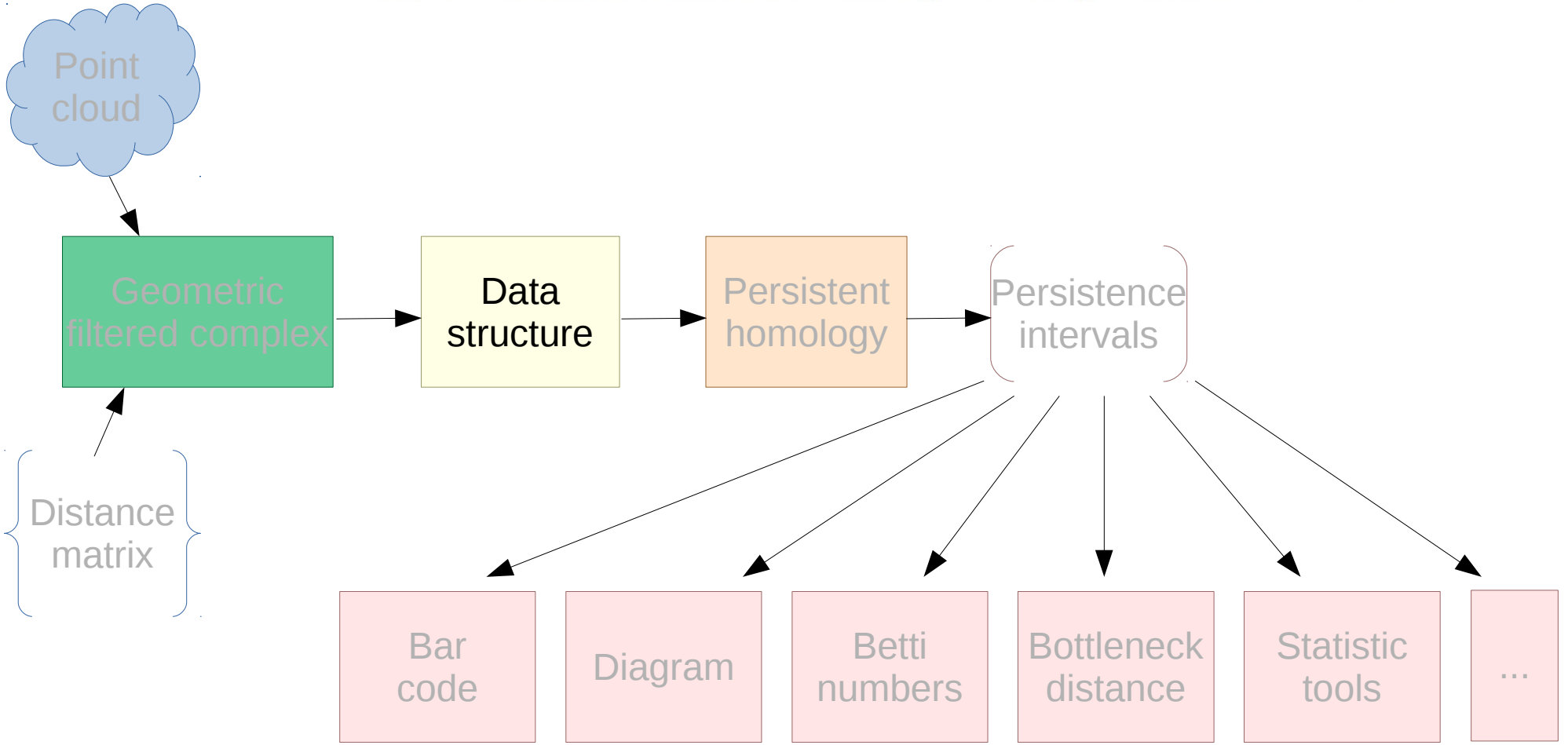

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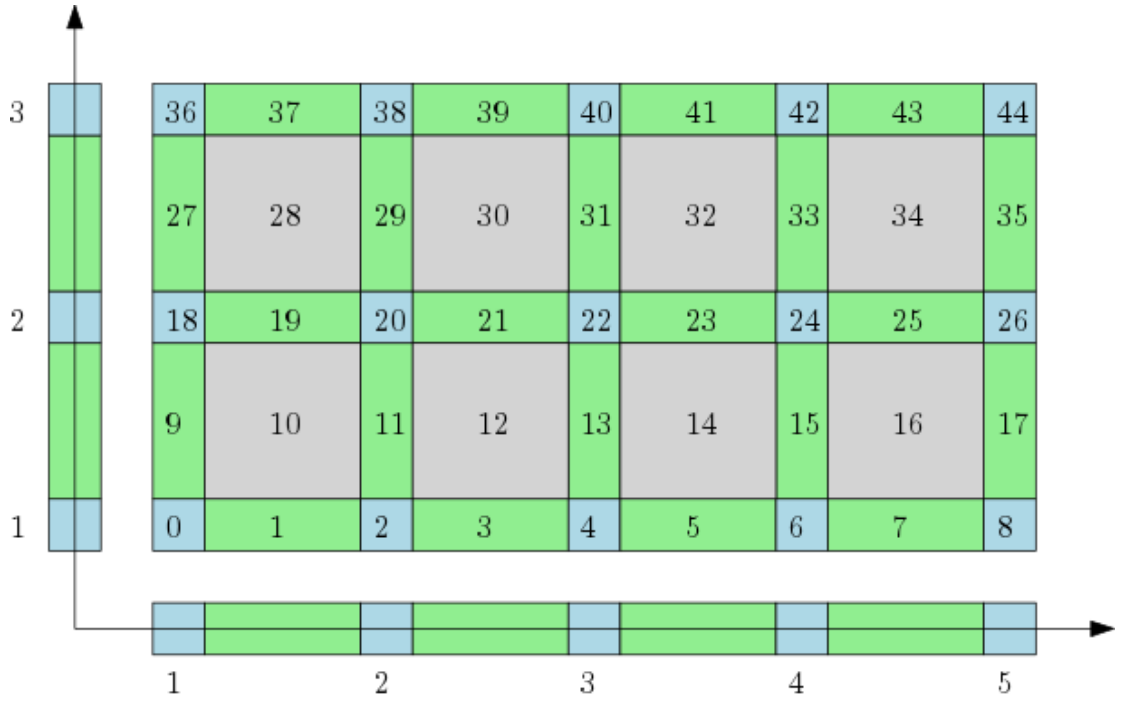
Data structure

Cubical complexes – bitmap



- Represented as a vector of filtration values.
- (Co)boundary computed based on the position in this vector.
- Used in analysis of grid-type data.

