Applications of High Dimensional Clustering on Riemannian Manifolds in Brain Measurements

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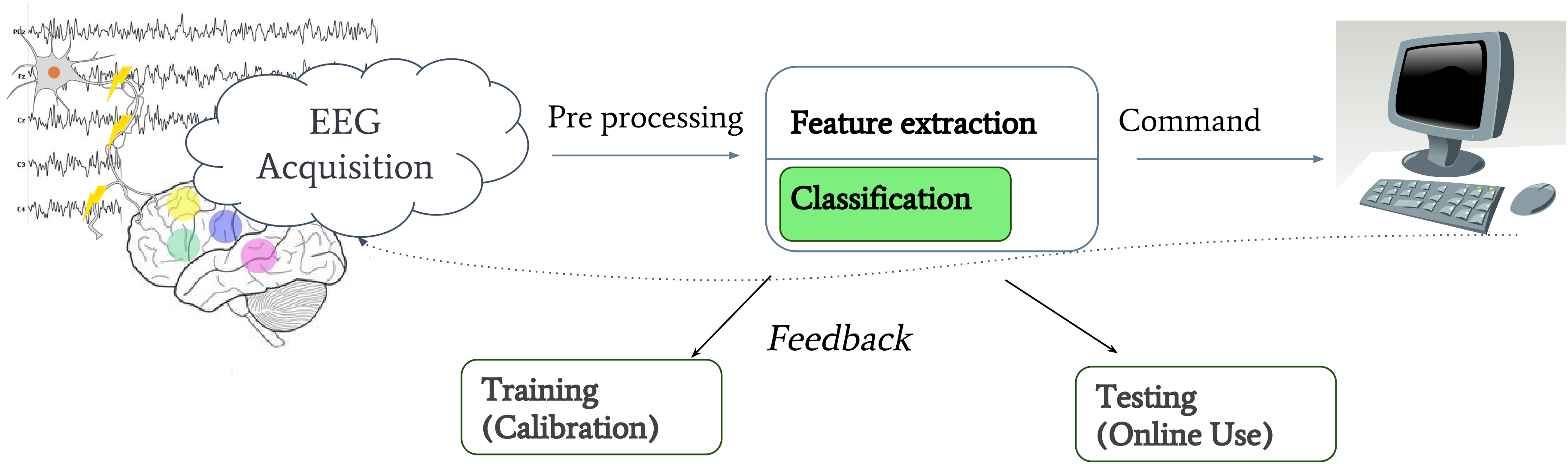
Introduction

Introduction: Brain Computer Interfaces (BCI)

“A brain–computer interface is a communication system that does not depend on the brain’s normal output pathways of peripheral nerves and muscles.”

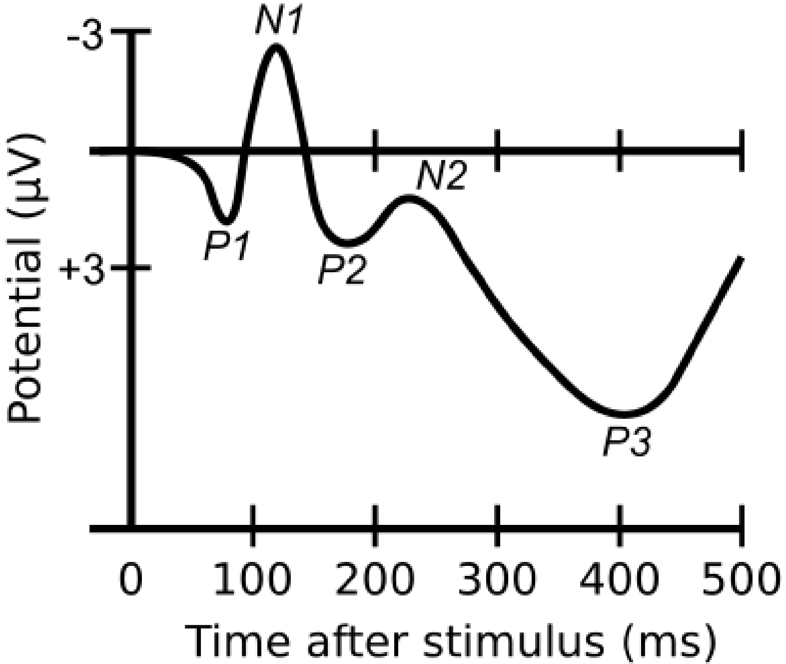
- definition by Wolpaw et al. [1], 2003



[1] Wolpaw, Jonathan R., et al. "Brain-computer interface technology: a review of the first international meeting." IEEE transactions on rehabilitation engineering 8.2 (2000): 164-173.

