Symposium of the International Association for Boundary Element Methods

Paris, June 26-28 2018

Book of abstracts (Version June, 20th 2018)

Preface

Dear colleagues,

A very warm welcome to the IABEM symposium 2018 in Paris.

This symposium series has a long history and serves as an informal meeting for researchers working in the wide field of boundary integral/element methods. I am quite happy to see in the program that this meeting also perpetuates a very unique tradition of having participants from both applied mathematics and engineering. This allows fruitful interdisciplinary discussions and helps initiating interdisciplinary collaborations.

The IABEM community is an open community without any fees or strict memberships. The main focus of IABEM is to bring together researchers, stimulate scientific interactions, and provide with the symposium a stage for presenting new ideas for BIE/BEM. The first IABEM symposium, initiating this series, took place in Rome in 1990. The IABEM community has already been in Paris in 1998 and I have a pleasant memory of this symposium. Now, after 20 years we are back in Paris and two young colleagues, Stéphanie Chaillat and Xavier Claeys, have taken the burden to organise the symposium. Let us thank both for their excellent preparation of this meeting. The community also thanks their organisations for supporting us: the Institut National de Recherche en Informatique et Automatique (INRIA), Sorbonne Université, Laboratoire Jacques-Louis Lions, and the Agence Nationale de la Recherche (ANR).

I wish all participants a very informative meeting, good discussions, and a pleasant time in Paris.

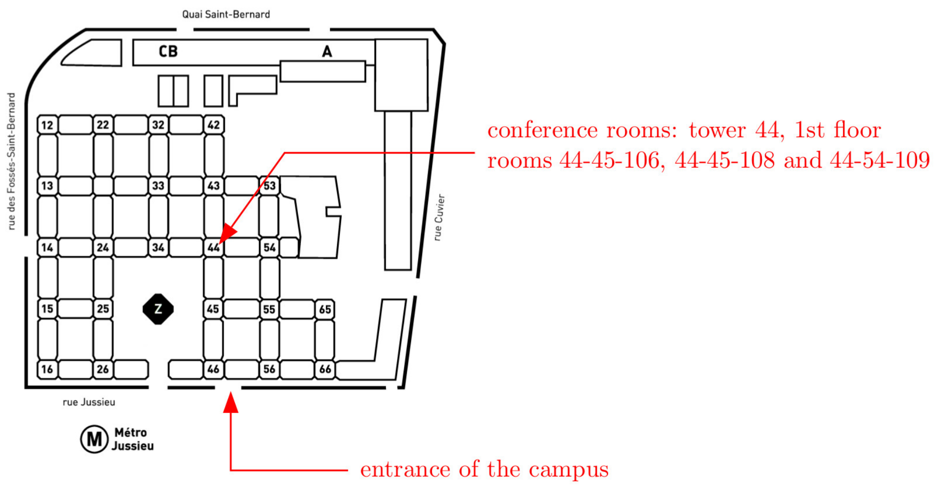
Martin Schanz (President of IABEM)

Acknowledgements

We would like to thank the following institutions for their financial support in the organisation of IABEM 2018: Institut National de Recherche en Informatique et Automatique (INRIA), Sorbonne Université and Laboratoire Jacques-Louis Lions (LJLL), and Agence National de la Recherche (ANR).

How to access the conference rooms on the campus of Jussieu

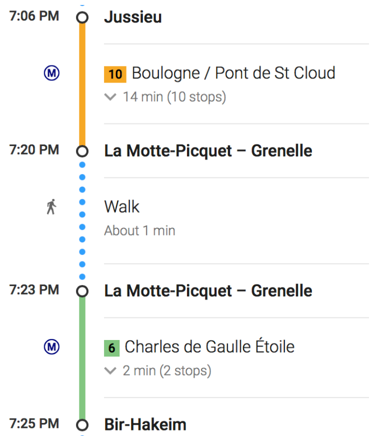
The conference will take place in rooms 44-45-106, 44-45-108 and 44-54-109 of the Jussieu campus. These rooms can be accessed by entering Tower 44 at ground level, taking the stairs or the elevator, and going to the first floor.



How to go to the conference dinner

The conference dinner will take place at the restaurant «Bistro Parisien» (`https://www.bateauxparisiens.com/en/le-bistro-parisien.html`) located at the foot of the Eiffel Tower and alongside the Seine river:

Bistro Parisien
Port de la Bourdonnais, 75007 Paris



The closest metro station is the stop Bir-Hakeim of line 6. From the Jussieu station, take line 10 bound for Boulogne, get off at La Motte-Picquet - Grenelle, then take line 6 bound for Charles de Gaulle Etoile and get off at Bir-Hakeim.

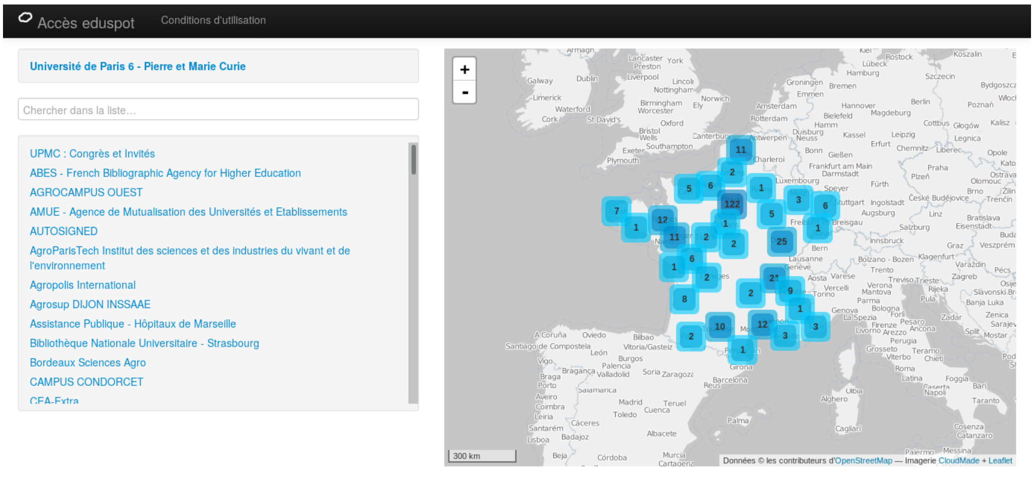
Wifi and internet access

In the conference rooms you can access internet via eduroam. If, for some reason, eduroam connection fails, you can connect via the eduspot network:

1) connect to the network named "eduspot" (SSID access)
2) open a browser, you should be redirected to an authentication portal
3) on this portal you first have to select an institution, choose "UPMC: Congrès et Invités" (see picture below)
4) you will then be asked for a login and password that you can obtain at the registration desk

With eduspot network, if you encounter difficulties for accessing the authentication portal (step 2 above), try entering `http://www.upmc.fr` in the url bar of your browser.

After a long connection without any interruption, the eduspot connection might break down. In this situation, turn off your connection to the eduspot network for 15 minutes and try to connect anew.



Conference program

Tuesday June 26 2018
Morning Sessions

08h30 - 08h50 Welcome of participants
08h50 - 09h00 Opening Session, Room 44-45-106

09h00 - 10h40 Room 44-45-106 - Chairperson: E. Darrigrand

09h00- S.E. Mikhailov: *Boundary-Domain Integral Equations for Stokes and Brinkman Systems with Variable Viscosity in* L_p*-based spaces on Lipschitz Domains*
09h25- A. Ayala: *Linear time CUR approximation of BEM matrices*
09h50- R. Haqshenas: *A fast coupled boundary element formulation for trans-abdominal high-intensity focused ultrasound therapy*
10h15- A. Dansou: *Modeling crack propagation in 3D heterogeneous multi-cracked roads by Fast Multipole Symmetric Galerkin Boundary Element Method*

09h00 - 10h40 Room 44-45-108 - Chairperson: M. Karkulik

09h00- R. Hiptmair: *First-Kind Galerkin Boundary Element Methods for the Hodge-Laplacian*
09h25- M. Bonnet: *Asymptotic expansion of the Maxwell integral equation formulation for the eddy current regime*
09h50- G. Of: *Some boundary element methods for multiply-connected domains*
10h15- P. Musolino: *Converging expansions for Lipschitz self-similar perforations of a plane sector*

10h45 - 11h15 Coffee Break

11h15 - 12h55 Room 44-45-106 - Chairperson: C. Jerez-Hanckes

11h15- S. Falletta: *Wavelets and convolution quadrature for a time domain boundary integral formulation of the wave equation*
11h40- E.P. Stephan: *Time Domain BEM for Fluid-Structure Interaction*
12h05- D. Pölz: *Collocation Methods for Retarded Potential Boundary Integral Equations with Space-Time Trial Spaces*
12h30- S. Dohr: *Parallelized space-time boundary element methods for the heat equation*

11h15 - 12h55 Room 44-45-108 - Chairperson: M. Darbas

11h15- E. Demaldent: *Multi-trace boundary integral formulations with eddy current models*
11h40- S. Adrian: *Refinement-Free Preconditioning Strategies for the Electric Field Integral Equation*
12h05- P. Escapil-Inchauspé: *Fast Calderón Preconditioning for the EFIE*
12h30- R. van Venetië: *Optimal preconditioning of operators of negative order*

13h00 - 14h10 Lunch

Tuesday June 26 2018
Afternoon Sessions

14h10 - 15h50 Room 44-45-106 - Chairperson: M. Bonnet

14h10- F. Le Louër: *Material derivatives of boundary integral operators in electromagnetism and applications*
14h35- K. Nakamoto: *A shape and topology optimisation using the BEM and an explicit boundary expression with the level set method*
15h00- M.L. Rapun: *Solving inverse multiple scattering problems in three-dimensional electromagnetism by topological gradient methods*
15h25- H.B. Chen : *Toward the optimization of acoustic performance using boundary element method*

14h10 - 15h50 Room 44-45-108 - Chairperson: B. Thierry

14h10- C. Erath: *Approximation of a parabolic-elliptic interface problem with a non-symmetric FEM-BEM and backward Euler coupling approach*
14h35- R. Schorr: *Stable non-symmetric coupling with the boundary element method for a convection-dominated parabolic-elliptic interface problem*
15h00- V. Dominguez: *An overlapped BEM-FEM coupling for simulating acoustic wave propagation in unbounded heterogeneous media*
15h25- L. Desiderio: *BEM-FEM coupling for estimating anchor losses in MEMS*

15h50 - 16h20 Coffee Break

16h20 - 18h00 Room 44-45-106 - Chairperson: N. Nishimura

16h20- E. Rejwer: *On increasing effciency of kernel-independent fast multipole method in 2D and 3D problems*
16h45- J. Dölz: *Interpolation-based H^2-compression of Higher Order Boundary Element Methods on Parametric Surfaces*
17h10- C. Jelich: *Inverse Fast Multipole Method Applied to the Galerkin Boundary Element Method*
17h35- G. Martinsson: *Accelerated Direct Solvers for Boundary Integral Equations*

16h20 - 18h00 Room 44-45-108 - Chairperson: N. Heuer

16h20- M. Ganesh: *A class of forward and inverse algorithms for a stochastic wave propagation model*
16h45- T. Hirai: *An isogeometric BEM for a 3D doubly-periodic PEC surface in electromagnetism*
17h10- G. Beer: *Isogeometric boundary element method for problems with inelastic inclusions*
17h35- F. Wolf: *Isogeometric Boundary Element Methods for Electromagnetic Problems: Discretisation and Numerical Examples*

18h00 - 19h00 IABEM General Assembly

Wednesday June 27 2018
Morning Sessions

08h30 - 08h45 Welcome of participants

08h45 - 10h25 Room 44-45-106 - Chairperson: A. Gillman

08h45- P. Zaspel: *Scalable parallel BEM solvers on many-core clusters*
09h10- Y. Matsumoto: *A Fast Direct Solver for the One-Periodic Transmission Problems Formulated with the Multi-Trace Boundary Integral Equation*
09h35- J. Zapletal: *Vectorized approach to the evaluation of boundary integral operators*
10h00- F. Kpadonou: *Efficient parallel implementation of H-matrix based solvers for 3D Helmholtz and elastodynamic oscillatory kernels*

08h45 - 10h25 Room 44-45-108 - Chairperson: S. Chandler-Wilde

08h45- T. Chaumont-Frelet: *High frequency behaviour of corner singularities in Helmholtz problems*
09h10- E. Spence: *The Helmholtz h-BEM: what can be proved about the pollution effect and the behaviour of GMRES?*
09h35- S. Baydoun: *A Pollution Effect Induced by Numercial Damping in the Acoustic Boundary Element Method for Duct Problems*
10h00- E. Parolin: *A Hybrid Numerical-Asymptotic Collocation BEM for High-Frequency Scattering by 2D Planar Screens*