by Paweł Dłotko

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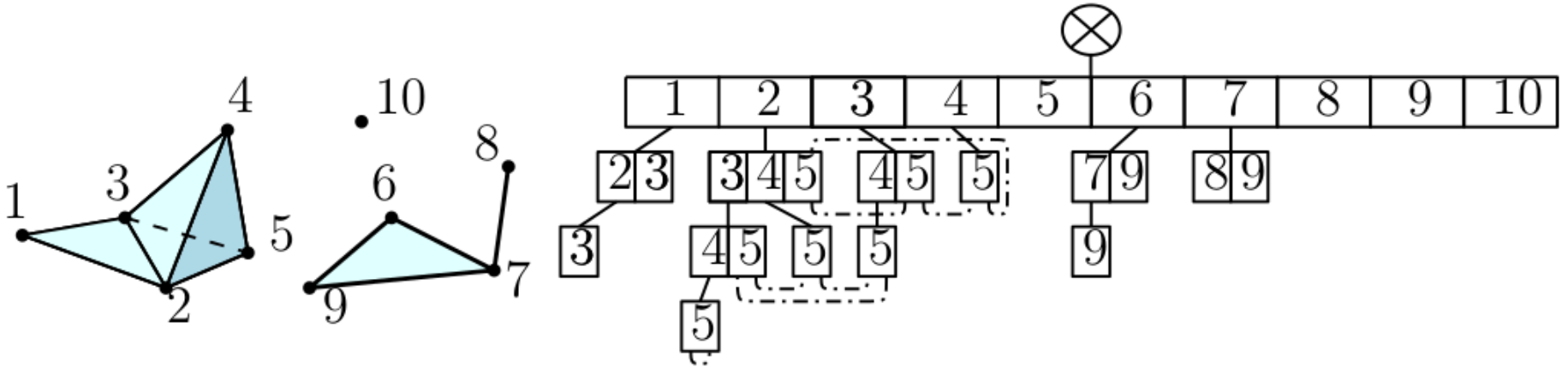
Data structure

Filtered simplicial complexes – Simplex tree



- Memory and time–efficient data structure to store simplicial complexes.
- Every simplex is a word stored in the tree.
- The nodes corresponding to simplices of the same dimension having the same maximal vertex are stored in a cyclic list.
- It is a base of all algorithms to compute persistence of weighted simplicial complexes in GUDHI.

by Clément Maria

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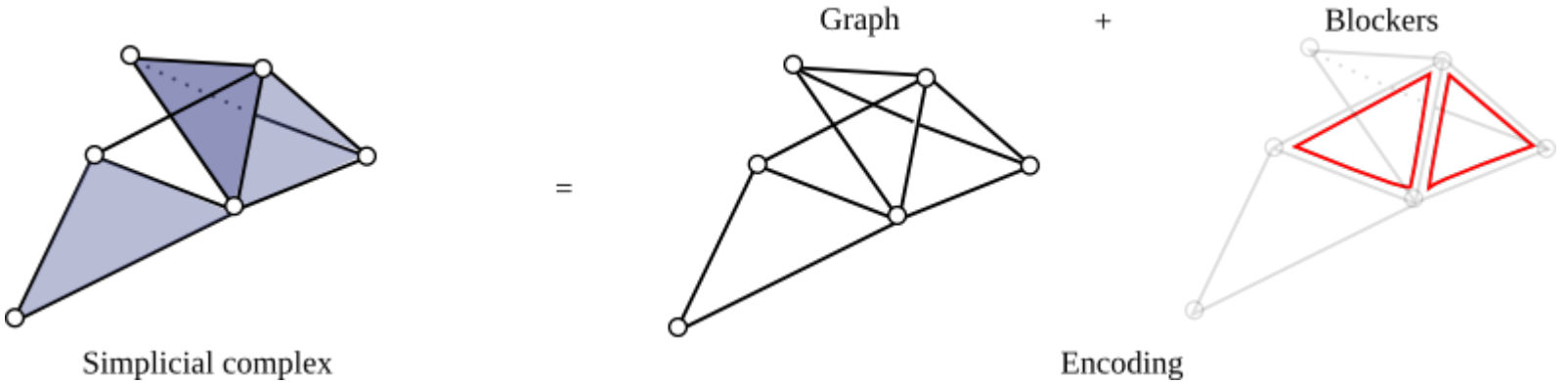
Data structure

Simplicial complexes – Skeleton blocker



- A data structure for very large simplicial complexes.
- We store the 1−skeleton and the minimal simplices which are not present in the complex.
- The rest is generated from cliques in the 1−skeleton.
- Used in edge contraction toolbox (details later).

by David Salinas

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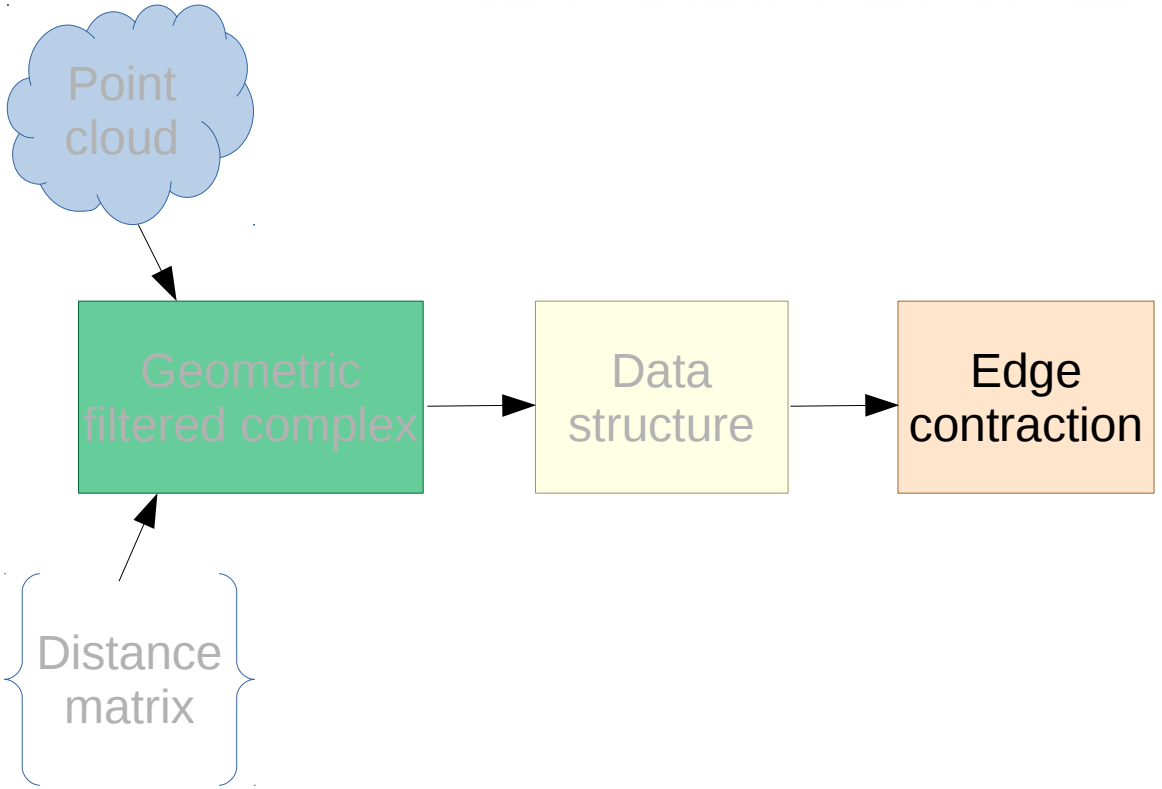