Event-Related Potentials

- Event Related Potentials are brain responses elicited from a **stimulus**
  - Auditory
  - Sensory
  - **Visual**



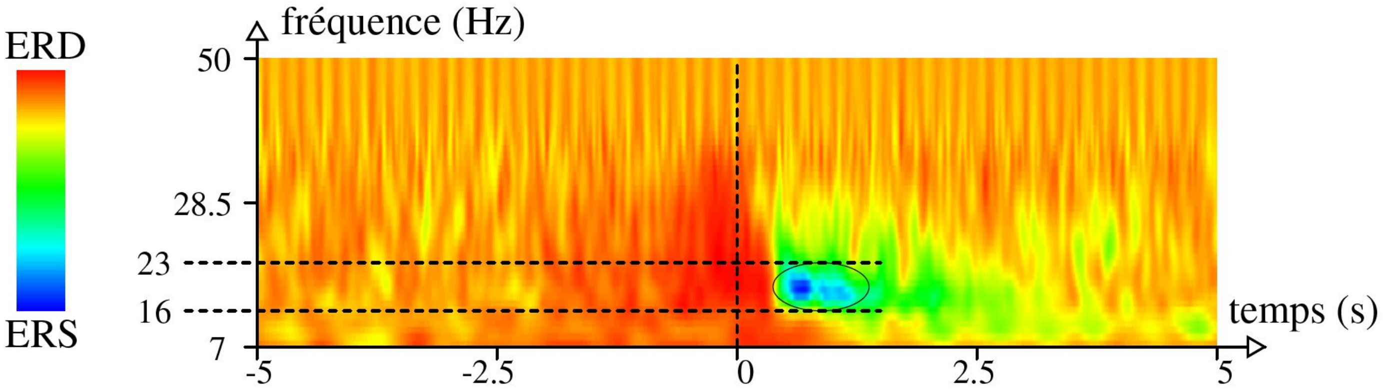
The P300 Speller

- The P300 wave:
  few “target” stimuli among a train of “nontarget” stimuli.
- A non-invasive EEG-based Brain Computer Interface.
- Spell a letter by counting number of flashes.

Coadapt P300 speller, calibration.
By Maureen Clerc, Dieter Devlaminck & Loïc Mahé @ Inria Sophia Antipolis-Méditerranée.
Code developed by Inria & Inserm, funded by CoAdapt project (ANR-09-EMER-002).

Event-Related Synchronization / Desynchronisation

- ERD/ERS are brain responses elicited from an **Imagined Movement**
  - Observed in the beta (12-30Hz) and mu (7.5 - 12.5Hz) frequencies.

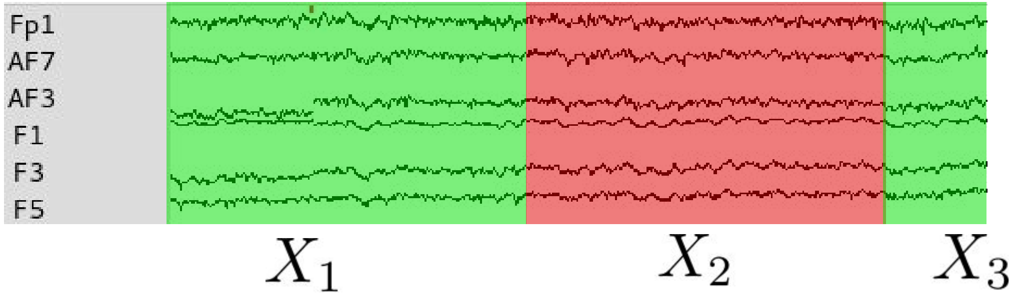


Pre processing

- Noise filtering
  - Bandpass filtering to eliminate uninformative frequencies.
- EEG segmentation into **I** trials from stimulus onset

$$X_i \in \mathbb{M}(C, N)$$

  - **C** denotes the number of sensors, **N** denotes the time points.
  - Signals have a **very low** Signal to Noise Ratio