08h45 - 10h25 Room 44-54-109 - Chairperson: A. Sellier

08h45- J. Ravnik: *Boundary element based solution of Navier-Stokes equations with variable material properties*
09h10- J. Tibaut: *Acceleration of the boundary-domain integral representation of the velocity-vorticity form of Navier-Stokes equations*
09h35- J. Watson: *Boundary elements for surfaces in contact in three dimensions*
10h00- L. Gray: *Volume Integration for the 3D Stokes Equation*

10h30 - 11h00 Coffee Break

Wednesday June 27 2018
Morning Sessions

11h00 - 12h40 Room 44-45-106 - Chairperson: F. Andriulli

11h00- M. Darbas: *Analytic preconditioners for 3D high-frequency elastic scattering problems*
11h25- B. Thierry: *Single scattering preconditioner applied to boundary integral equations*
11h50- C. Urzua-Torres: *Preconditioning for the Electric Field Integral Equation on Screens*
12h15- A. Molavi Tabrizi: *Modeling multiscale interface phenomena using nonlinear transmission conditions*

11h00 - 12h40 Room 44-45-108 - Chairperson: M. Ganesh

11h00- D. Hewett: *Scattering by Fractal Screens and Apertures: I - Functional Analysis*
11h25- S. Chandler-Wilde: *Scattering by Fractal Screens and Apertures: II - Numerical Computation*
11h50- A. Zemlyanova: *Singular integral equations method for a fracture problem with a surface energy in the Steigmann-Ogden form on the boundary*
12h15- T. Maruyama: *Application of numerical continuation method to BIE for steady-state wave scattering by a crack with contact acoustic nonlinearity*

11h00 - 12h40 Room 44-54-109 - Chairperson: J. Ravnik

11h00- A. Sellier: *Fundamental coupled MHD creeping flow and electric potential for a conducting liquid bounded by a plane slip wall*
11h25- H. Fendoglu: *MHD flow in a rectangular duct with a perturbed boundary*
11h50- K. Yang: *Radial integration BEM for nonlinear heat conduction problems with temperature-dependent conductivity*
12h15- J. Zhang: *How to achieve the goal of 5aCAE based on BIE*

12h40 - 14h00 Lunch

Wednesday June 27 2018
Afternoon Sessions

14h00 - 16h05 Room 44-45-106 - Chairperson: O. Steinbach

14h00- F. Amlani: *Anisotropic mesh adaptation for 3D accelerated high-order boundary element methods in acoustics*
14h25- S. Schimanko: *Adaptive BEM with inexact PCG solver yields almost optimal computational costs*
14h50- A. Haberl: *Adaptive BEM for the Helmholtz equation*
15h15- H. Harbrecht: *Adaptive Wavelet Boundary Element Methods*
15h40- Y. Zhang: *An efficient adaptive solution technique for periodic Stokes flow*

14h00 - 16h05 Room 44-45-108 - Chairperson: S. Rjasanow

14h00- N. Heuer: *A non-conforming domain decomposition approximation for the Helmholtz screen problem with hypersingular operator*
14h25- P. Marchand: *Two-level preconditioning for BEM with GenEO*
14h50- A.S. Bonnet-Ben Dhia: *Coupling BEMs in overlapping domains when a global Green's function is not available*
15h15- B. Caudron: *An optimized domain decomposition method between interior and exterior domains for harmonic, penetrable and inhomogeneous electromagnetic scattering problems*
15h40- V. Mattesi: *A Padé-localized absorbing boundary condition for 2D time-harmonic elastodynamic scattering problems*

14h00 - 16h05 Room 44-54-109 - Chairperson: M. Schanz

14h00- A. Sellier: *Particle-particle interactions in axisymmetric MHD creeping flow*
14h25- M. Ancellin: *Recent developments of the linear potential flow solver NEMOH and its application for the design of wave energy converters*
14h50- E. Darrigrand: *FastMMLib: a generic library of fast multipole methods*
15h15- E. van't Wout: *Using boundary element methods to analyse the low-frequency resonance of fish schools*
15h40- M. Aussal: *FEM-BEM coupling using Gypsilab*

16h05 - 16h30 Coffee Break

Wednesday June 27 2018
Afternoon Sessions

16h30 - 18h10 Room 44-45-106 - Chairperson: D. Hewett

16h30- A. Gibbs: *A new toolbox for highly oscillatory and singular integrals*
16h55- S. Langdon: *Hybrid numerical-asymptotic boundary element methods for high frequency scattering by penetrable convex polygons*
17h20- H. Gimperlein: *Higher-order and adaptive boundary elements for the wave equation*
17h45- B. Gilvey: *Evaluation of highly oscillatory Partition of Unity BEM integrals arising in 2D wave scattering simulations*

16h30 - 18h10 Room 44-45-108 - Chairperson: V. Dominguez

16h30- O. Steinbach: *On the coupling of space-time finite and boundary element methods*
16h55- F.J. Sayas: *Time Domain Boundary Integral Equations for scattering by obstacles with locally homogeneous material properties*
17h20- J. Tausch: *Fast Galerkin BEM for parabolic moving boundary problems*
17h45- L. Desiderio: *A stable 2D energetic Galerkin BEM approach for linear elastodynamic problems*

16h30 - 18h10 Room 44-54-109 - Chairperson: V. Mantic

16h30- K. Kuzmina: *The Hierarchy of Numerical Schemes for Boundary Integral Equation Solution in 2D Vortex Methods at Airfoil Polygonal Approximation*
16h55- S. Veerapaneni: *Integral equation methods for electro- and magneto-hydrodynamics of soft particles*
17h20- C. Jerez-Hanckes: *Boundary Integral Formulation for Helmholtz and Laplace Dirichlet Problems On Multiple Open Arcs*
17h45- M. Scroggs: *Weak imposition of boundary conditions using a penalty method*

20h00 - 23h00 Conference Dinner at "Bistrot Parisien"

Thursday June 28 2018
Morning Sessions

08h30 - 08h45 Welcome of participants

08h45 - 10h25 Room 44-45-106 - Chairperson: G. Of

08h45- V. Mantic: *Complex variable BEM for a Gurtin-Murdoch material surface in the form of a circular arc in an elastic plane under far-field loads*
09h10- A. Furukawa: *A Boundary Element Method for Antiplane Wave Analysis of Frozen Porous Media*
09h35- J.W. Lee: *Combination of the CHIEF and the self-regularization technique for solving 2D exterior Helmholtz equations with fictitious frequencies in the indirect BEM and MFS*
10h00- P. Fedeli: *Application of Boundary Integral Equations to MEMS working in near vacuum*

08h45 - 10h25 Room 44-45-108 - Chairperson: F.J. Sayas

08h45- M. Schanz: *Elastodynamic BE formulation with Runge-Kutta based Generalised Convolution Quadrature Method*
09h10- M. Zank: *Space-Time Variational Formulations for the Wave Equation*
09h35- C. Jerez-Hanckes: *Multiple Traces Formulation and Semi-Implicit Scheme for Modelling Biological Cells under Electrical Stimulation*
10h00- V. Arnautovski-Toseva: *Solving Electromagnetics Problems by Using Mixed Potential Integral Equation*

10h30 - 11h00 Coffee Break

11h00 - 12h40 Room 44-45-106 - Chairperson: E. Spence

11h00- B. Quaife: *A Boundary Integral Equation for the Clamped Bi-Laplacian Eigenvalue Problem*
11h25- T. Führer: *On the coupling of DPG and BEM*
11h50- M. Karkulik: *The inverse of a finite element discretization of the fractional Laplacian can be approximated by H-matrices*
12h15- S. Rjasanow: *Matrix-valued radial basis functions for the BEM treatment of the Lamé system*

11h00 - 12h40 Room 44-45-108 - Chairperson: J. Tausch

11h00- M. Leitner: *Uncoupled Thermoelastic Boundary Element Formulation with Variable Time Step Size*
11h25- K. Niino: *Computation of layer potentials in the BEM with the space-time method for the heat equation in 2D*
11h50- A. Haider: *Data-Sparse Boundary Element Methods for Elastic Waves*
12h15- N. Dumont : *Conceptually Consistent Formulation of the Boundary Element Method and Arbitrarily High Accurate Numerical Integrations for the Analysis of 2D Problems of General Topology and Shape*

12h40 - 14h00 Lunch

Thursday June 28 2018
Afternoon Sessions

14h00 - 15h40 Room 44-45-106 - Chairperson: G. Martinsson

14h00- A. Gillman: *A fast direct solver for boundary value problems on evolving geometries*
14h25- N. Nishimura: *Optimisation of Electromagnetic Metamaterials Using Periodic FMM and Cylindrical-Hole Topological Derivatives*
14h50- M. Oneil: *A Fast Boundary Integral Method for Generating High-order Surface Meshes*
15h15- G. Oelker Silva: *Quantification of the Impact of Small Random Perturbations in Electromagnetic Scattering from Reflective Gratings*

14h00 - 15h40 Room 44-45-108 - Chairperson: R. Hiptmair

14h00- A. Kleanthous: *Electromagnetic scattering by ice crystals and implementation using Bempp*
14h25- M. Issa: *Boundary Element Method for Conductive Thin Layer in 3D Eddy Current Problems*
14h50- Q. Sun: *Wavelength stable field-only boundary regularised integral solution of electromagnetic scattering based on the Helmholtz equation*
15h15- X.W. Gao: *Non-Conventional Boundary Elements and Their Applications in BEM Analysis of Structurally Multi-Scale Problems*

15h40 - 16h15 Coffee Break

16h15 - 17h55 Room 44-45-106 - Chairperson: F. Le Louer

16h15- O.I. Yaman: *Reconstruction of surface impedance functions from the acoustic far field pattern*
16h40- K. Matsushima: *A topology optimisation of elastic wave absorber with the BEM and H-matrix method*
17h05- H. Isakari: *A topology optimisation for photonic crystals using a fast boundary element method*
17h30- A. Lefebvre-Lepot: *The Sparse Cardinal Sine Decomposition (SCSD) and its application to the simulation of suspensions*

16h15 - 17h55 Room 44-45-108 - Chairperson: J. Zhang

16h15- J.R. Poirier: *Boundary Integral Equations and Hierarchical Matrices for a Waveguide Mode Solver*
16h40- C. Ju: *A binary-tree subdivision method for evaluation of nearly singular integrals and singular integrals in 3D BEM*
17h05- B. Chi: *A binary-tree subdivision method for volume integrals in BEM*
17h30- H.F. Peng: *Radial Integration BEM for solving convection-conduction problems*

Abstracts

Refinement-Free Preconditioning Strategies for the Electric Field Integral Equation

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Keywords: Calderón preconditioning, EFIE, integral equation

Different preconditioning techniques have been devised in the past for curing the h-refinement ill-conditioning of the electric field integral equation (EFIE). Among the most popular techniques are multilevel/pre-wavelet preconditioners, which typically lead to a logarithmic bound on the condition number for the asymptotic limit $h \to 0$ (see [1] and references therein), and analytic or Calderón-identity based preconditioners, which lead to a constant bound on the condition number for $h \to 0$ [2,3].

When Calderón techniques are used in a standard Galerkin scheme, the discretization of two EFIE operators and the use of dual elements defined on the barycentric refinement [4] is often required. This, however, increases the computational costs due to the higher number of barycentric degrees of freedom. Although fast matrix-vector multiplication algorithms can be used to alleviate this problem, the necessity of adapting a fast scheme to two different stiffness matrices is still a source of computational overburdens.

In this talk we tackle this problem by presenting a strategy which does not require the use of dual basis functions, and associated barycentric refinements, and still leads to an optimally preconditioned EFIE. The approach is based on implicit quasi-Helmholtz decompositions of the EFIE operator which allow to exploit scalar-instead-of-vector Calderón identities. This allows to link, via spectral equivalences, two primal mesh EFIE matrices with discretizations of the forward and inverse Laplace-Beltrami operators. The overall scheme results in order zero Helmholtz components without use of barycentric meshes.

References

[1] S. B. Adrian, F. P. Andriulli and T. F. Eibert, A Hierarchical Preconditioner for the Electric Field Integral Equation on Unstructured Meshes Based on Primal and Dual Haar Bases, *Journal of Computational Physics* **330** (2017), pp. 365–379.

[2] O. Steinbach, W. Wendland, The Construction of Some Efficient Preconditioners in the Boundary Element Method, *Advances in Computational Mathematics* **9** (1998), pp. 191–216.

