T-coercivity: a practical tool for the study of variational formulations.
Application to the magnetostatic model

Patrick Ciarlet [1]
[1] POEMS, CNRS, INRIA, ENSTA Paris,
Institut Polytechnique de Paris, Palaiseau, France
E-mail: patrick.ciarlet@ensta-paris.fr

Variational formulations are a popular tool to analyse linear PDEs (eg. neutron diffusion, Maxwell equations, Stokes equations …), and it also provides a convenient basis to design numerical methods to solve them. Of paramount importance is the **inf-sup condition**, designed by Ladyzhenskaya, Necas, Babuska and Brezzi in the 1960s and 1970s. As is well-known, it provides sharp conditions to prove well-posedness of the problem, namely existence and uniqueness of the solution, and continuous dependence with respect to the data. Then, to solve the approximate, or discrete, problems, there is the **(uniform) discrete inf-sup condition**, to ensure existence of the approximate solutions, and convergence of those solutions to the exact solution. Often, the two sides of this problem (exact and approximate) are handled separately, or at least no explicit connection is made between the two.

In this talk, I will focus on an approach that is completely equivalent to the inf-sup condition for problems set in Hilbert spaces, the **T-coercivity approach**. This approach relies on the design of an *explicit* operator to realize the inf-sup condition. If the operator is carefully chosen, it can provide useful insight for a “natural” definition of the approximation of the exact problem. Two kinds of results can be derived. On one hand, the derivation of the discrete inf-sup condition often becomes elementary, at least when one considers conforming methods, that is when the discrete spaces are subspaces of the exact Hilbert spaces : both the exact and the approximate problems are considered, analysed and solved at once. On the other hand, the knowledge of the operator may lead to a variational formulation that is simpler to solve than the original one.

In itself, T-coercivity is not a new theory, however it seems that some of its strengths have been overlooked, and that, if used properly, it can be a simple, yet powerful tool to analyse and solve linear PDEs.

This claim will be illustrated on the magnetostatic model.