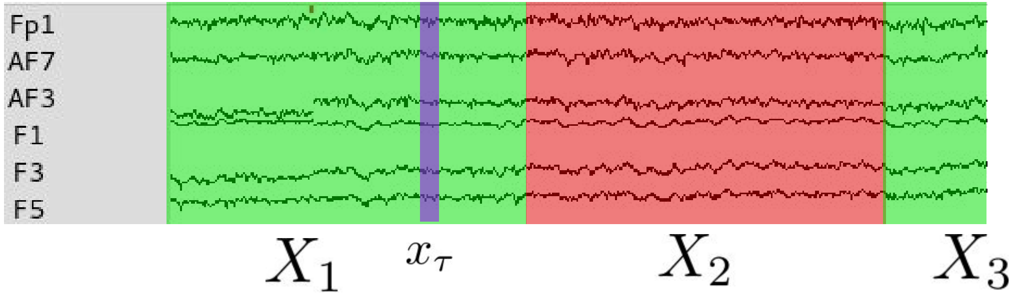


Assumptions

- Choice of N: each trial is a stationary process:

$$x_\tau \sim \mathcal{N}(0, \Sigma^i)$$

  - Trials corresponding to the *target* class follow distribution $x_\tau \sim \mathcal{N}(0, \Sigma^1)$
  - Trials corresponding to the *nontarget* class follow distribution $x_\tau \sim \mathcal{N}(0, \Sigma^2)$



We assume that $\Sigma^1 \neq \Sigma^2$

Feature Extraction

- The features are the elements of the Sample Covariance Matrix

$$\Sigma_i = \frac{1}{N-1} X_i X_i^T$$

- *Assumption:* Σ_i is a Symmetric Positive Definite (SPD) matrix
    - Σ_i lies on the **Statistical Manifold**, also called the **manifold of SPD matrices**.

The Riemannian Manifold of SPD Matrices

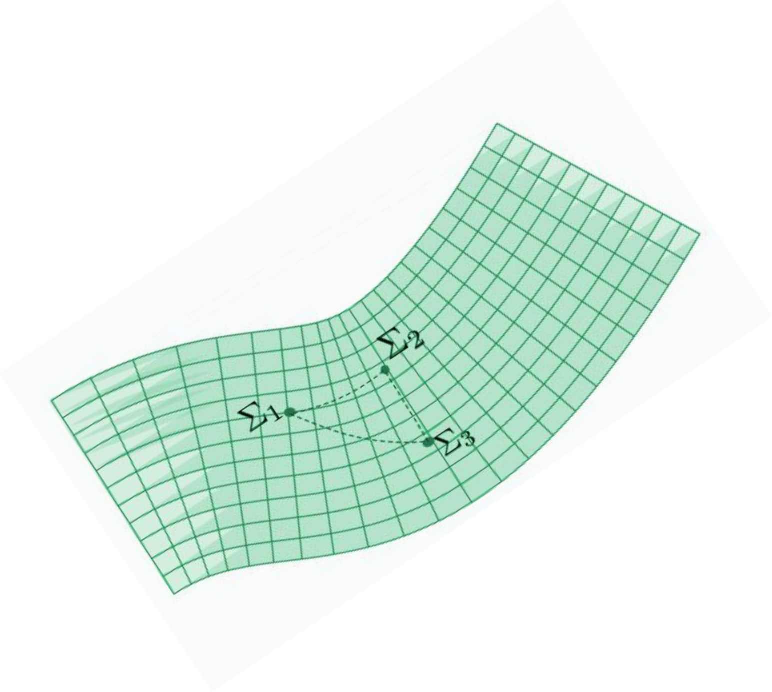
- Embedded with a Riemannian metric

$$d_R(\Sigma_1, \Sigma_2) = \|log(\Sigma_1^{-1}\Sigma_2)\|_F = \sqrt{\sum_{i=1}^n log^2 \lambda_i}$$

  - λ_i are the eigenvalues of $\Sigma_1^{-1}\Sigma_2$

- Some properties
  - Hadamard Manifold [2]
  - Derived from information geometry [3].
  - Invariant to linear transformations.

$$d_R(\Sigma_1, \Sigma_2) = d_R(W\Sigma_1 W^T, W\Sigma_2 W^T)$$



[2] Pennec, Xavier. "Statistical computing on manifolds: from Riemannian geometry to computational anatomy." Emerging Trends in Visual Computing. Springer Berlin Heidelberg, 2009. 347-386.
[3] Skovgaard, Lene Theil. "A Riemannian geometry of the multivariate normal model." Scandinavian Journal of Statistics (1984): 211-223.

Analysis

Materials

EEG signals recorded during
P300 speller sessions.
20 Subjects, 3 Sessions per subject (calibration).