[3] X. Antoine, A. Bendali, and M. Darbas, Analytic Preconditioners for the Electric Field Integral Equation, *Advances in Computational Mathematics* **9.61** (2004), pp. 1310–1331.

[4] A. Buffa and S. Christiansen, A Dual Finite Element Complex on the Barycentric Refinement, *Mathematics of Computation* **76.260** (2007), pp. 1743–1769.

Anisotropic mesh adaptation for 3D accelerated high-order boundary element methods in acoustics

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Keywords: accelerated boundary element methods, anisotropic mesh adaptation, high-order methods

There are a number of factors—such as integral representation, numerical implementation and discretization strategy—that determine the effectiveness of boundary element methods (BEMs) for the solution of wave scattering problems in computational physics. Although BEM formulations exactly account for radiation conditions at infinity and advantageously restrict the descritization of a problem domain to that of the boundary alone, standard implementations lead to dense and (possibly) nonsymmetric linear systems whose solutions become prohibitively expensive for large-scale problems. However, several algorithms–such as fast multipole methods [1] and their algebraic counterpart, hierarchical ($\mathcal{H}$-) matrices [2]–have been developed to enable drastic reductions in computational time and memory requirements by approximating the system matrix.

Nevertheless, the exactness of a numerical solution as effected by mesh discretization still poses a challenge to the BEM community. Anisotropic features of a solution (e.g. some elastic materials) as well as discontinuities near geometric singularities (e.g. corners and edges) are difficult to capture and diminish the regularity of the boundary solution and subsequent performance of a BEM. This is particularly true when uniform meshes are employed. To this end, iterative mesh refinement schemes have been constructed to transform an initial mesh into an improved one according to error estimates calculated at each step, with the hope of reducing the number of degrees of freedom required to resolve a solution within a desired level of accuracy. Fewer studies on these strategies have been made for BEMs, and most current BEM adaptivity methods, like those relying on Dörfler marking, have been confined to isotropic techniques. In addition, most works are restricted to Galerkin discretizations and are formulated specifically for a system of underlying equations [3].

In this contribution, we will present recent developments of an *anisotropic* mesh adaptation (AMA) strategy using a metric-based error estimator whose effectiveness was first demonstrated for volumetric (finite element) methods [4] and only recently for BEMs [5]. The methodology is independent of discretization technique as well as the choice of PDE and integral equation formulation, iteratively constructing meshes refined in size, shape and orientation according to an "optimal" metric relying on a reconstructed Hessian of the boundary solution. The resulting adaptation is anisotropic in nature and accommodates geometric complexities that include engineering detail. Realistic examples will be explored in the context of frequency-space scattering problems in acoustics and elastodynamics, demonstrating optimal convergence rates using various BEM accelerations, integral equation representations and finite-element orders.

References

[1] E. Darve, The fast multipole method: Numerical implementation, *J. Comput. Physics* **160** (2000), pp. 195-240.

[2] S. Chaillat et al, Theory and implementation of H-matrix based iterative and direct solvers for Helmholtz and elastodynamic oscillatory kernels. *J. Comput. Physics* **351** (2017), pp. 165-186.

[3] C. Erath et al, Simple error estimators for the Galerkin BEM for some hypersingular integral equation in 2D, *Applicable Analysis* **92** (2013), pp. 1194-1216.

[4] A. Loseille et al, Optimal 3D highly anisotropic mesh adaptation based on the continuous mesh framework, *Proc. in 18th Intl Meshing Roundtable, Salt Lake City, USA, Oct 2009,* pp. 575-594

[5] S. Chaillat et al, Metric-based anisotropic mesh adaptation for 3D acoustic boundary element methods. *Submitted.*

Recent developments of the linear potential flow solver NEMOH and its application for the design of wave energy converters.

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Keywords: potential flow, diffraction-radiation problems, water waves, renewable energy

The simplest model to describe the interaction of Wave Energy Converters (WECs) with ocean waves is linear potential flow theory. Several numerical solvers using the Boundary Element Method exist. The code NEMOH, developed at École Centrale de Nantes, is the only open source among them [1]. Since its release in 2014, it has been widely used for the study of WECs [2], although improvements of the code still need to be made.

In this talk the theory of radiation-diffraction problem for sea keeping studies will be briefly recalled. Then the main characteristics of NEMOH will be introduced, along with its limitations.

The presentation will focus in particular on a recent optimization of the code for symmetric bodies. Unlike the traditional problems of ship-building, the WECs often present symmetric structures: axi-symmetrical buoy, cylindrical shapes or regular array of floating bodies. This symmetry can be used to speed up the resolution, by using the symmetric block Toeplitz structure of the influence matrix [3].

References

[1] A. Babarit and G. Delhommeau, Theoretical and numerical aspects of the open source BEM solver NEMOH, in *Proceedings of the 11th European Wave and Tidal Energy Conference (EWTEC2015)*, 2015.

[2] M. Penalba Retes, T. Kelly and J. Ringwood, Using NEMOH for modelling wave energy converters: A comparative study with WAMIT, in *Proceedings of the 12th European Wave and Tidal Energy Conference (EWTEC2017)*, 2017.

[3] M. Karimi, P. Croaker and N. Kessissoglou, Boundary element solution for periodic acoustic problems, *Journal of Sound and Vibration*, **360**, 2016, pp. 129–139.

Solving Electromagnetics Problems by Using Mixed Potential Integral Equation

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Keywords: electromagnetics, integral equation, frequency domain

Mathematical model

The electromagnetic interactions related to specified boundary value problem leads to mathematical model that is derived by integral equation. Although the approach is quite general, it has to be re-derived based on the type of objects geometries and the corresponding medium properties. Integral equations are firstly used for solving electromagnetic problems of antennas and scatterers. This, so called rigorous electromagnetic model is confirmed as theoretically most accurate, and valid in wide frequency range. In the paper the authors will give an overview of their experience in solving high frequency electromagnetic problems of grounding systems in presence of finitely conducting homogeneous or layered soil. One of the pioneer work in this field is [1] where the mathematical model of a grounding conductor is based on the Electric Field Integral Equation (EFIE). In [2] the authors develop improved mathematical model for high frequency analysis of horizontal grounding systems in layered soil, that is later applied to vertical grounding rods. Both mathematical models are derived in frequency domain by using integral equation formulation for the electric field due to currents and charges along the wire conductors in terms of magnetic vector and electric scalar potentials, so called Mixed Potential Integral Equation (MPIE) [3]. The unknowns in the corresponding integral equation are the currents and charges that are related through the continuity equation. The rigorous treatment of the specific boundary value problem that include the effect of the various medium properties is based on the use of Sommerfeld formulations for the Green's functions of the corresponding Hertz horizontal or vertical dipole as an elementary source of the field [4]. In this work the authors focus on development of mathematical model of complex grounding structures consisted of horizontal, vertical and arbitrarily oriented conductors. Also, the authors are interested on derivation of simplified closed form solutions of Sommerfeld integrals based on quasi-dynamic approximate formulations of the appropriate Green's functions by using method of images and complex images.

Solving the model

The solution of the mathematical model is developed by using the method of moments (MoM) with Galerkin triangular basis functions (dipoles) for the current and pulse functions for the charge, and also triangular testing functions [2]. This leads to a system of linear equations, where the unknown are the current coefficients of the basis distribution functions. The matrix equation may be easily solved by standard numerical procedures.

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FEM-BEM coupling using Gypsilab

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Keywords: FEM/BEM coupling, $\mathcal{H}-$matrix, Matlab prototyping.

Introduction

A large class of problem in multiphysics may be solved numerically using a coupled FEM-BEM formulation. As an example of such, we may consider the electromagnetic scattering when different materials are involved (PEC and dielectric for instance). However, FEM-BEM coupling still suffers from the lack of available softwares and especially high level generic programming languages able to handle such formulations. One of the possible reasons is that FEM and BEM respectively often need different numerical and algorithmic tools (sparse matrix software for the FEM, compression techniques for the BEM) that still require some expertise and are not yet widespread in the respective communities.

Gypsilab

Gypsilab [4] is a recent prototyping environment developed by the authors, designed to solve various numerical problems including 3D variational formulations coming from finite element formulations (FEM) and/or integral equations (BEM). Entirely written in Matlab and available in open-source (GPL 3.0), it provides the user with a complete environment and is able to solve numerical problems of reasonable sizes (up to millions of unknowns in FEM and hundreds of thousands unknowns in BEM). A simple interface allows the user to write high level variational formulations, such as in [1–3], and solve complex problems without entering inside the architecture of the software, and with a uniform language that takes benefit of Matlab strengths.

In order to achieve good performances, every functionality has been vectorized, and a specific hierarchical compression library, including the full algebra, openHmx, has been implemented [5]. A new type of matrix, the $\mathcal{H}$-Matrix, becomes available completing transparently the classical Matlab matrix classes (full or sparse).

In this talk, we show how Gypsilab can solve multi-physics problems, for example in acoustics, vibro-acoustics, electromagnetism, fluid mechanics, etc. Moreover, we will focus on a new approach offered by Gypsilab that permits to mix the sparse and full matrices coming from the FEM and BEM parts of the problem respectively, into a unique $\mathcal{H}$-matrix, that can be LU-factorized and exactly inverted. In particular, we will insist on the modularity and the genericity of the approach.

Acknowledgment

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Symposium of the International Association for Boundary Element Methods

Linear time CUR approximation of BEM matrices

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Keywords: CUR decomposition, Linear time algorithms, BEM matrices.

In this talk we propose a linear cost CUR decomposition for admissible matrices obtained from the Hierarchical form of Boundary Element matrices. We propose a new approach using the gravity points of clusters in admissible interaction blocks to select the most significant columns and rows. This strategy is tailored to Boundary Element Methods (BEM) since it uses directly and explicitly the cluster tree containing information from the problem geometry. Our algorithm has a precision comparable with the truncated QR factorization and when compared to the well-known Adaptive Cross Approximation (ACA) with partial pivoting, we show that our algorithm improves, in general, the convergence error and overcomes some cases where ACA fails. A theoretical bound on the approximation error is provided and numerically compared with the bounds from the maximal volume and maximal projective volume algorithm. The performance of our algorithm is also compared with state of the art algorithms on traditional BEM problems defined over different geometries. Finally, we provide a brief analysis on its randomized version.

Acknowledgement

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A Pollution Effect Induced by Numercial Damping in the Acoustic Boundary Element Method for Duct Problems

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Keywords: Kirchhoff-Helmholtz integral equation, collocation boundary element method, pollution effect, numerical damping

A brief review of the pollution effect in the boundary element method

The pollution effect is a widely known phenomenon occurring when using the finite element method (FEM) for wave problems [3]. It refers to a numerical error growing with the number of waves in the computational domain. In case of time-harmonic FEM, pollution is caused by numerical dispersion, which is introduced by the wavenumber in the discrete setting and results in an accumulated phase error. For what concerns the acoustic boundary element method (BEM), numerical dispersion is rather small and negligible [5]. However, the BEM suffers from a different kind of pollution effect, which is the result of numerical damping [1, 2, 4].

Investigation of the pollution effect induced by numerical damping on the example of acoustic duct problems

In this paper, numerical damping in the collocation BEM is investigated for plane sound waves in rectangular ducts subjected to rigid and absorbing boundary conditions. Different lengths of the duct and different meshes of linear and quadratic continuous elements are studied. The extent of numerical damping is quantified based on a damping model with exponential decay. In the case of rigid boundary conditions, the full width at half maximum method is applied to the resonance peaks of the sound pressure amplitude. For traveling waves, numerical damping is determined by relating the amplitude decay to the analytical solution. It is found that the observed damping increases exponentially with respect to frequency and that upper bounds can be found for constant element-to-wavelength ratios. The results show that numerical damping can lead to excessive errors in case of large domains accommodating a large number of waves. Therefore, the influence of numerical damping needs to be considered when evaluating the appropriateness of boundary element meshes, and the common practice to employ a certain number of elements per wavelength should be applied with care.

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Isogeometric boundary element method for problems with nonlinear inclusions

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Keywords: Isogeometric analysis, Inclusions

In isogeometric analysis the same basis functions as in Computer Aided Design (CAD) are used for the analysis. These are Non-Uniform Rational B-Splines or NURBS. The advantage of using these functions is that geometry information can be taken directly from CAD data with the possibility of avoiding mesh generation. In addition, the use of these functions for the approximation of the unknowns offers greater flexibility in the refinement procedures and also result in a reduction of problem size. The authors have published previously on the subject of isogeometric BEM (IGABEM) and have shown that isogeometric procedures can also be used for the evaluation of volume integral that arise when non-linear inclusions are considered [1–4].