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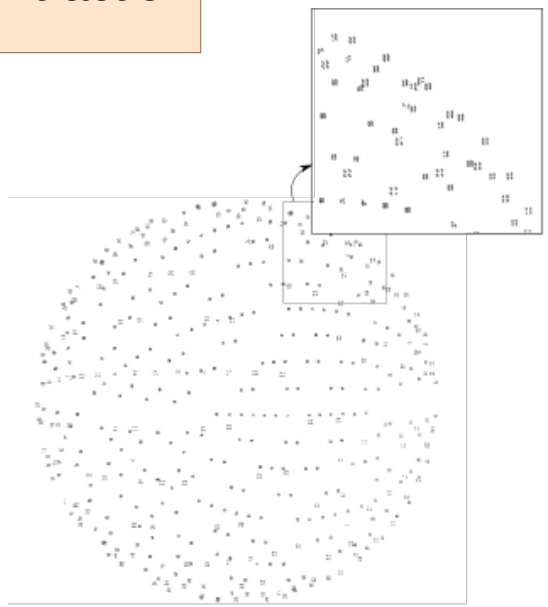
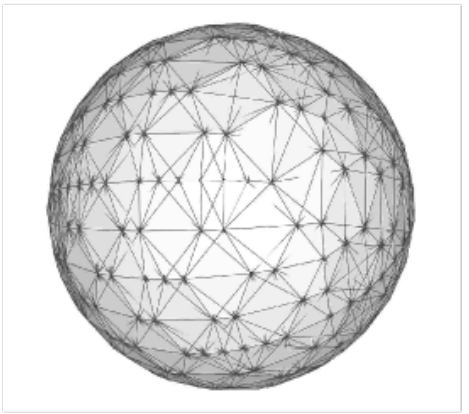
Edge contraction

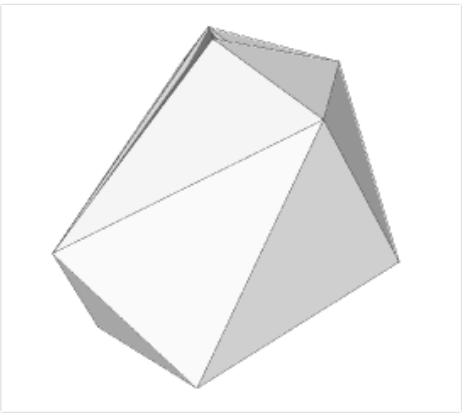
Toolbox – edge contraction



Point cloud sampling SO3
(points are in R^9 but projected into R^3
for vizualization)



Rips complex built uppon these points
20 millions simplices



Simplicial complex obtained after simplification
714 simplices

- Efficient on a skeleton blocker data structure.

by David Salinas

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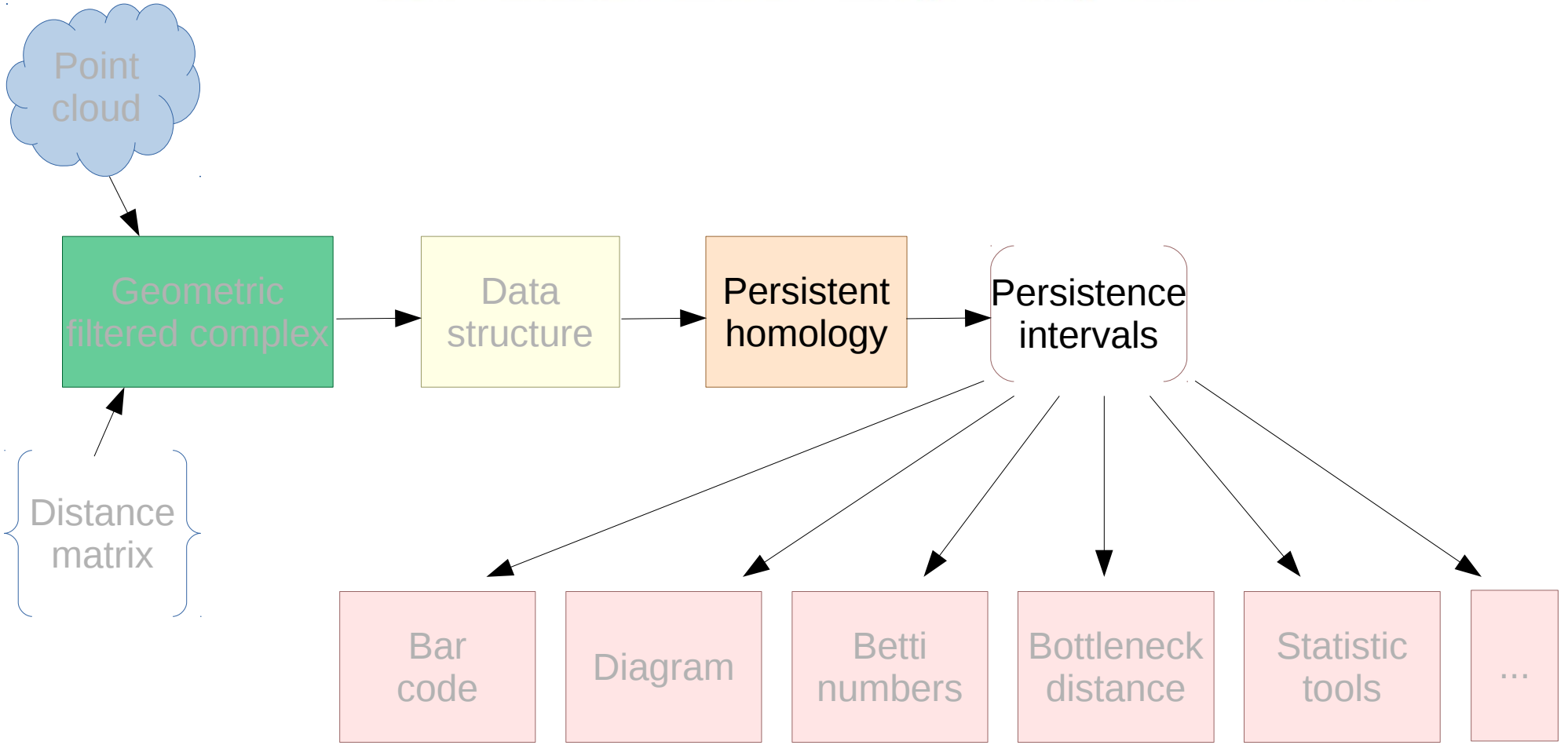