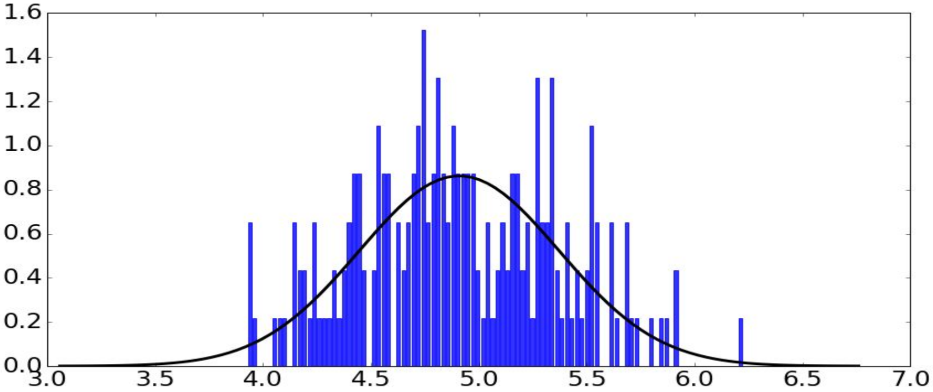
C = 12 electrodes
Sampling rate = 256, epoch = 0,5s
N = 128

Bandpass butterworth filter applied.
(5th order, between 1.0 and 2.0)

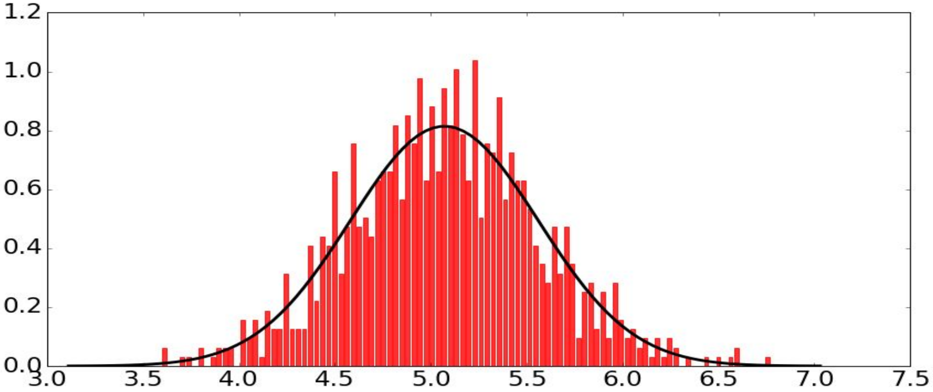
Two classes:
Target (T), Nontarget (N)

Distribution of distances on M

- Observations:
    - Points are almost equidistant to the mean of their class



Class Target, distribution of distances to class mean.



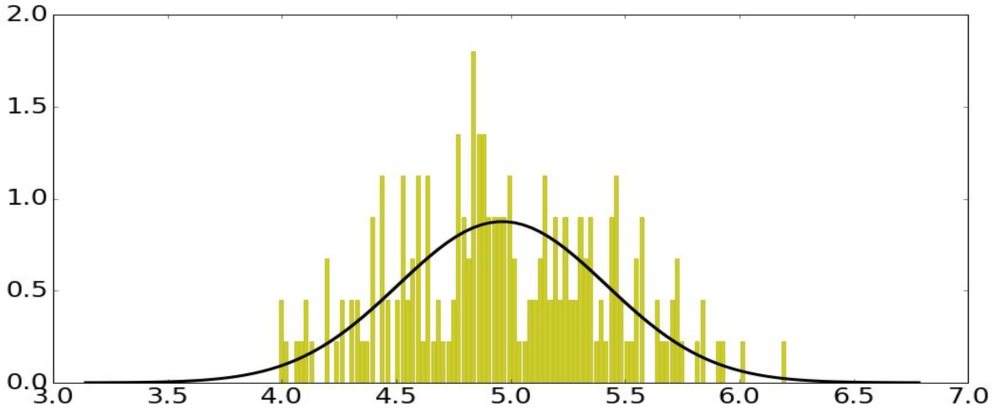
Class Nontarget, distribution of distances to class mean.