The aim of the paper is to show, on a number of practical examples, the advantages of the IGABEM in terms of:

- Description of smooth geometries with few parameters
- Obtaining good quality solutions with few unknowns
- Easy definition of inclusions and areas of material non-linear behaviour

The examples include 2-D and 3-D simulations in geomechanics and in viscous fluid flow and a comparison is made with conventional BEM approaches.

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Asymptotic expansion of the Maxwell integral equation formulation for the eddy current regime

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Keywords: Maxwell's equations, PMCHWT integral equation, eddy currents, asymptotic expansion

The current advent of mid-frequency testing for complex media (e.g. made of fibres with diverse conductivities) spurs the development of computational formulations allowing transition from the full Maxwell equation system to modified versions of eddy current (EC) models, for which the dielectric permittivity is classically set to zero. For testing simulations based on the boundary element method (BEM), insight into such model transition is necessary for addressing the scaling disparities between components of the surface currents (primary unknowns in e.g. BEM formulations of PMCHWT type, see [1, Chap. 4]) that severely affect solution accuracy in the low-frequency, high-conductivity limit. For instance, splitting currents into zero-divergence (loop) and complementary (star) components [4] and then applying blockwise solution algorithms to the BEM system [3] allow the Maxwell BEM problem to perform reliably under EC-type conditions.

Another approach, which is the subject of this communication, consists in deriving asymptotic expansions of the Maxwell PMCHWT integral problem with respect to a non-dimensional parameter γ, set here to $\gamma := \sqrt{\omega\varepsilon_0/\sigma}$. We have rigorously proved that with this choice the integral problem for the classical EC model [2] is obtained in the limiting case $\gamma \to 0$, with our study including estimates in γ of the expansion remainders. The leading and remainder orders in γ of the surface current densities depend on the current component. We plan to state the main theoretical results and to demonstrate them on illustrative numerical examples such as that of Fig. 1, where mathematically established error estimates in γ are seen to be reproduced by the numerical results. Additionally, an extension of our asymptotic model to testing configurations involving a second medium (taken as non-conducting and with a high magnetic permeability) will be discussed.

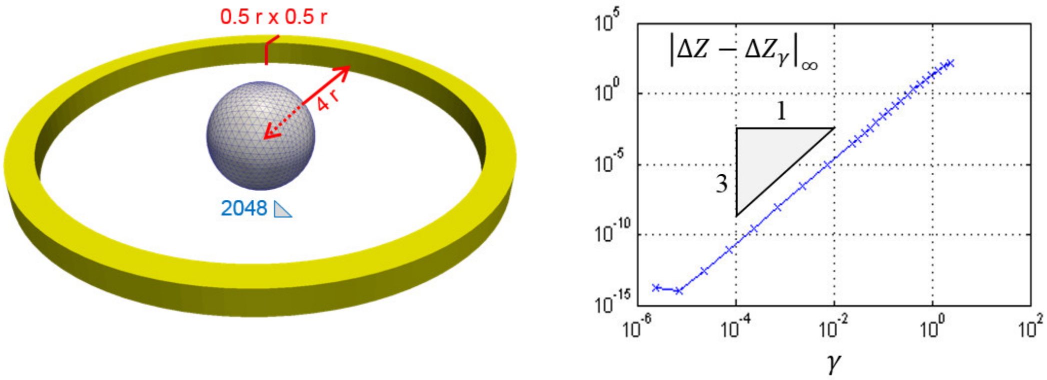

Figure 1: *Conducting sphere excited by a coil: test configuration (left); error on the impedance variation ΔZ_γ, evaluated using the asymptotic model as a function of γ, against its full-Maxwell counterpart ΔZ (right). The observed $|\,\Delta Z_\gamma - \Delta Z\,| = O(\gamma^3)$ behavior matches its theoretical prediction.*

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Coupling BEMs in overlapping domains when a global Green's function is not available

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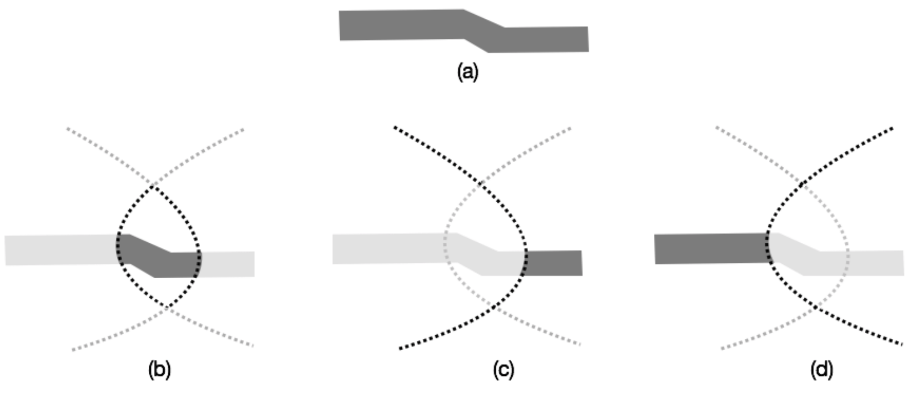
Keywords: Green's function, complex domain, overlapping

We consider in this work problems for which the Green's function is not available, so that classical Boundary Integral equation methods are not applicable. Let us mention for instance the junction of two different stratified media (tapered optical fibers in integrated optics or junction of two topographic elastic surfaces in geophysics).

To this end, we propose a generalization of the Half-Space Matching method [1, 2]. Initially proposed for the scalar wave propagation in a locally perturbed 2D homogeneous medium, this approach consists in coupling several plane-wave representations of the solution in half-spaces surrounding the defect, with a Finite Element computation of the solution around the defect. Ensuring that all these representations match, in particular in the infinite intersections of the half-spaces, leads to a formulation which couples, via Fourier-integral operators, the solution in a bounded domain including the defect and the Dirichlet traces of the solution on the edge of the half-planes. In this version of the Half-Space Matching method, the use of a partial Fourier transform restricts the exterior sub-domains to be Half-Spaces.

In this work, by replacing the Fourier representations by integral representations, we are able to use more general unbounded overlapping sub-domains. We choose the sub-domains in such a way that an explicit Green's function is available for each subdomain. For instance, for the configuration described above (figure (a)), it suffices to introduce two infinite sub domains, each of them containing only one stratification (figures (c) and (d)) and a bounded domain containing the junction (figure (b)). The formulation couples the solution in the bounded domain with the single and double layer potentials on each boundary of the sub-domains. The main drawback is that these boundaries are infinite but it is well-handled with the Half-space Matching Method. The approximation relies on a FE discretisation of the volume unknown and a truncation and a discretization of the boundary/surface unknowns. The choice of the discretization parameters will be discussed and numerical results will be shown.



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An optimized domain decomposition method between interior and exterior domains for harmonic, penetrable and inhomogeneous electromagnetic scattering problems

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Keywords: Harmonic electromagnetic scattering, FEM/BEM coupling, Domain decomposition

Introduction

This ongoing work focuses on the numerical resolution of three-dimensional harmonic electromagnetic scattering problems for which the scatterer is dielectric and inhomogeneous. Denoting by Ω_- the scatterer, Ω_+ the exterior domain, Γ the surface of the scatterer, $\mathbf{n}$ the outward-pointing unit normal vector to Ω_- and $(\mathbf{E}_\pm;\mathbf{H}_\pm)$ the electromagnetic fields within $\Omega_\pm$, such problems read:

$$\begin{matrix} \mathbf{rot}\,\mathbf{E}_\pm - ik_\pm Z_\pm \mathbf{H}_\pm = \mathbf{0} \\ \mathbf{rot}\,\mathbf{H}_\pm + ik_\pm Z_\pm^{-1}\mathbf{E}_\pm = \mathbf{0} \end{matrix} \ \text{in } \Omega_\pm \quad , \quad \begin{matrix} \mathbf{E}_- \wedge \mathbf{n} = \mathbf{E}_+ \wedge \mathbf{n} + \mathbf{E}_i \wedge \mathbf{n} \\ \mathbf{H}_- \wedge \mathbf{n} = \mathbf{H}_+ \wedge \mathbf{n} + \mathbf{H}_i \wedge \mathbf{n} \end{matrix} \ \text{on } \Gamma. \tag{1}$$

The interior wave number, k_-, and impedance, Z_-, could be functions on Ω_-, $(\mathbf{E}_i;\mathbf{H}_i)$ is the incident wave and the scattered field $(\mathbf{E}_+;\mathbf{H}_+)$ should satisfy the Silver-Müller radiation condition. A standard approach to solve (1) consists in combining integral equations for the exterior domain and a variational formulation for the interior domain resulting in a strong coupling of the finite (FEM) and boundary (BEM) element methods. However, there is a drawback to this approach. Implementing the strong FEM/BEM coupling by combining two pre-existing solvers, a FEM solver for arbitrary interior problems and a BEM solver for arbitrary exterior problems, will most-likely require serious modification of the pre-existing solvers' source codes. We introduce a weak FEM/BEM coupling, based on domain decomposition (DD), allowing to couple the pre-existing solvers with minimal implementation effort.

The weak FEM/BEM coupling

The weak FEM/BEM coupling is an equivalent reformulation of the transmission problem (1):

$$(\mathbf{Id}-\mathbf{S}_\pi)\begin{pmatrix}\mathbf{g}_-\\ \mathbf{g}_+\end{pmatrix} = \begin{pmatrix}\mathbf{H}_i\wedge\mathbf{n}+\mathbf{T}_-(\mathbf{E}_i\wedge\mathbf{n})\\ -\mathbf{H}_i\wedge\mathbf{n}+\mathbf{T}_+(\mathbf{E}_i\wedge\mathbf{n})\end{pmatrix} \quad , \quad \begin{matrix}\mathbf{S}_\pi = \begin{pmatrix}0 & \mathbf{S}_+\\ \mathbf{S}_- & 0\end{pmatrix} \\ \mathbf{S}_\pm = \mathbf{Id}\pm(\mathbf{T}_+ + \mathbf{T}_-)\mathbf{R}_\pm\end{matrix} \quad , \quad \mathbf{g}_\pm = (\mathbf{H}_\pm\wedge\mathbf{n})\mp\mathbf{T}_\pm(\mathbf{E}_\pm\wedge\mathbf{n}).$$

The operators $\mathbf{T}_\pm$ transfer information between Ω_- and Ω_+ in a DD-like manner, $\mathbf{g}_\pm$ should be understood in terms of traces on Γ and $\mathbf{R}_\pm$ are the continuous analogs of the pre-existing solvers:

$$\mathbf{R}_\pm\mathbf{g} = \tilde{\mathbf{E}}_\pm\wedge\mathbf{n} \quad , \quad \begin{matrix}\mathbf{rot}\,\tilde{\mathbf{E}}_\pm - ik_\pm Z_\pm\tilde{\mathbf{H}}_\pm = \mathbf{0}\\ \mathbf{rot}\,\tilde{\mathbf{H}}_\pm + ik_\pm Z_\pm^{-1}\tilde{\mathbf{E}}_\pm = \mathbf{0}\end{matrix} \ \text{in } \Omega_\pm \quad , \quad (\tilde{\mathbf{H}}_\pm\wedge\mathbf{n})\mp\mathbf{T}_\pm(\tilde{\mathbf{E}}_\pm\wedge\mathbf{n}) = \mathbf{g} \text{ on } \Gamma.$$

The weak FEM/BEM coupling should be solved iteratively, typically using the GMRES method. Choosing $\mathbf{T}_\pm$ as proper Padé approximations of approximate Magnetic-to-Electric operators [1]:

$$\frac{1}{Z_\pm}\left(\mathbf{Id}+\frac{\mathbf{\Delta}_\Gamma}{\tilde{k}_\pm^2}\right)^{-\frac{1}{2}}\left(\mathbf{Id}-\frac{1}{\tilde{k}_\pm^2}\mathbf{rot}_\Gamma\mathrm{rot}_\Gamma\right)(\cdot\wedge\mathbf{n}) \quad , \quad \tilde{k}_\pm = k_\pm + i\epsilon_\pm,$$

the regularizing functions $\epsilon_\pm$ being strictly positive, ensures a good GMRES convergence.

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Scattering by Fractal Screens and Apertures: II - Numerical Computation

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Keywords: Helmholtz equation, fractal screens, Sobolev spaces, BEM

We consider time-harmonic acoustic scattering in $\mathbb{R}^{n+1}$, $n = 1, 2$, by planar screens and apertures. When the screen/aperture $\Gamma \subset \mathbb{R}^n$ is a Lipschitz open set, the associated boundary integral equation formulations are classical: the boundary integral operator for the Dirichlet screen problem is the single layer operator $S : \tilde{H}^{-1/2}(\Gamma) \to H^{1/2}(\Gamma)$, and for the Neumann screen problem it is the hypersingular operator $T : \tilde{H}^{1/2}(\Gamma) \to H^{-1/2}(\Gamma)$.