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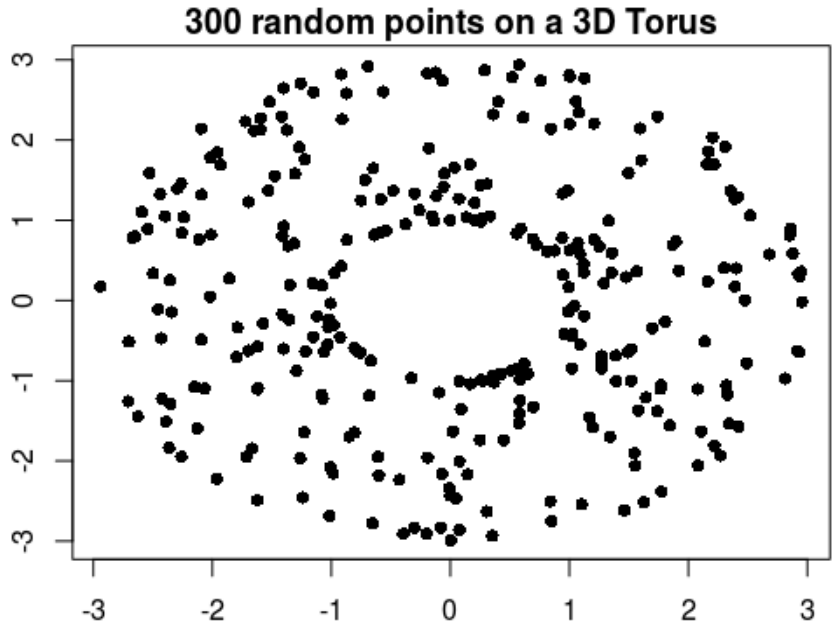
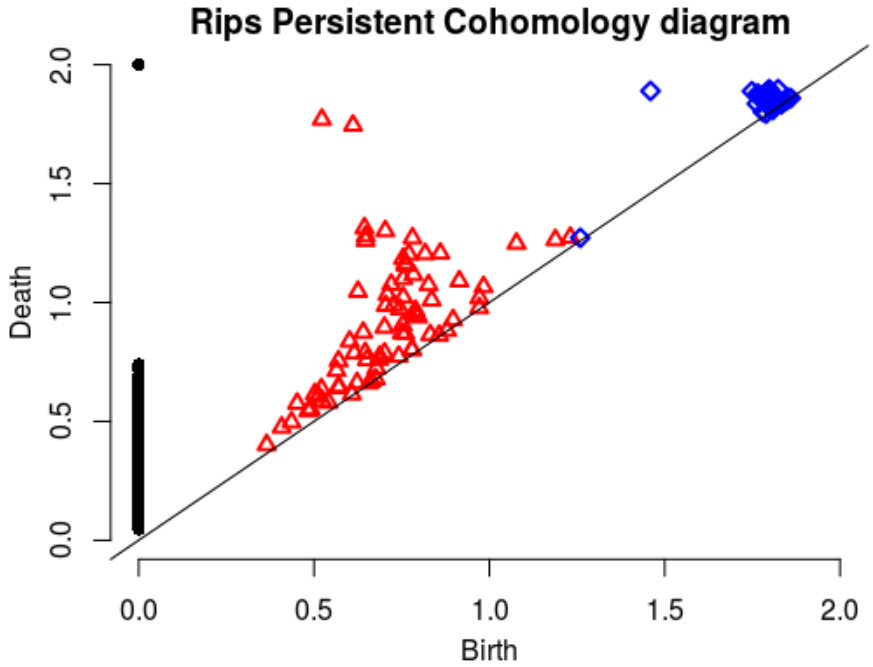
Persistent homology → Persistence intervals

Toolbox – Persistence cohomology





- Standard persistence cohomology computations by using compressed annotation matrix.
- Multi–field persistence (detection of torsion coefficients).

by Clément Maria

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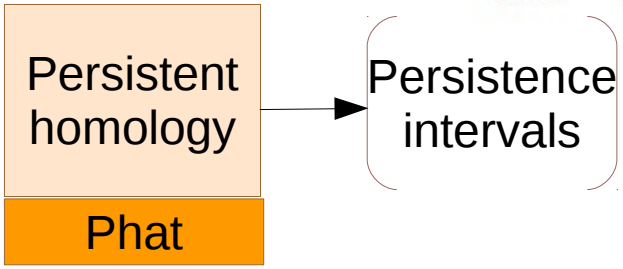

Toolbox – Phat persistence interface

- Computing persistence with Phat (Phat by Ulrich Bauer, Michael Kerber, Jan Reininghaus and Hubert Wagner).

by Paweł Dłotko

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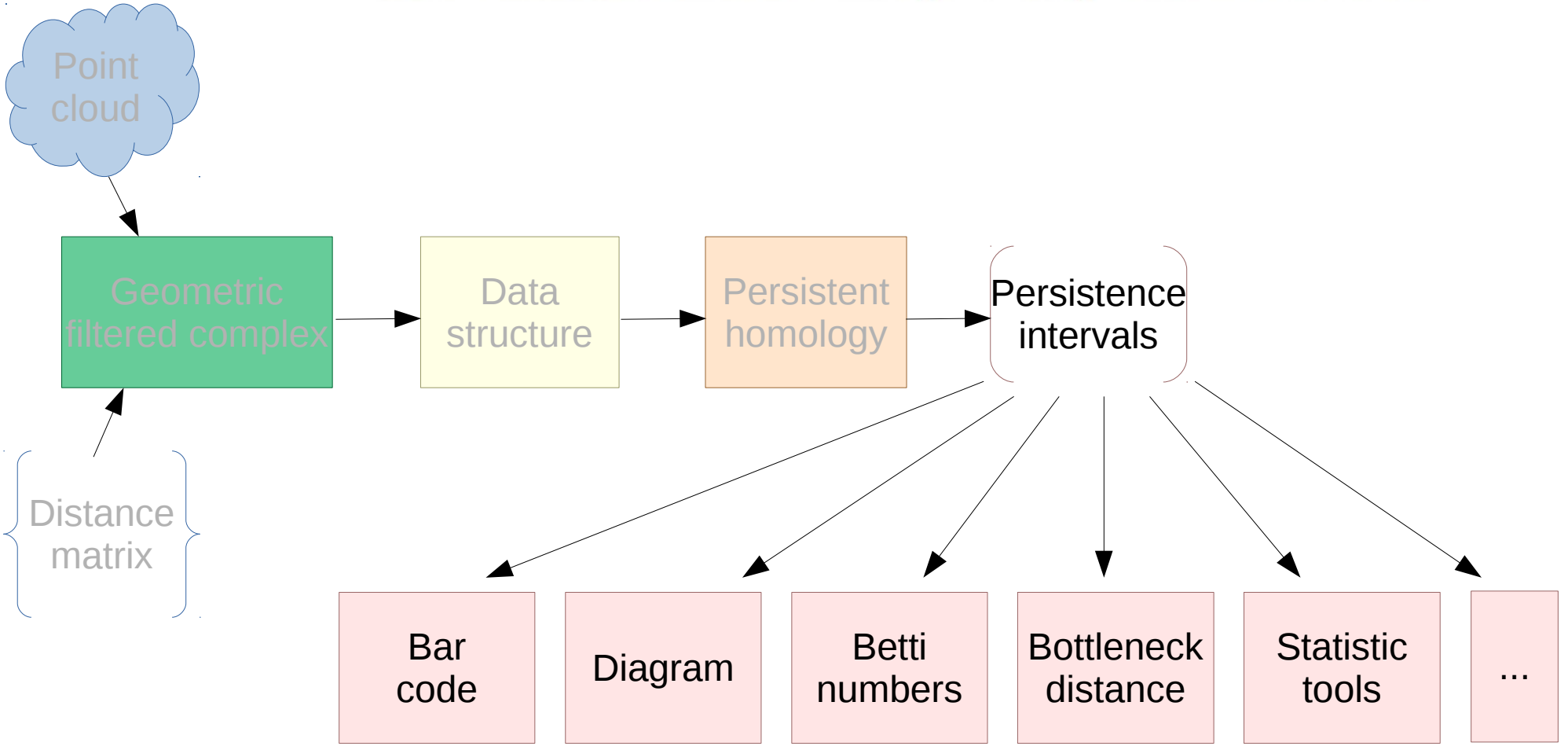