Example: Subject 6, Session 1

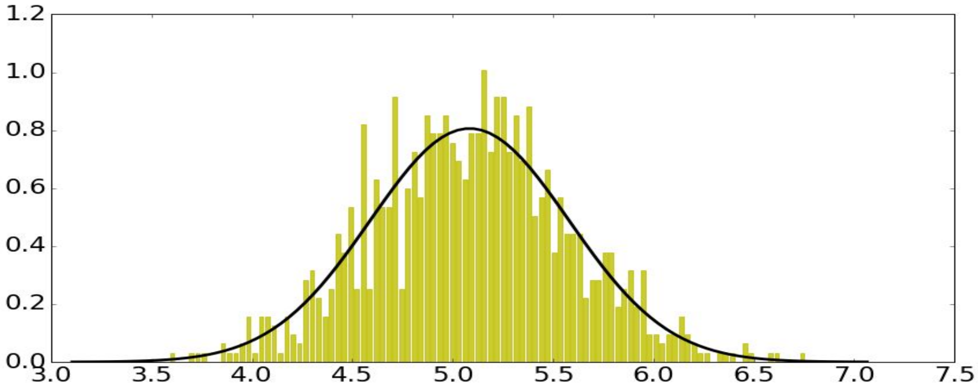
Distribution of distances on M

- Observations:
  - Points are almost equidistant to the mean of their class
  - Points are almost equidistant to the mean of their class and to the mean of the other class.
  - Riemannian distance between the two means

    $d_R(\bar{\Sigma}^T, \bar{\Sigma}^N) = 0,657$



Class Target, distribution of distances to other class mean.

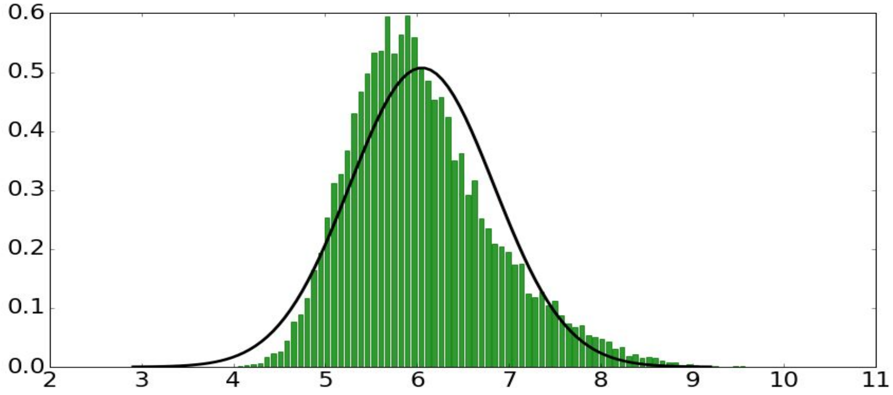


Class Nontarget, distribution of distances to other class mean.

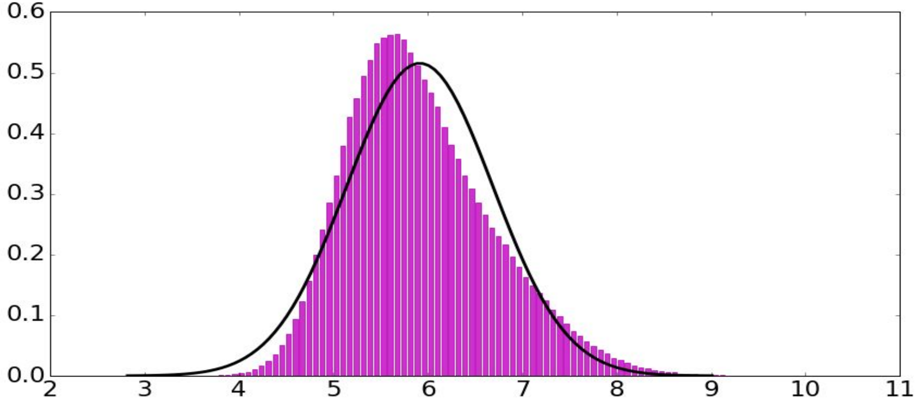
Example: Subject 6, Session 1

Distribution of distances on M

- Observations:
  - Points are equidistant to each other on the Riemannian manifold (Riemannian distance) and to the Tangent space at the mean of all points (Euclidean distance).



Distribution of distances between points on the manifold.