However, when Γ is open but non-Lipschitz (e.g. with a fractal boundary, like the Koch snowflake) the classical formulations may fail to be well posed. And when Γ is a closed set with empty interior (e.g. a fractal set such as a Cantor dust or Sierpinski triangle) then it is not obvious how one should even impose boundary conditions, let alone formulate the integral equations.

It turns out that for an arbitrary $\Gamma \subset \mathbb{R}^n$ there are in general an uncountably infinite number of possible formulations as boundary integral equations, all producing distinct scattered fields. The physically correct choice can be determined by limiting geometry principles, viewing the rough set Γ as a suitable limit of a sequence of smoother sets Γ_j.

In this talk, a follow-up to the companion talk given by D.P. Hewett on functional analytic questions, we will show BEM numerical simulations, illustrating convergence of numerical solutions as the sequence of smoother sets Γ_j converges to the rough limiting set Γ. A particular emphasis will be on cases where the sequence of smoother sets Γ_j is a sequence of pre-fractals converging to a limit set Γ that is fractal or has fractal boundary, including examples where the limit set is a fractal with no interior points and zero surface measure, e.g. Γ is a Cantor dust or Sierpinski triangle.

These numerical simulations, of interest in their own right, will also illustrate theorems in [1, 2] on convergence as $j \to \infty$ of solutions of boundary integral equations on smoother sets Γ_j to limiting solutions on a limiting rough set Γ. More particularly, they will illustrate recently proved theorems [3], using ideas of Mosco convergence, on convergence of the BEM solution computed with a step-size h_j on the smoother set Γ_j, in the limit as $j \to \infty$ and $h_j \to 0$, with the size of h_j dependent on j in a carefully controlled way.

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High frequency behaviour of corner singularities in Helmholtz problems

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Keywords: Corner singularities, Pollution effect, Helmholtz problems

We study the acoustic Helmholtz equation set in the exterior of a convex polygon $K \subset \mathbb{R}^2$:

$$\begin{cases} -k^2 u - \Delta u &= 0, & \text{in } \mathcal{O} = \mathbb{R}^2 \setminus K, \\ u &= e^{ik\mathbf{d}\cdot\boldsymbol{x}}, & \text{on } \partial K, \\ \boldsymbol{\nabla} u \cdot \dfrac{\boldsymbol{x}}{|\boldsymbol{x}|} - iku &= o\left(|\boldsymbol{x}|^{-1/2}\right), & \text{as } |\boldsymbol{x}| \to \infty. \end{cases}$$

It is well known that, in general, u exhibits singularities at the vicinity of the corners of K [2]. Specifically, if we denote by $\{\boldsymbol{x}_j\}_{j=1}^N$ the vertices of K, we have

$$u = \sum_{j=1}^N c_j s_j + u_R,$$

where $u_R \in H^2_{loc}(\mathcal{O})$, $s_j \in H^{1+\alpha_j}_{loc}(\mathcal{O})$ with $1/2 < \alpha_j < 1$ and $c_j \in \mathbb{C}$. The function s_j is k independent, and represents the singularity associated with $\boldsymbol{x}_j$. It has regularity $1 + \alpha_j$, where α_j depends on the angle of ∂K at $\boldsymbol{x}_j$. The coefficients c_j depend linearly on the right-hand-side, and implicitly on the wavenumber.

In the first part of the talk, we analyze the behaviour of c_j and u_R with respect to the wavenumber. Specifically, we show that

$$\|u_R\|_{2,B} \leq Ck^2, \quad |c_j| \leq Ck^{1/2+\alpha_j}, \tag{2}$$

where B is a ball containing K and $j = \{1, \ldots, N\}$. The key observation is that the singularities (which are the most complex part to numerically approximate) have a better scaling than the regular component with respect to the wavenumber.

In the second part of the talk, we examine the impact of (2) on error estimates for numerical discretizations. We focus on finite element methods (the Sommerfeld condition being replaced by a first-order absorbing condition), but estimate (2) also permits to obtain sharp error estimates for boundary element methods. Our key result is the error estimate

$$\frac{|u - u_h|_{1,\Omega}}{|u|_{1,\Omega}} \leq C\left(k^{-1/2}k^\alpha h^\alpha + kh + k^3h^2\right), \tag{3}$$

for linear Lagrange finite elements on uniform meshes (without refinements close to the corners of K). The term $k^{-1/2}k^\alpha h^\alpha$ (with $\alpha = \min_j \alpha_j$) is not present when ∂K is smooth [1], and represents the impact of the singularities of u. It follows that if the number of discretization points per wavelength is bounded from below ($kh \leq C$), then the additional term tends to zero as $k^{-1/2}$ for high-frequency problems.

From (3) and numerical experiments, we conclude that for high-frequency problems the effect of the singularities on numerical discretizations is invisible (at least for low order discretization methods), unless very high accuracy is required.

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Toward the optimization of acoustic performance using boundary element method

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Keywords: topology optimization, shape optimization, boundary element method, fast multipole method, isogeometric analysis

Passive noise and vibration control of structures is of great concern in engineering problems, which consists in the problem of acoustic optimization [1, 2]. This work aims at improving the acoustic performance of system using optimization technique, where the boundary element method (BEM) is employed for acoustic analysis. Our work consists of two main parts. In part I, we conduct porous material distribution and shape optimization of structures for sound scattering problems. Based on the solid isotropic material with penalization (SIMP) method, the topology optimization is performed by setting the artificial element densities of porous material and damping material as design [3]. The shape optimization is performed via isogeometric analysis (IGA) [4], where IGA provides exact geometric representations. Furthermore, refinements and shape changes for the design model are easily implemented without mesh regeneration, which significantly reduces subsequent communication with the original description. In this part, the fast sensitivity analysis approach based on the fast multipole method (FMM) and analytical method, including direct differentiation method and adjoint variable method, is developed to calculate the sensitivities of objective function with respect to design variables. The FMM is applied to accelerate the matrix-vector and vector-matrix products to improve overall computational efficiency. After the acoustic state and sensitivity information are obtained, the method of moving asymptotes (MMA) is used for solving the optimization problem to find the optimal solution. We validate the proposed optimization approach through a number of numerical simulations for 2D shaped noise barrier. In part II, a topology optimization approach is proposed for the optimal design of bi-material distribution on underwater shell structures. The coupled finite element method/boundary element method scheme is used for the system response analysis, where the strong interaction between the structural domain and acoustic domain is considered. The design variable is the artificial density of design material element in a bi-material model constructed by the SIMP method, and the minimization of sound power level is chosen to be the design objective. The adjoint variable method is employed to calculate the sensitivity of the objective function with respect to the design variables. Similarly, the MMA solver is also adopted here. Numerical examples are provided to illustrate the correctness of the sensitivity analysis approach and the validity of the proposed optimization procedure. In the further work, the efficiency improvement of the topology optimization of the coupled structural-acoustic system by the FMM and the reduced finite element model will be investigated.

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Modeling crack propagation in 3D heterogeneous multi-cracked structures by Fast Multipole Symmetric Galerkin Boundary Element Method.

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Keywords: Symmetric Galerkin BEM, Fast Multipole Method, Fracture Mechanics.

Context and Motivatons

In a context where road networks are aging, glass fiber grids reinforcement is one of the only effective solutions to reinforce highly cracked roads. The development of these solutions is currently hampered, due to the lack of widely-accepted methods for design. The main objective of this work is to develop a code to calculate multi-cracked structures in general, and multi-cracked roads taking into account fiberglass grids, in particular. Over recent decades, the Boundary Element Method has become an important technique in the computational solution of a number of physical problems: electromagnetic, elastodynamic [3]... The approach adopted in our work is based on the use of Galerkin's (3D) integral equations accelerated by the Fast Multipole Method (FMM) introduced by Greengard and Rokhlin [2]. The development of the Fast Multipole Symmetric Galerkin BEM (FM-SGBEM) in three-dimensional elastostatics was first introduced by Yoshida [1]. Our work is the continuation of that of Q. Trinh [4]. We use the FM-SGBEM for the calculation of real multi-cracked structures. Thus, we deal with different problems including multi-zone domain, crack contact, crack propagation, etc.

Main Contributions

In crack propagation, at the end of each cycle of propagation, new elements are created and the elastostatic calculation is resumed. To avoid wasteful operations, the old parts of the matrix are kept constant, only the parts related to the newly added elements are computed. This has led to an important reduction of the duration of the preparation phase.

During the propagation, the solution vector does not change much in theory. So we reused part of the current vector solution as the initial guess for the next increment. This has led to a reduction of the number of iterations and thus the duration of the iterative resolution.

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Analytic preconditioners for 3D high-frequency elastic scattering problems

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Keywords: time-harmonic elastic waves, Boundary Element Method, Fast Multipole Method, Analytical Preconditioner, approximate local DtN map.

Motivations

The aim of this work is to solve numerically 3D high-frequency elastic scattering problems by a bounded rigid obstacle, namely the exterior Navier problem with a Dirichlet boundary condition. To deal with the unbounded characteristic of the computational domain, we choose to apply the integral equation method. The advantage is to reformulate equivalently, through the potential theory, the exterior boundary-value problem as an integral equation on the boundary of the scatterer. The dimension of the problem is thus reduced by one. However, the discretization by BEM of boundary integral equations leads to the solution of large and fully-populated complex linear systems. The solution of these systems is handled by the GMRES iterative method. To decrease the overall cost of the solver, two complementary ways are investigated: fast methods for the computation of matrix-vector products and preconditioners to speed up the convergence of the solver.

Methodology and results

We combine an approximate DtN map as an analytic preconditioner with a FM-BEM solver. The approximations of the DtN map are derived using tools proposed in [2]. They are expressed in terms of surface differential operators, square-root operators and their inverse. Complex Padé rational approximants provide local and uniform representations of the square-root operators. The numerical efficiency of the different proposed preconditioned CFIEs is illustrated for several more or less complex geometries. An analytical study for the spherical case underlines an "ideal" eigenvalue clustering around the point $(1, 0)$ for the preconditioned CFIEs. This is not the case for the standard CFIE which has small eigenvalues close to zero. The number of GMRES iterations is drastically reduced when the preconditioned CFIEs are considered. In particular, the number of iterations is shown to be completely independent of the number of degrees of freedom and of the frequency for convex obstacles numerically [2].

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FastMMLib: a generic library of fast multipole methods

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Keywords: Integral equations, BEM, Fast Multipole Method, generic library FastMMLib

The fast multipole method (FMM) was introduced in the 80's [3, 4] and is nowadays widely used but the application of the method to a new configuration or new code is always quite challenging. At the Research Institut of Mathematics of Rennes, we are developing a generic fast method library named FastMMLib. The library is developed on the basis of a set of generic expressions for the kernel to be efficiently evaluated. The interaction with the user is made such that the library deals entirely and only with the FMM ingredients. This means that the user do not need to know about FMM and, on the other side the library interacts with the FE framework of the user or whatever dicretization he uses. The library is written in C++ and contains a specific class to ensure the consideration of the user framework (FE, quadrature rules, ...). FastMMLib includes the regularized FMM [2] which means that the FMM boxes can overlap and the distributed particles or degrees of freedom can belong to several boxes.

The library can interact with any code in C/C++ or Fortran. The generic aspect and the squeleton of FastMMLib are designed in such a way that the library could also contain other kinds of fast methods like H-matrices [5], the high-order method [1], and even some families of kernel-independent FMM [6]. First validation results and examples of use will be presented.

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Multi-trace Boundary Integral Formulations with Eddy Current Models

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Keywords: Boundary integral equations, multi-trace formalism, eddy current approximation

Motivations

We are interested in boundary integral formulations adapted to the solution of low frequency inductive electromagnetics (low frequency, high conductivity, loop source current) [1] in the case where the geometry is partitioned in (potentially irregular) subdomains. The multi-trace formalism (MTF) [2, 3] provides boundary integral formulations for Maxwell's equations posed at the interfaces between different media, with the unknowns associated to one medium a priori decoupled from the unknowns associated to other media. This makes MTF a comfortable paradigm for integral equation based domain decomposition.

Key results

We first explored the practical aspects of using the MTF within the Maxwell integral transmission problem on simple 3D test cases. In particular the simplicity of its implementation with a nonconforming discretization (conform on each side of the boundary but not from one side to the other) has been confirmed, here with mixed high-order edge functions. The MTF has then been extended to an integral formulation of the eddy current problem, leading to a new characteristic constraint that requires a quasi-Helmholtz decomposition and results to a (targeted) simplification of the integral vacuum-side block. However extra-diagonal cross identity terms are now predominant and require an appropriate discretization and, therefore, mixing primal with dual (low order) basis functions [4] in each subdomain. We conclude discussing the optimization of the eddy current MTF to overcome this unexpected complexity.