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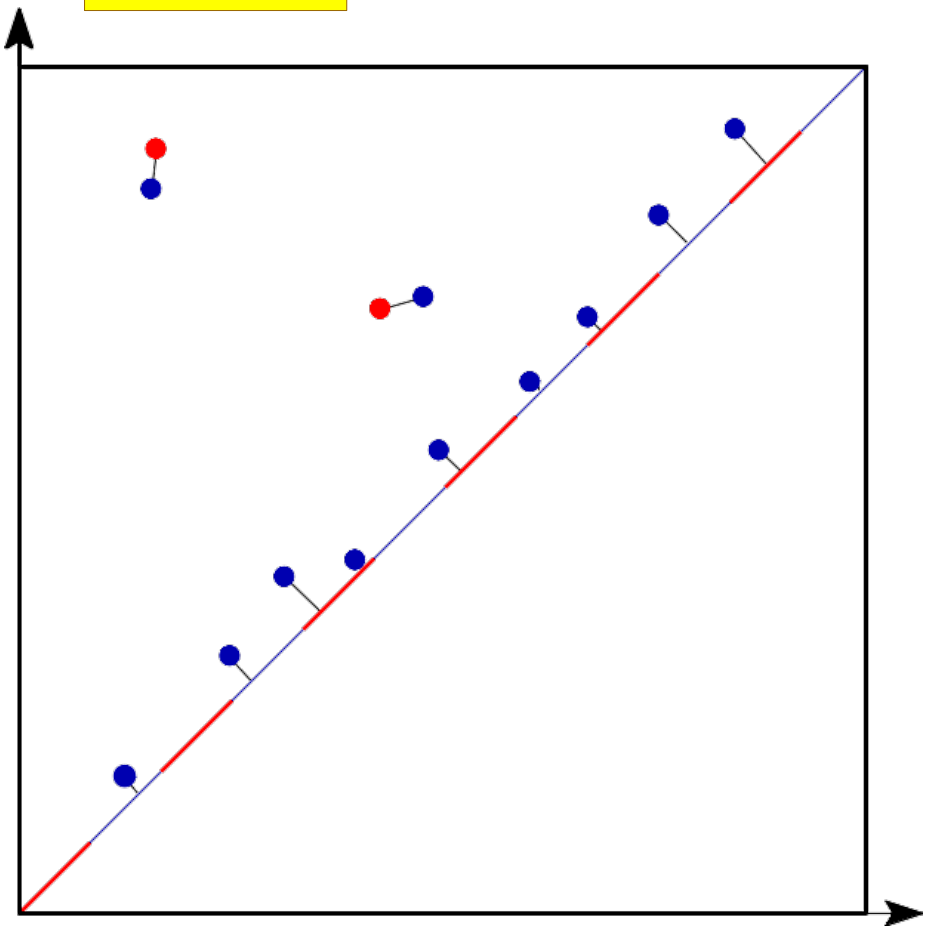
Persistence intervals → Bottleneck distance

CGAL

Toolbox – Bottleneck distance



by François Godi

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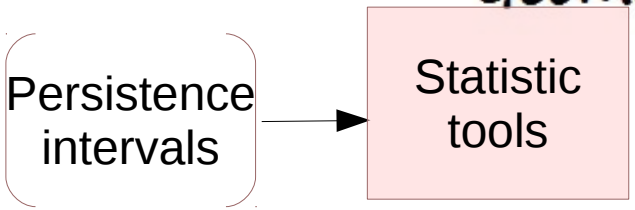

Toolbox – GUDHI stat

- Statistics on persistence diagrams
- Distance to measure

by Paweł Dłotko

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Our website:
http://gudhi.gforge.inria.fr

Documentation will be available here:
http://gudhi.gforge.inria.fr/doc/latest/

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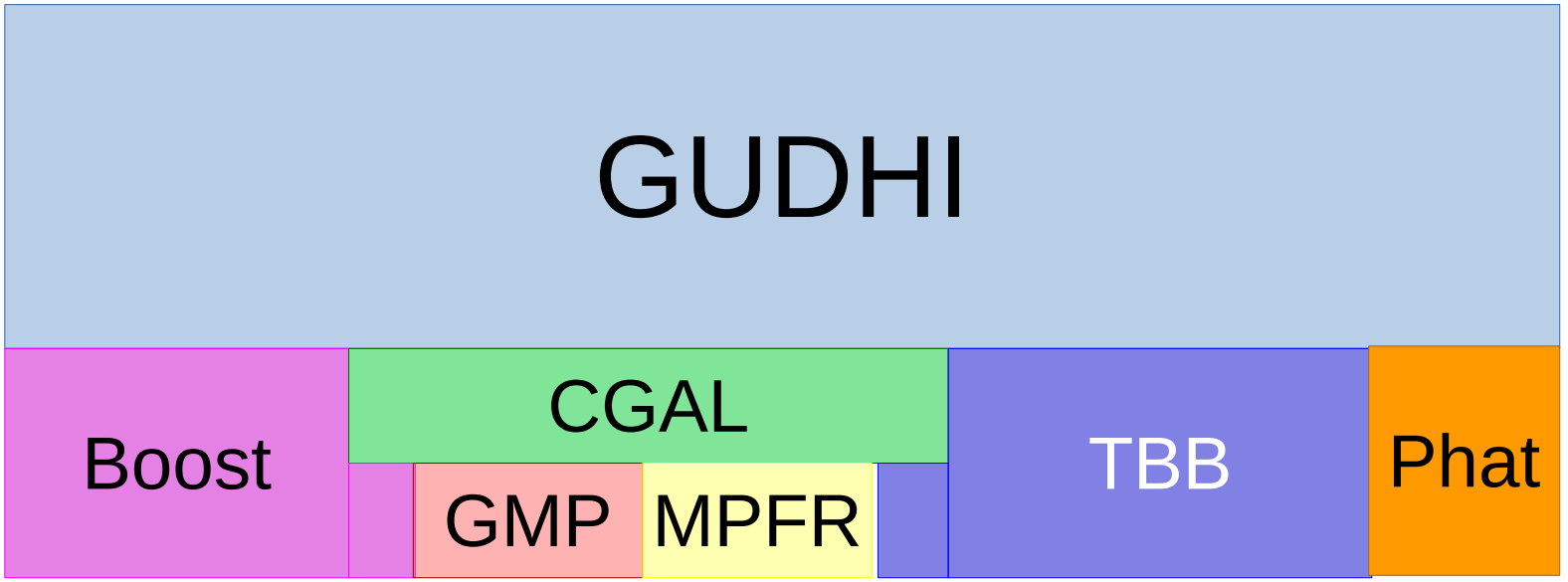
Third party libraries



