A posteriori goal-oriented error estimators based on equilibrated flux and potential reconstructions

Many engineering problems require the calculation of certain quantities of interest, which are usually defined by linear functionals depending on the solution of a partial differential equation. Examples include the local or global mean value of a temperature, or the magnetic flux density at a given point of an electromagnetic device. In this talk, we focus on estimating the error of such functionals using a wide variety of numerical methods (finite elements, discontinuous Galerkin and finite volumes), within a unified framework for elliptic and parabolic problems. The key point lies in solving a dual problem and using guaranteed equilibrated estimators for the primal and the dual problems, computed using flux and potential reconstructions. In all cases, we prove that the goal-oriented error can be split into a fully computable estimator and a remainder term that can be bounded above by computable energy-based estimators. We present some numerical tests to underline the capability of this goal-oriented estimator in different contexts: reaction-diffusion problems, heat equation, and harmonic formulation of eddy current problems.

These results are based on joint works with E. Creusé (DMATHS, CERAMATHS, UPHF) and Z. Tang (L2EP, ULille).