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A stable 2D energetic Galerkin BEM approach for linear elastodynamic problems

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Keywords: elastic wave propagation, space-time approach, energy identity, boundary element method.

For many decades, researchers have made considerable efforts to develop efficient numerical methods to solve linear elastodynamic problems, since this demand arises in a variety of real life applications, like engineering problems, geotechnical evaluations, environmental studies, hydrogeological investigations, seismic risk assessment, archeology, etc. Recent advances are due to Joly et al. [4], who have decoupled the pressure and the shear wave inside a homogeneous isotropic media and exploited this strategy for numerical computation by Finite Element Method (FEM). A displacement field approach is ideally suited to successful applications of Boundary Integral Equation (BIE) techniques and to discretization by Boundary Element Method (BEM). An excellent review of the application of the BIE methods and BEM to elastic wave propagation problems can be found in [3]. Frequently claimed advantages over domain approaches are the dimensionality reduction, the easy implicit enforcement of radiation conditions at infinity and the high accuracy achievement. According to the different approximate solution strategies in space-time for treating elastodynamic wave propagation problems, BEM generally follows two approaches, namely, time-domain [5] and frequency-domain approaches [2]. Since all numerical inversion formulas from frequency-domain to time-domain depend on a proper choice of their parameters, a direct evaluation in time-domain seems to be preferable; besides, it is more natural to work in the actual time-domain and observe the phenomenon as it evolves. In this last approach, the construction of BIEs via representation formula in terms of single and double layer potentials, uses the fundamental solution of the hyperbolic partial differential equation. Numerical results have shown that the standard BEM formulation can be unstable in some applications. Starting from a recently developed energetic space-time weak formulation of the BIEs related to scalar wave propagation problems [1], we focus on the 2D elastodynamic extension of the above wave propagation analysis, with the aim of testing the stability and accuracy properties of the so-called energetic Galerkin BEM in this context. Preliminary numerical simulations will be presented and discussed.

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BEM-FEM coupling for estimating anchor losses in MEMS

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Keywords: Anchor losses; BEM-FEM coupling; MEMS; Resonators; Wave dissipation

In a resonating structure the quality factor is defined as $Q = 2\pi W/\Delta W$, where ΔW and W are the energy lost per cycle and the maximum value of energy stored in the resonator, respectively. The Q factor has a peculiar importance in micro-structures (MEMS) where energy issues become dominant. Among the sources of damping that affect their performance, the most relevant are: thermoelastic dissipation, air damping, intrinsic material losses, electrical loading due to electrode routing, anchor losses.

The focus of the present contribution is set on anchor losses which become particularly meaningful for resonators working at pressure in the order of the microbar [1, 2]. Anchor losses are due to the scattering of elastic waves from the resonator into the substrate. Since the latter is typically much larger than the resonator itself, it is assumed that all the elastic energy entering the substrate through the anchors is eventually dissipated. In this work, the response of a resonating MEMS attached to a substrate is computed by using a classical Finite Element Method (FEM), while a Boundary Integral Equation (BIE) approach and its discretization by Boundary Element Method (BEM) is discussed, in order to simulate dissipation of radiated waves. A similar technique for space-time wave propagation problem was formulated and implemented in [3,4]. Even if its reduced dimensionality and high accuracy have made BEM particularity suitable for time-harmonic elastodynamics [5], solving one frequency-domain equation in 3D domain using classic BEM is computationally very costly. Since addressing a fully 3D analysis of the proposed approach for the extraction of the quality factor of a resonator requires in general the solution of a large-scale generalized complex symmetric eigenvalue problem, we consider an $\mathcal{H}$-matrix based approach to solve the BEM system, whose efficiency and accuracy have been recently tested in the context of 3D elastodynamics [6].

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Parallelized space-time boundary element methods for the heat equation

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Keywords: Space-time boundary element methods, heat equation, a priori error estimates, parallelization

The standard approach in space-time boundary element methods for discretizing variational formulations of boundary integral equations is using space-time tensor product spaces originating from a separate decomposition of the boundary $\partial\Omega$ and the time interval $(0, T)$. This space-time discretization technique allows us to parallelize the computation of the global solution of the whole space-time system. Instead of using tensor product spaces one can also consider an arbitrary decomposition of the whole space-time boundary $\Sigma = \partial\Omega \times (0, T)$ into boundary elements. This approach additionally allows adaptive refinement in space and time simultaneously. In this talk we consider the heat equation as a model problem and compare these two discretization methods. Moreover we introduce a parallel solver for the space-time system. Due to the structure of the matrices one has to design a suitable scheme for the distribution of the matrix blocks to the computational nodes in order to get an efficient method. We present numerical tests to confirm the theoretical results and evaluate the efficiency of the proposed parallelization approach.

The presented parallel solver is based on joint work with G. Of from TU Graz, J. Zapletal and M. Merta from the Technical University of Ostrava.

Interpolation-based $\mathcal{H}^2$-compression of Higher Order Boundary Element Methods on Parametric Surfaces

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Keywords: boundary element methods; parametric surfaces; higher-order; $\mathcal{H}^2$-matrices

We propose a black-box higher order fast multipole method for solving boundary integral equations on parametric surfaces in three dimensions. Such piecewise smooth surfaces are the topic of recent studies in isogeometric analysis. Due to the exact surface representation, the rate of convergence of higher order methods is not limited by approximation errors of the surface. An element-wise clustering yields a balanced cluster tree and an efficient numerical integration scheme for the underlying Galerkin method. By performing the interpolation for the fast multipole method directly on the reference domain, we reduce the cost complexity in the polynomial degree by one order. This gain is independent of the application of either $\mathcal{H}$- or $\mathcal{H}^2$-matrices. In fact, we point out several simplifications in the construction of $\mathcal{H}^2$-matrices, which are a by-product of the surface representation.

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An overlapped BEM-FEM coupling for simulating acoustic wave propagation in unbounded heterogeneous media

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Keywords: heterogeneous media, Helmholtz model, FEM-BEM overlapping coupling.

In this work we develop, analyze, and implement a new algorithm for simulating time-harmonic acoustic wave propagation in an unbounded medium, with heterogeneity comprising non-homogeneous bounded obstacles. The method is based on introducing two artificial boundaries Σ and Γ that facilitate in dividing the medium into a bounded heterogenous domain and an exterior unbounded homogeneous region, and the resulting interior and exterior regions are overlapped. The interior domain with the polygonal boundary Σ includes all of the heterogeneity in the medium and also the curve/surface Γ, and the exterior unbounded simply-connected region has the smooth boundary Γ.

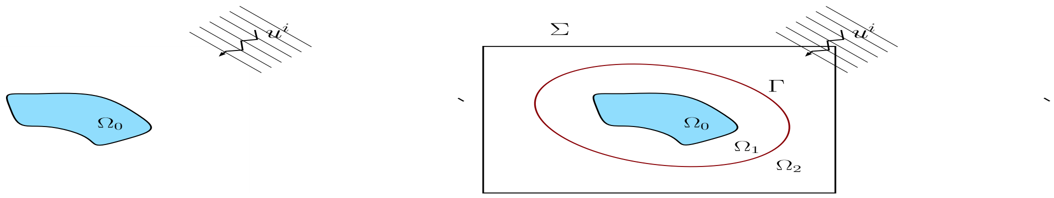

Figure 2: *Original domain and auxiliary domains*

Using the above framework, we propose a high-order discretization algorithm that is based on approximating solutions in the bounded heterogenous domain and unbounded homogeneous region, respectively, by finite and boundary elements. Thus we combine advantages of both the celebrated discretizations: the natural treatment of the Helmholtz boundary problem in the exterior of the smooth boundary Γ by boundary elements and efficient treatment of heterogeneity by the finite elements in the polygonal domain. A full analysis of the algorithm is presented for the two-dimensional case using standard $\mathbb{P}_k$ finite elements and the Kress Nyström methods as solvers in the corresponding auxiliary problems. We show that the combined FEM-BEM algorithm is stable, convergent and that, as consequence of the well-posedness of the continuous equation, the linear system which determines the components of the solutions can be solved by GMRES with a number of iterations that is independent of the level of discretization. We conclude this work with some numerical experiments in two dimensions, substantiating the theoretical results. The computational results include a complex geometry with smooth and non-smooth regions (such as a "pikachu-shaped" obstacle) and we achieve high-order (several-digits) accuracy using $\mathbb{P}_k$, elements with $k = 2, 3, 4$, respectively, for low-, medium-, and high-frequency wave propagation in $\mathbb{R}^2$ with discontinuous heterogeneities.

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Conceptually Consistent Formulation of the Collocation Boundary Element Method and Arbitrarily High Accurate Numerical Integrations for the Analysis of 2D Problems of General Topology and Shape

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Keywords: Boundary elements, numerical integration, quasi-singularity issues

The proposed contribution is the sequel and confluence of two initially independent developments made in the area of boundary element methods. The first one is best described in References [1] and [2]. It is proposed in [2] that, in the evaluation of the single-layer potential matrix of the collocation BEM for 2D or 3D problems, traction forces (as for elasticity) or normal fluxes (as for potential problems) must be interpolated as surface attributes, for generally curved boundaries, thus including in the denominator the Jacobian of the transformation from global to local, parametric variables. Only recently did the author realize that a general convergence theorem for generally curved boundaries in the collocation BEM comes out from [2], as presented at a 2017 conference [3]. However, the most important outcome is the fact that the integrand of the single-layer potential matrix simplifies to the extent that its accurate evaluation – for generally 2D problems with curved boundaries – may be carried out exclusively in terms of Gauss-Legendre quadrature (thus dispensing with a quadrature rule for logarithmic singularity). The second development referred to above is related to a unified numerical proposition for the evaluation of regular, improper, quasi-singular, singular and hypersingular integrals that may occur in the boundary element methods, as proposed in [4] and [5] (References [1] and [5] come from presentations at the IABEM 1997 workshop in Seville, Spain). The relevant result (for 2D problems, as up to now) is that, in the evaluation of the matrices of the collocation BEM as well as in retrieving results at internal points, only the Gauss-Legendre quadrature turns out to be required together with some adequate correction terms that are locally dependent from singularity type and element shape. Such corrections are computationally simple and involve no approximations – the evaluation errors come exclusively from the Gauss-Legendre quadrature of the locally regularized integrals. Several numerical examples illustrate the applicability of the proposed procedure to problems of complicated topology, or with high stress gradients. It is shown that, for a constant number of integration points along curved boundary segments, the higher the singularity (as for source points approaching a given boundary segment), the more accurate the numerical results become.

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Approximation of a parabolic-elliptic interface problem with a non-symmetric FEM-BEM and backward Euler coupling approach

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Keywords: parabolic-elliptic interface problem, finite element method, boundary element method, non-symmetric coupling, method of lines, convergence, quasi-optimality, optimal error estimates

In 1987 MacCamy and Suri [2] proposed a time-dependent interface problem for two-dimensional eddy currents with an unbounded exterior domain. However, the analysis of the model problem and their semi-discretization with a non-symmetric finite element method (FEM) and boundary element method (BEM) coupling need a smooth coupling boundary to apply a compactness argument. To overcome this restriction [3] considers a symmetric FEM-BEM coupling version in space. Additionally, they provide an analysis for a full discretization of the model with the usual regularity assumptions on the model data and solution needed for the time stepping scheme. In our presentation of the work [1], we consider the same model problem and prove well-posedness also for Lipschitz interfaces. Furthermore, we apply a classical method of lines to get a fully discrete system. For the semi-discretization with the non-symmetric FEM-BEM coupling method we establish well-posedness for problems with polygonal interfaces and prove quasi-optimality for this semi-discretization under minimal regularity assumptions. A variant of the implicit Euler method for the time stepping scheme allows us to prove well-posedness and quasi-optimality for the fully discrete scheme again under minimal regularity assumptions. The analysis does not use duality arguments and corresponding estimates for an elliptic projection which are not available for the non-symmetric coupling method. Instead, we use estimates in appropriate energy norms. Error estimates with optimal order follow directly for both, the semi- and the full discretization. Numerical examples illustrate the predicted (optimal) convergence rates and underline the potential for practical applications.