Installing GUDHI:
http://gudhi.gforge.inria.fr/doc/latest/installation.html

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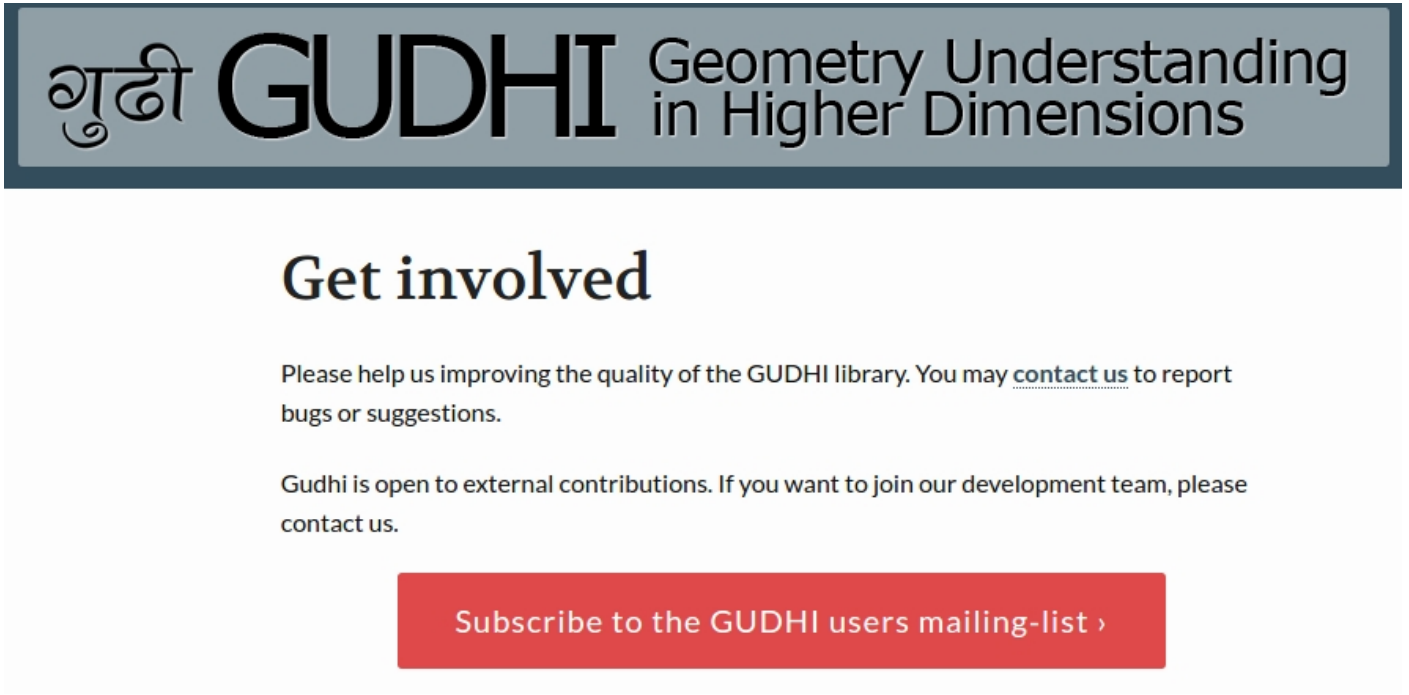
If you want to join the GUDHI users community:
http://gudhi.gforge.inria.fr/getinvolved/



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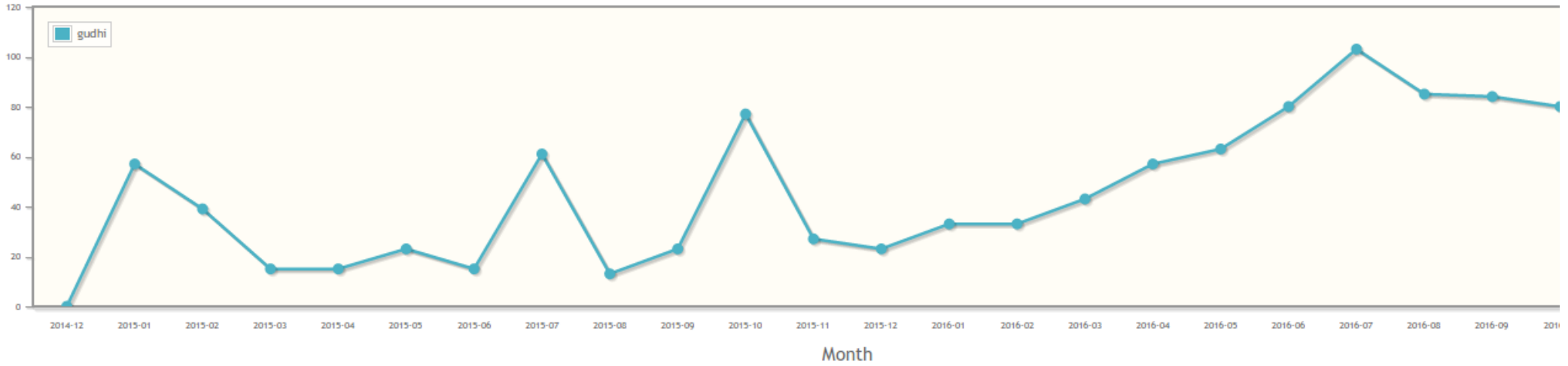
GUDHI downloads:



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GUDHI is open to external contributions.

- Examples driven development
- Documentation is required
- Unitary tests are required
- Some conventions to write code
- Peer review process
- All the packages come with the names of their authors