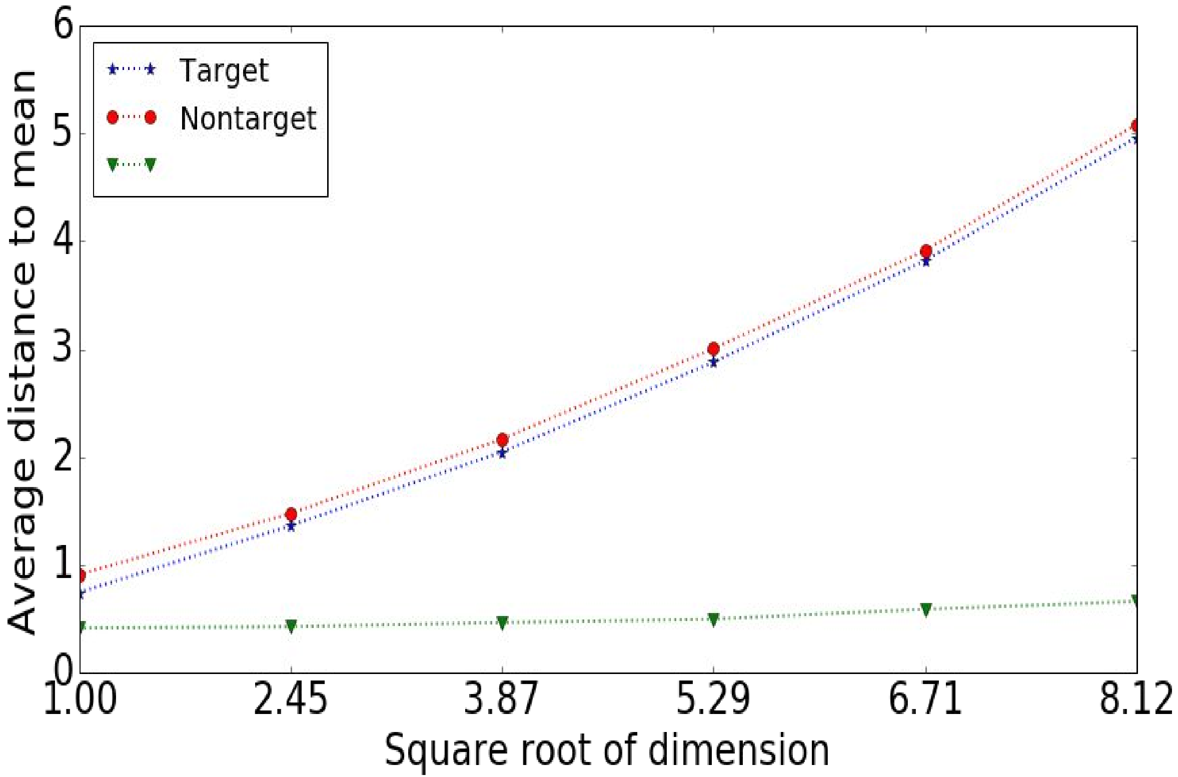


Distribution of distances between points on the Tangent Space

Example: Subject 6, Session 1

Distribution of distances on M

- Observations:
  - The distance increases as the dimension increases, **but not the distance between class means.**



Methods

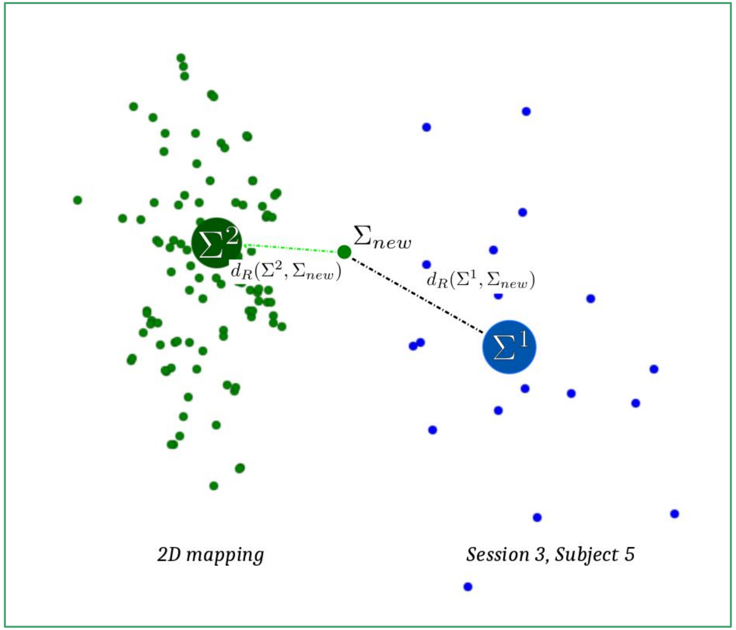
The Minimum Distance to Riemannian Mean Algorithm [4]