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Fast Calderón Preconditioning for the Electric Field Integral Equation

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Keywords: Electromagnetic Scattering, Electric field integral equation, Calderón preconditioning, hierarchical matrices, fast resolution methods

Calderón preconditioning for the Electric Field Integral Equation (EFIE) was successfully introduced at the beginning of the century with immediate applications in Industry and launching a great deal of research [1–3, 5–7]. Highly praised by supporters due to its solid mathematical background, critics point out the dramatic increase in dimensions related to its construction over barycentrically refined meshes and consequent inadequacy to deal with high frequencies. In the present work, we introduce a fast implementation of the algorithm that preserves the good properties of the original operator based on compression through Hierarchical Matrices [4], optimization of accuracy error and compression rates that significantly improve the standard preconditioning technique with results comparable to those by high performance algebraic preconditioners such as Near Field-based preconditioners [4]. Numerical experiments are presented to validate our claims and future research lines are presented.

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A BEM-wavelet method for the time dependent wave equation

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We consider the Dirichlet problem associated to the wave equation in two dimensions, for the solution of which we use a BEM approach in the space-time domain. The resulting integral equation is discretized in time by a convolution quadrature formula based on a BDF method of order 2, and by a Galerkin method based on wavelet type approximating functions in space. Such an approach allows to approximate the integral operators that appear in the formulation, with arbitrary precision, by highly sparse matrices. As a consequence, we obtain a substantial reduction of the computational spatial complexity of the method, with respect to a standard approach that uses Lagrangian bases.

Application of Boundary Integral Equations to MEMS working in near vacuum

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Keywords: Boundary Integral Equations, MEMS, near vacuum

The increasingly rapid growth of smart electronics is essentially due to the just as rapid development of Micro Electro Mechanical Systems (MEMS). These microsystems typically consist of a collections of fixed parts and vibrating shuttles separated by variable gaps of few microns. One important issue in the design of these devices is the evaluation of mechanical dissipation. In several applications like gyroscopes, resonators, frequency-modulated accelerometers, or magnetometers, the devices are packaged in near vacuum with a getter. The length scale and the working pressure are such that the collisions between molecules can be neglected. This regime is known as free-molecule flow [1]. We address a Boundary Integral Equation (BIE) approach for the analysis of gas dissipation, which provides the most meaningful contribution in this regime. The deterministic model, proposed in [2,3], is based on first principles of the kinetic theory of rarefied gases and on the simple diffuse model for wall-molecule interaction, which is realistic for polysilicon surfaces. The proposed approach has been validated with a dedicated experimental campaign [4] and with results taken from the literature, confirming the expected accuracy of the formulation. Recent advancements in the model implementation [5] allow to simulate almost realistic MEMS structures on standard hardware. In particular, inspired by an analogy with the radiosity equation in computer graphics, which is a tool for the generation and the manipulation of images on computer screens [6, 7], we introduce an efficient way to compute the visible domain of integration. Indeed, one of the key elements in the integral equation is the presence of the visibility operator limiting the integration to the visible portion of surfaces. Moreover, when integrating over visible close portions of the surface, issues associated to the singular nature of integral kernels require particular care. In this case we develop analytical formulas valid for triangles and piecewise constant representation of the unknown field. Finally, we discuss the application of multiple-expansions to the kernels of the BIE to further improve the overall performance and present some preliminary numerical results.

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MHD flow in a rectangular duct with a perturbed boundary

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Keywords: DBEM, MHD flow, Perturbed boundary

In this paper, a numerical study is carried for solving the unsteady magnetohydrodynamic (MHD) flow of a viscous, incompressible and electrically conducting fluid in a rectangular duct with a perturbed boundary subjected to an external magnetic field applied in y-direction. A small boundary perturbation of magnitude ε is applied on the upper wall of the duct which is encountered in the visualization of the vein anatomy and blood flow in constricted arteries. The governing MHD flow convection-diffusion type equations are coupled in the velocity and the induced magnetic field. No-slip conditions are assumed on the boundary of the duct in which the vertical walls are insulated and the horizontal walls are perfectly conducting. The numerical method is based on the use of the domain boundary element method (DBEM) in spatial discretization and a backward finite difference scheme is employed in time integration. These MHD equations are decoupled first into two transient convection-diffusion equations, and then into two modified Helmholtz equations by using suitable transformations. Then, DBEM is used to transform these equations into equivalent integral equations by employing the fundamental solution of either steady-state convection-diffusion or modified Helmholtz equations. Thus, the resulting BEM integral equations contain a domain integral whose kernel involves the multiplication of the fundamental solution with the first order time derivative of the unknown, and it is treated by numerical integration. The velocity and the induced magnetic fields are visualized in terms of equivelocity and current lines at transient and steady-state levels for several values of Hartmann number and the boundary perturbation parameter. The validity of the code is ascertained by comparing the obtained results with the ones given in literature [2]. The results reveal that the well-known characteristics of MHD flow are captured, that is, as M increases the velocity decreases and becomes stagnant at the center of the duct and a boundary layer formation is observed for both the velocity and the induced magnetic field. The perturbation parameter and the shape of the curved boundary significantly affect the behavior of the flow and cause an increase in the magnitude of induced magnetic field. DBEM with the fundamental solution of convection-diffusion equation gives better results compared to the ones obtained with the fundamental solution of modified Helmholtz equation in the sense of increasing M.

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On the coupling of DPG and BEM

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Keywords: transmission problem, DPG method with optimal test functions, boundary elements, least-squares method, coupling, ultra-weak formulation, Calderón projector

In this talk we present results of our recent works [2,3]: We develop and analyze strategies to couple the discontinuous Petrov-Galerkin method with optimal test functions to

(i) least-squares boundary elements and

(ii) various variants of standard Galerkin boundary elements.

The procedure (i) is somehow a natural approach, because the DPG method can also be equivalently written as a least-squares problem. However, the implementation involves, besides the computation of discrete boundary integral operators, the evaluation of non-local norms, which is not needed for the methods (ii). The derivation of the procedures (ii) relies on either one or both equations of the Calderón system and their analysis makes use of various ideas resp. results for the coupling of FEM and BEM [1, 4, 5]. The stability of these methods hinges on an additional parameter, i.e., a scaling of the trial-to-test operator or the test functions. Nevertheless, numerical experiments indicate that the stability does not depend on this parameter.

An essential feature of the methods (i)–(ii) is that, despite the use of boundary integral equations, optimal test functions have to be computed only locally.

We apply our findings to a standard transmission problem in full space and present numerical examples to validate our theory. Moreover, we give some ideas on how to extend the framework to a more challenging singularly perturbed transmission problem [2].

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A Boundary Element Method for Antiplane Wave Analysis of Frozen Porous Media

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Keywords: BEM, antiplane shear wave, wave scattering, frozen porous media

Frozen porous media have been proposed by Leclaire *et al* [1]. This model is known as the extended model based on the Biot's two phase model [2] and consists of three phases, i.e. solid skeleton, pore fluid, and ice matrix. The frozen porous media recently attract a great deal of attention from researchers in the field of the geophysical exploration because this model is adequate to describe wave propagation in seabed layer which involves methane hydrate. Elastic waves can play an important role in the estimation of the amount of hydrate. Thus not only understanding the wave properties but also developing a numerical method to analyse the waves in the frozen porous media can contribute to an accurate estimation. Waves in the frozen porous media propagate with dispersion and dissipation in the same manner as waves in the Biot's model and, besides that, antiplane shear wave has two propagation modes, i.e. S1- and S2-modes. These properties have been confirmed in several papers [3,4]. However, there are few reports which propose numerical computation methods for wave scattering by an inclusion which involves highly concentrated hydrate.

Thus, this study presents a boundary element method for wave scattering by the inclusion in the frozen porous media. The proposed method deals with antiplane shear waves in the frozen porous media and uses an integral equation in frequency domain. In the presented formulation, the boundary values are expressed by generalized displacement and traction. The generalized displacement consists of displacement components of the solid skeleton and the ice matrix and, on the other hand, the generalized traction is composed of the traction components of the two solid phases. Moreover, the fundamental solutions describe wave propagation of the antiplane shear waves in the frozen porous media. Several numerical examples are shown to provide the validity of the proposed method and scattering properties resulting from the numerical simulation.

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A class of forward and inverse algorithms for a stochastic wave propagation model

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Keywords: dielectric media, Maxwell equations, surface integral equations, Bayesian

In this work we consider the interaction of time harmonic electromagnetic waves with a homogeneous dielectric body $D \subseteq \mathbb{R}^3$. The material properties of the body D are characterized by the permittivity and permeability constants ϵ^- and μ^- respectively. In many applications of interest, the material properties of the body and properties of the incident electromagnetic wave, such as the position of its source or its frequency, are uncertain and must be modeled stochastically.

In our stochastic model, we let $\boldsymbol{\sigma} \in \Omega \subseteq \mathbb{R}^d$, $d \geq 4$ denote an outcome in a probability space $(\Omega, \mathcal{F}, P)$ where $\mathcal{F}$ is a Borel σ-algebra and the probabilities of the events in $\mathcal{F}$ are given by the probability measure P. Here d denotes the number of uncertain parameters in the Maxwell wave propagation model. In particular, in our model the electric permeability of D is then $\epsilon^-(\boldsymbol{\sigma})$ and the magnetic permeability of D is $\mu^-(\boldsymbol{\sigma})$.

The forward dielectric wave propagation model is concerned with developing an efficient computational model to simulate statistical moments of an important wave propagation quantity of interest (QoI). The QoI in the model is the far-field, induced by an incident wave (with uncertain parameters) impinging on the dielectric body with uncertain material properties. The inverse model is concerned with identifying a region in high-dimensional uncertain parameter space, with data comprising noisy measurements of the QoI at a few directions.

The unbounded and uncertainty nature of the model lead to several mathematical and computational challenges. The main focus of this work is to address these challenges by developing and implementing efficient deterministic and stochastic algorithms. A key ingredient to address these challenges is an all-frequency stable surface integral equation (SIE) reformulation of the deterministic wave propagation model. Such a continuous model reformulation of the Maxwell equations, with robust mathematical analysis, was developed by the authors in [1].

We use the deterministic SIE reformulation in [1] to develop an efficient high-order algorithm to simulate the deterministic dielectric model, and demonstrate our algorithm with a range of low to high frequencies. Using our algorithm, we develop a spectrally accurate forward stochastic wave propagation computational model to efficiently simulate statistical moments of the QoI. Using the Bayesian framework and our forward stochastic algorithm, we develop an efficient framework to construct and sample the posterior distribution to identify regions of the uncertain parameters from a few samples of the QoI. Numerical experiments demonstrate the efficiency of our forward and inverse algorithms.

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Non-Conventional Boundary Elements and Their Applications in BEM Analysis of Structurally Multi-Scale Problems

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Keywords: trans-accuracy element, tube element, disk element, multi-scale problem, boundary element method

When solve structurally multi-scale problems with small or slender components using BEM, different sized boundary elements are inevitably required to simulate all kinds of related geometries. In this paper, a family of non-conventional boundary elements are constructed based on Lagrange interpolation formulation [1] for solving multi-scale problems, including high order serendipity and trans-accuracy boundary elements as shown in Fig.3.

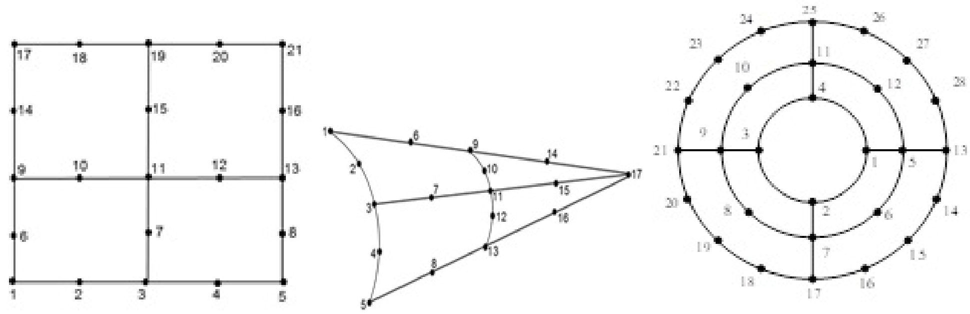

Figure 3: *21-node and 17-node serendipity elements and 28-node trans-accuracy element*

These elements not only can simulate the whole or a part of surfaces of cylinders accurately using very few nodes, but also can fulfil the transition between small and large components. A BEM analysis method for structurally multi-scale problems (MSBEM) is proposed, in which the singular boundary integrals over the constructed various non-conventional elements are evaluated using the element subdivision technique [2]. A few numerical examples for media including different numbers of slender components such as fibers are given to validate the correctness and demonstrate the potential of the proposed methods.