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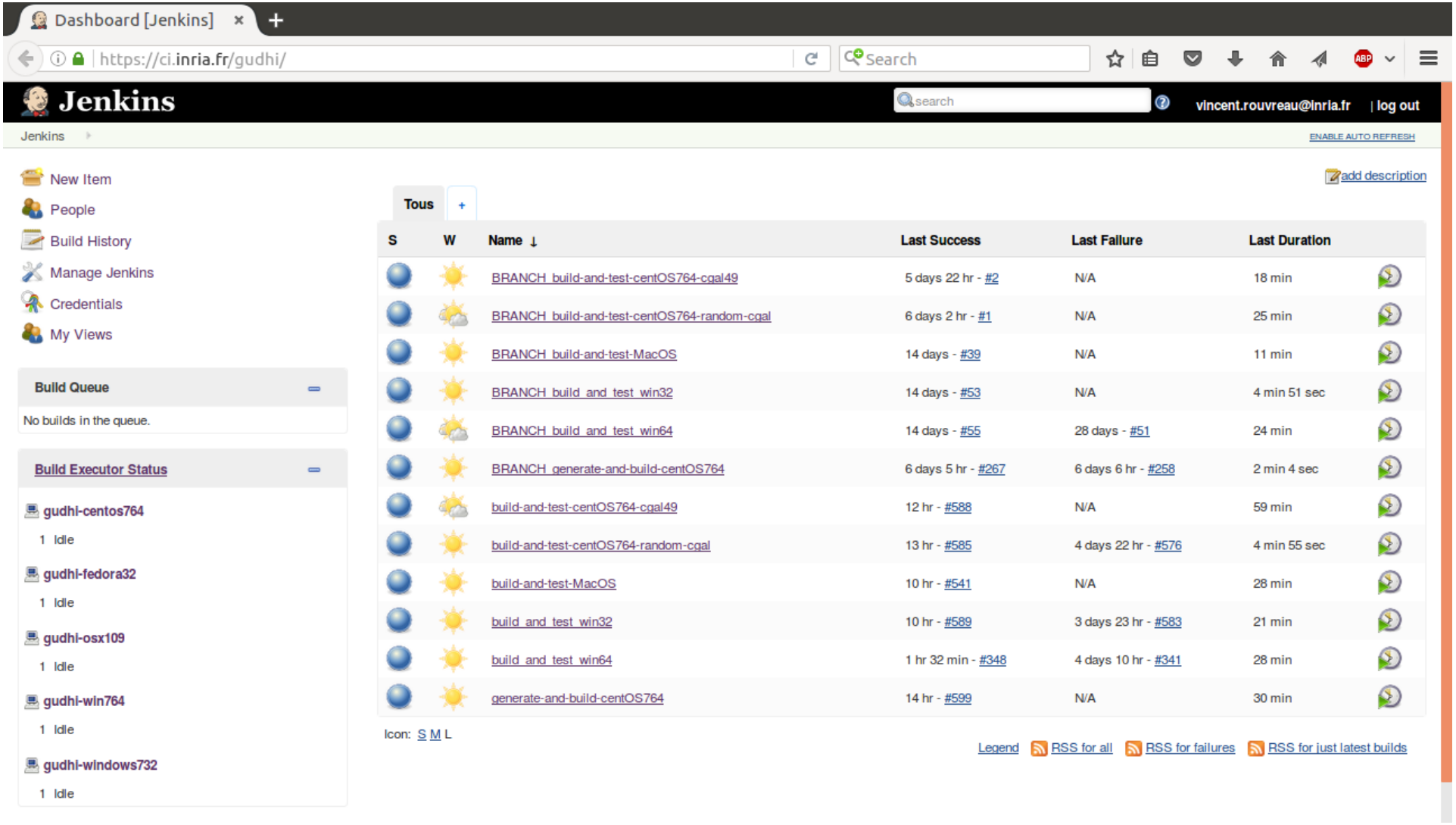

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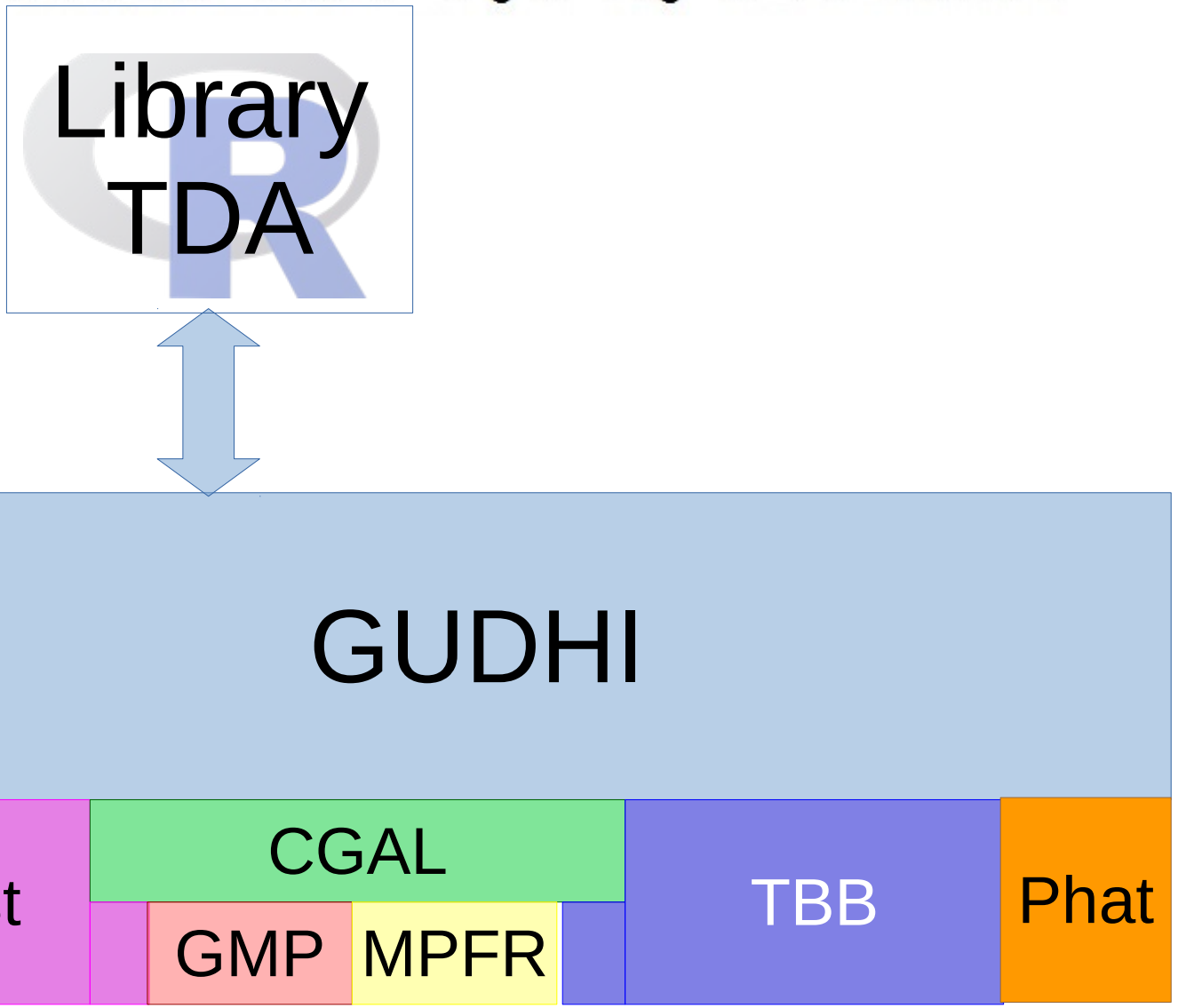
Interfaces