- Calibration
  - Compute the Riemannian Center of Mass of each class [5]

$$\bar{\Sigma} = \underset{\Sigma}{\operatorname{argmin}}\left(\frac{1}{I} \sum_{i=1}^{I} d_R^2(\Sigma, \Sigma_i) \right)$$

- Online Use
  - Compute the Riemannian distance to each center of mass
  - The minimum distance defines the classification result

$$class = \underset{c}{\operatorname{argmin}}\ d_R(\Sigma^c, \Sigma_{new})$$



[4] Barachant, A, et al. "Riemannian geometry applied to BCI classification." International Conference on Latent Variable Analysis and Signal Separation. Springer Berlin Heidelberg, 2010.

[5] Fréchet, M. "Les éléments aléatoires de nature quelconque dans un espace distancié." Annales de l'institut Henri Poincaré. Vol. 10. No. 4. 1948.

Tangent Space Projection

Tangent space projection of Σ_i at $\bar{\Sigma}$:

$$S_i = \Sigma^{1/2} log(\Sigma^{-1/2} \Sigma_i \Sigma^{-1/2}) \Sigma^{1/2}$$

Projecting S_i back to the manifold:

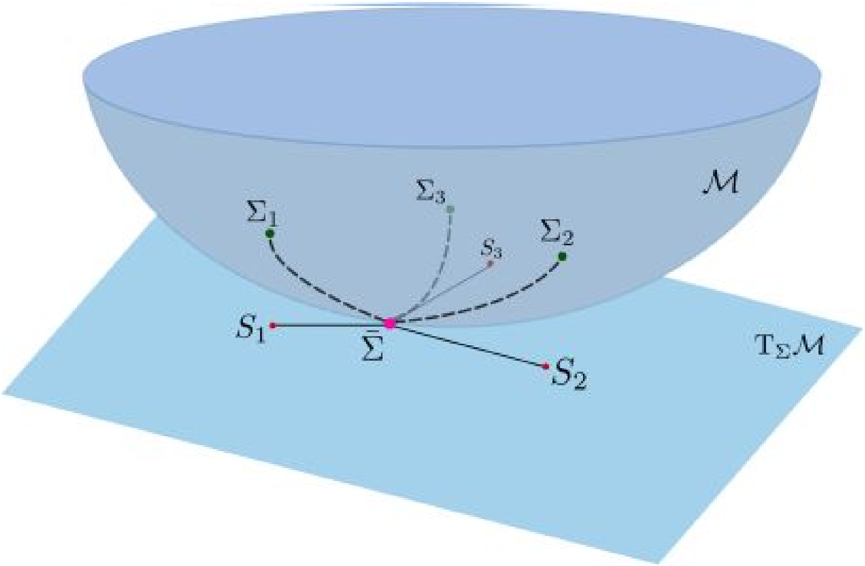
$$\Sigma_i = \Sigma^{1/2} exp(\Sigma^{-1/2} S_i \Sigma^{-1/2}) \Sigma^{1/2}$$

Let Σ_1, Σ_2 and Σ_3 be three points on the manifold ***M***

Compute their riemannian mean $\bar{\Sigma}$

Project the three points on the tangent space at $\bar{\Sigma}$.

On the Tangent space $T_{\bar{\Sigma}}M$, these points are S_1, S_2, S_3



[6] Pennec, X., Fillard, P., and Ayache, N. "A Riemannian framework for tensor computing." International Journal of Computer Vision 66.1 (2006): 41-66.

Tangent Space Projection

- Feature extraction

  Transform the feature space into a ***Euclidean*** *space.*

  - Compute the Riemannian mean **Σ**
    - *Where?* Center of Mass of all the Features
  - Project the features onto the **Tangent Space** of the manifold at **Σ**
- Train the appropriate classifier