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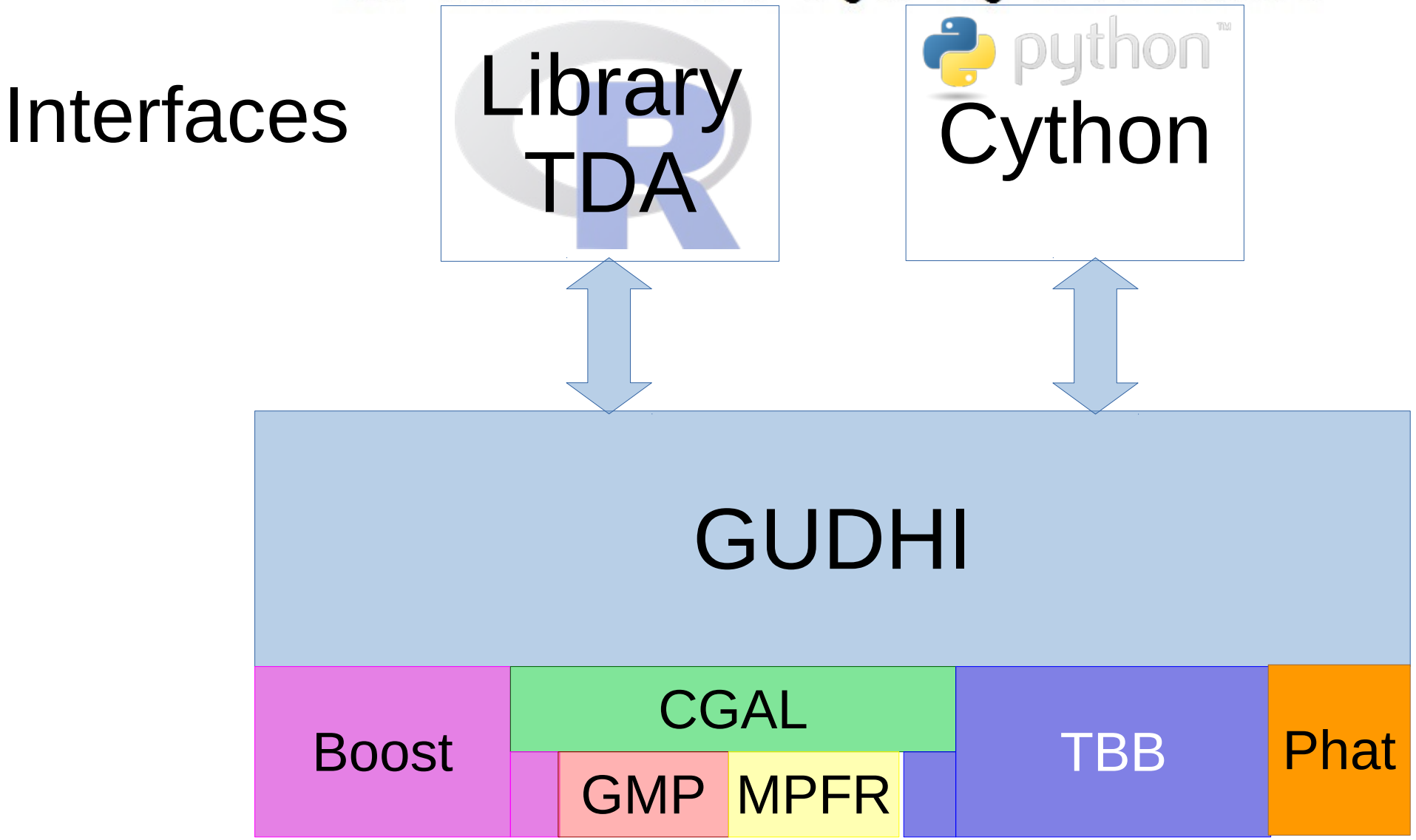
Interfaces



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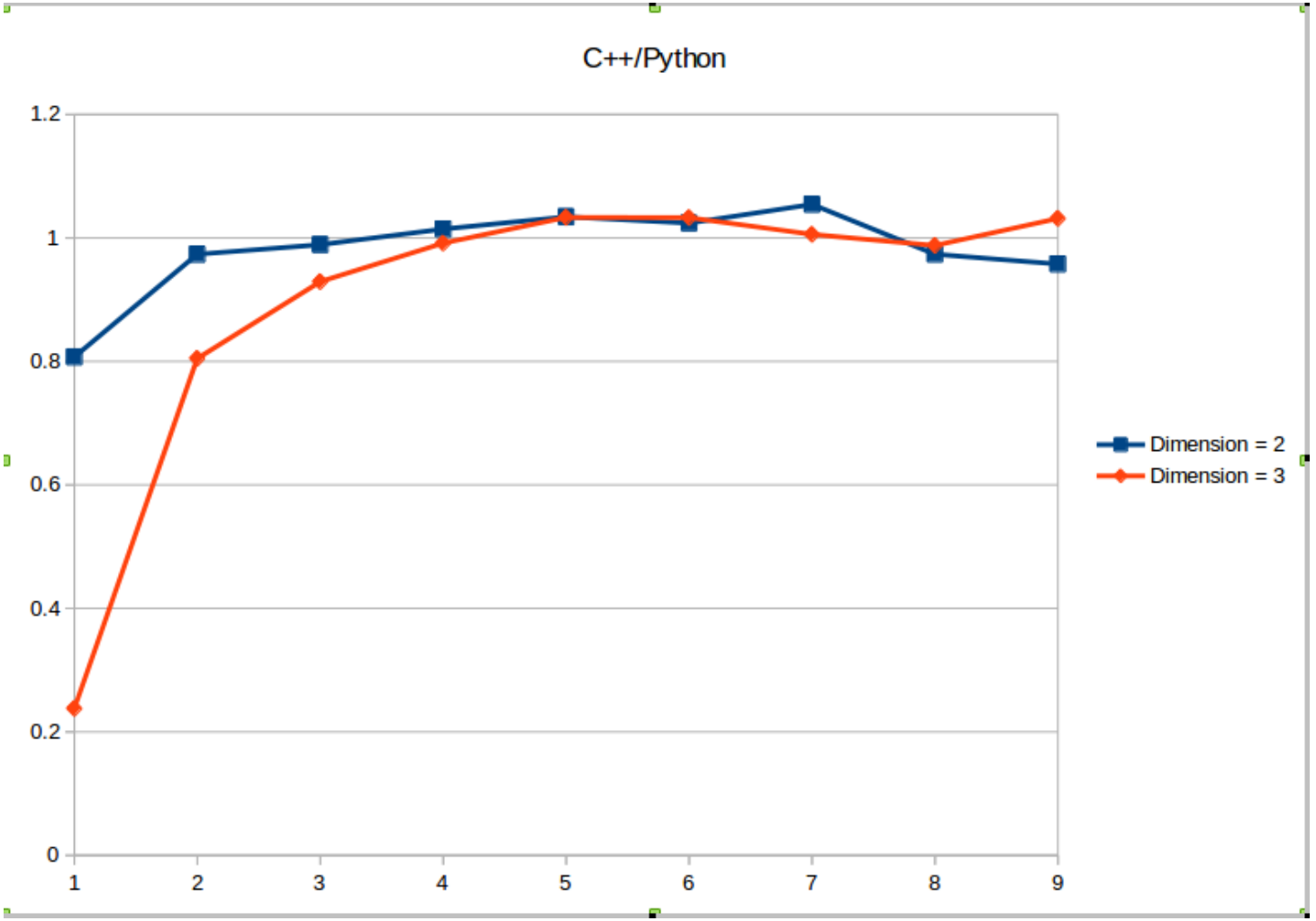

Dim2: random cubical complex 400 × 400 to 3600 × 3600, dx = 400

Dim3: random cubical complex 20 × 20 × 20 to 180 × 180 × 180, dx = 20

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- Documentation will be available here.

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What will arrive after GUDHI 1.4.0 ?

- S.A.L.

Thank you !