Results - Discussion

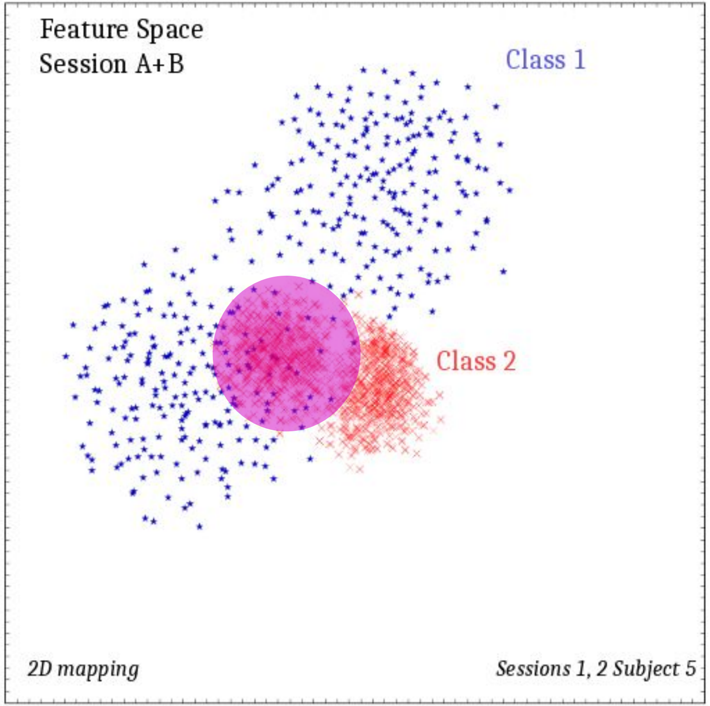
The Problem: Calibrating a P300 Speller

Session 1 - Day 1:

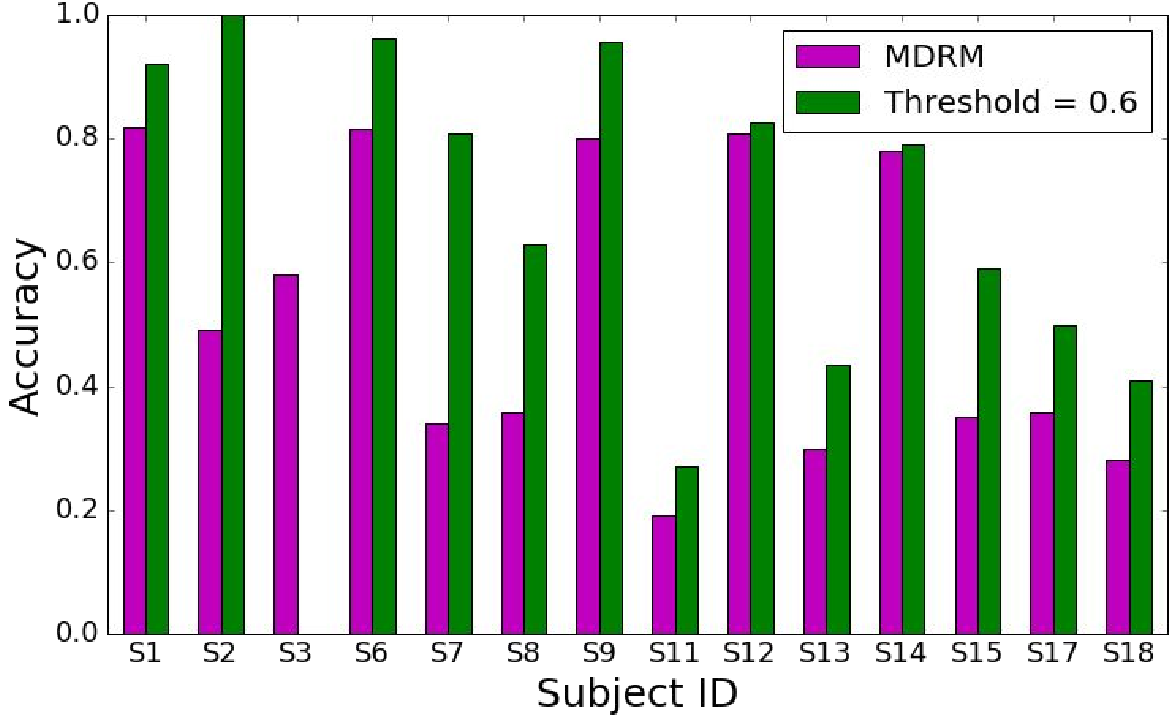
The user is asked to spell a specific word

Session 2 - Day 2:

The user is asked to spell the same word



Results after performing a statistical analysis



Training: Sessions 1 & 2, Testing: Session 3

- *Current methods improve classification results*
- Dimensionality Reduction on the manifold
- Robust features, less sensitivity to outliers
- Classification Algorithms using Differential Geometry

Discussion

Thank you for your attention!

Questions?