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A new toolbox for highly oscillatory and singular integrals

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Keywords: High frequency scattering, Helmholtz, highly oscillatory quadrature

When modelling problems of high frequency wave scattering, a common approach is to reformulate as a boundary integral equation and approximate using a BEM with a basis enriched with oscillatory functions. For example, in the Hybrid Numerical Asymptotic BEM of [1], it is shown that the size of such an approximation space does not need to grow with frequency to maintain a given accuracy. However, a consequence of such an approach is that the discrete system will contain many integrals of the form

$$\int_{\Omega} f(\mathbf{x}) \mathrm{e}^{\mathrm{i}\omega g(\mathbf{x})} \,\mathrm{d}s(\mathbf{x}), \quad \Omega \subset \mathbb{R}^N, \tag{4}$$

for $N \in \{1, 2, 3, 4\}$, where $\omega \gg 0$ is proportional to the frequency of the scattered wave, f may be weakly singular, the support Ω is over many wavelengths and the phase g may contain stationary points inside of Ω. If a standard quadrature routine (e.g. Gaussian / Clenshaw-Curtis) is used to evaluate (4), the number of weights and nodes required must grow like $O(\omega^N)$ to remain accurate. Therefore beyond a certain frequency, if a standard numerical integration routine is used, this routine is the most expensive part of the scattering model.

We present a MATLAB toolbox for evaluating integrals of the form (4) with a computational cost that does not increase with ω, hence a wave scattering sovler which combines an oscillatory basis (as in [1]) with this toolbox may have an overall cost which is independent of frequency. The algorithm uses Generalised Gaussian quadrature (see e.g. [2]) to handle singularities of f, alongside numerical steepest descent (see e.g. [1, Chapter 2]) to resolve the oscillations. Classically, computation of the steepest descent path requires knowledge of stationary points of g, alongside g', g^{-1} and $(g^{-1})'$ (which may be multi-valued). In contrast, our toolbox requires the user to provide only g and its derivatives, for coordinates in some complex neighbourhood of Ω. In the first step of the algorithm, the argument principle is used to automatically determine the stationary points. The problem of computing the components of the connecting path is decomposed into a series of initial value problems, where the solution of each problem is a truncated steepest descent path, starting at a, b or a stationary point. This removes the need for explicit representation of g^{-1} and $(g^{-1})'$. Finally, the problem of choosing the optimal steepest descent path is solved as a shortest path problem, by considering each truncated steepest descent path as a node on a graph.

Recent theoretical developments will also be presented, in particular error estimates which are explicit in the number of quadrature points and in ω. Through a handful of examples, we will explore the type of integrals currently solvable using the toolbox, alongside (if there's time) an example using the package to solve high frequency scattering problems.

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A fast direct solver for boundary value problems on evolving geometries

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Keywords: fast direct solver, evolving geometries, boundary value problems

Fast direct solvers invert a discretized boundary integral equation by exploiting structure in the matrix with a cost that grows linearly (or nearly linearly) with the number of discretization points. Since an inverse is computed, additional boundary conditions that arise frequently in design problems can be processed rapidly.

Since the constant prefactor for constructing fast direct solvers is high, these solution techniques have not been applied to problems where the geometry changes such as fluid simulations or optimal design problems. Recently developed solvers [1, 2] are able to handle problems where the changes in the geometry are localized.

This talk presents a fast direct solver for problems involving globally changing geometries. Roughly speaking, the idea is to re-use as much information from a direct solver constructed for a model geometry as possible. The resulting direct solver will be constructed for a fraction of the cost of building a direct solver from scratch. Numerical results will illustrate the performance of the method.

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Evaluation of highly oscillatory Partition of Unity BEM integrals arising in 2D wave scattering simulations

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Keywords: PU-BEM, Filon method, highly oscillatory integrals

The Partition of Unity Boundary Element Method (PU-BEM) [1] is an enriched numerical method in which the fundamental wave behaviour is included within the element formulation. PU-BEM offers highly accurate results, requiring significantly fewer degrees of freedom than conventional BEM. However, the enrichment introduces oscillatory behaviour into the integrals that arise which can provide a challenge for traditional quadrature schemes. Attempts to reduce this expense using numerical steepest descent in [2] have enjoyed some success but only in the absence of stationary points, thus indicating the requirement for a more robust method.

The authors propose the use of integration schemes that rely on asymptotic expansions as opposed to the local Taylor expansions employed when formulating conventional quadrature schemes. In particular, an extended version of the Filon-type method of Iserles and Norsett [3]. This involves integrating the highly oscillatory component of the integrals analytically by parts and multiplying the results by the coefficients of a Hermite polynomial, fitted to the non-oscillatory component of the integral. This method offers a considerable reduction in computational expense when compared with Gauss-Legendre quadrature and an increase in accuracy when compared with the traditional Asymptotic method via repeated integration by parts. Moreover, with the inclusion of an error function of a complex argument, the method is successfully extended to stationary point cases. This renders it a viable alternative integration scheme to treat 2D PU-BEM integrals for polygonal scatterers.

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Higher-order and adaptive boundary elements for the wave equation

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Keywords: time domain boundary element method, wave equation, a posteriori error estimates, graded meshes, p-version

We discuss higher-order and adaptive time domain boundary element methods for the wave equation in singular geometries, in particular graded meshes, adaptive mesh refinements and a p-version in space-time. First, we discuss asymptotic expansions near edge and corner singularities for a Dirichlet problem for the wave equation. Time independent graded meshes lead to efficient numerical approximations, as confirmed by numerical experiments for wave scattering from screens. A general a posteriori error estimate is discussed, which leads to adaptive mesh refinement procedures. The convergence rates recover those known for boundary element discretizations of time-independent problems. Finally, a p-version of the time-domain boundary element method is presented.

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Volume Integration for the $3D$ Stokes Equation

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Keywords: Viscous flow, volume integration, regular grid

Based upon a previously developed elasticity algorithm [1], a regular grid volume integration method is constructed for the nonhomogeneous $3D$ Stokes problem

$$\begin{aligned} \mu\nabla^2\mathbf{u} - \nabla p &= -\rho\mathcal{F} \\ \nabla\cdot\mathbf{u} &= 0\ . \end{aligned}$$

The domain integral to be evaluated is

$$\int_\Omega \mathcal{U}_{kj}(Q,P)\mathcal{F}_j(Q)\,\mathrm{d}\Omega\ ,$$

where $\mathcal{U}$ is the well-known Stokeslet (Green's function) [2] and $\mathcal{F}$ the given source function. The key observation is that the Stokeslet can be written as $\mathcal{U} = \nabla^2\mathcal{H}$, with $\mathcal{H}$ given by a simple analytic expression. Using Green's Theorem, the volume integral is transformed into a boundary integral, together with a 'remainder' domain integral. Evaluation of the boundary integral is straightforward, as the kernel functions $\mathcal{H}(Q,P)$ and its normal derivative do not diverge at $Q = P$. For the remainder volume term, the integrand is everywhere zero on the boundary, and thus it can be continuously extended as zero outside the domain. Numerical evaluation is then carried out by employing linear interpolation over a regular grid of cuboid cells covering the domain, *without* any special consideration for 'partial cells' intersecting the boundary. This approach thereby avoids the construction of a 'body-fitted' volume mesh, and there is no approximation of $\mathcal{F}$ as in a reciprocity formulation [4]. Following [2, 3], it should therefore be possible to treat incompressible viscous flow problems by taking the nonlinear term (involving velocity derivatives [4]) as the source function $\mathcal{F}$ [5]. The regular grid algorithm will require the evaluation of velocity gradients on the boundary, and therefore this calculation will also be discussed.

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Adaptive BEM for the Helmholtz equation

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Keywords: adaptive mesh-refinement, convergence of adaptive BEM, optimal convergence rates, Helmholtz equation

We consider the weakly-singular integral equation $V_k\,\phi = f$ associated with the Helmholtz equation for arbitrary but fixed wavenumber $k > 0$. For a standard conforming BEM discretization with piecewise polynomials, usual duality arguments show that the underlying triangulation has to be sufficiently fine to ensure the existence and uniqueness of the Galerkin solution.

Extending the abstract approach of [2], we prove in [3] that adaptive mesh-refinement is capable of overcoming this preasymptotic behavior and eventually leads to convergence with optimal algebraic rates. Unlike previous works, one does not have to deal with the *a priori* assumption that the initial mesh is sufficiently fine.

By generalizing existing inverse-type estimates for the Laplace equation from [4] to arbitrary wavenumber $k > 0$, we prove in [1] that ABEM with the weighted-residual error estimator fits in the abstract setting of [3]. Thus, we show that ABEM does not only lead to linear convergence, but also guarantees optimal algebraic convergence behavior of the underlying *a posteriori* error estimator. The overall conclusion of our results thus is that adaptivity has stabilizing effects and can, in particular, overcome preasymptotic and possibly pessimistic restrictions on the meshes.

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Data-Sparse Boundary Element Methods for Elastic Waves

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Keywords: adaptive cross approximation, 3D elastodynamics, convolution quadrature

Propagation of elastic waves is a major concern in a variety of engineering applications. The governing equations may be treated by the boundary element method in time domain. The associated boundary integral equations are of convolution type in time. Hence, the discretization in time can be performed with the convolution quadrature method (CQM) proposed by Lubich [4]. A particular advantage of the method is that the fundamental solution has to be known solely in Laplace domain, however the time step size needs to be constant. This limitation does not apply to the generalized convolution quadrature method (gCQM) developed by Lopez-Fernandez and Sauter [3].

On the downside, these methods lead to dense system matrix, which causes excessive memory consumption and computing time. We intend to carry out a low-rank approximation to overcome this drawback. The adaptive cross approximation (ACA), see for instance [1], has already been applied successfully to problems such as the scalar wave equation. Messner and Schanz published in [5] an ACA for elastodynamics. In this paper, the problem is decomposed in each direction such that the matrix is partitioned into nine submatrices and each submatrix is approximated in standard fashion.

The talk comprises an extension of the ACA to handle three-dimensional elastic waves without reordering of the system matrix. In this context we focus on a suitable pivot strategy motivated by [2, 6]. To reinforce this strategy some numerical experiments are presented and discussed. They show that the occuring matrices can indeed be compressed significantly while retaining a good quality of the solution of the underlying problem.

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A fast coupled boundary element formulation for trans-abdominal high-intensity focused ultrasound therapy

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Keywords: HIFU treatment planning, Helmholtz transmission problem, coupled BEM-BEM

High-intensity focused ultrasound (HIFU) is a promising treatment modality for the non-invasive ablation of pathological tissue in many organs, including the liver. In the case of liver cancer, the first choice of therapy is either surgical resection or transplantation. The risks associated with resection make it unsuitable for the majority of patients. Thus the ability to non-invasively and precisely ablate liver tumors will have substantial clinical impact. Optimal treatment planning strategies based on high-performance computing numerical methods are expected to form a vital component of a successful clinical outcome in which healthy tissue is preserved and accurate focusing achieved, thus compensating for soft tissue inhomogeneity and the presence of ribs. The boundary element method (BEM) is an effective approach for this purpose because only the boundaries of the ribs and soft tissue regions require discretization, as opposed to standard approaches which require the entire volume around the ribcage to be meshed.

The current state of the art coupled BEM formulations is discussed. Subsequently, a coupled BEM-BEM formulation in combination with preconditioning and matrix compression techniques implemented in Bempp [3] is presented. This formulation is used to carry out simulations of Helmholtz transmission problem (HTP) in different scenarios. The simulation results of HTP in spheroids are used to investigate the speed, convergence and accuracy of the solution. Additionally, numerical experiments are performed to solve HTP in a domain comprising a human ribcage, an abdominal fat layer, and liver tissue subdomains in the acoustic field generated by a focused array transducer. These results reveal the challenges to overcome in order to develop a viable BEM formulation for trans-abdominal HIFU treatment planning.

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Adaptive Wavelet Boundary Element Methods

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Keywords: Boundary Element Method, Wavelets, Adaptivity

Many mathematical models concerning for example field calculations, flow simulation, elasticity or visualization are based on operator equations with *nonlocal operators*, especially boundary integral operators. This talk is concerned with developing numerical techniques for the adaptive application of such operators in wavelet coordinates. This is a core ingredient for a new type of adaptive solvers that has so far been explored primarily for partial differential equations. We shall show how to realize asymptotically optimal complexity in the present context of nonlocal operators. *Asymptotically optimal* means here that the solution is approximated at a desired target accuracy with a computational expense that stays proportional to the number of degrees of freedom (within the setting determined by an underlying wavelet basis) that would ideally be necessary for realizing that target accuracy if full knowledge about the unknown solution were given. The theoretical findings are supported and quantified by numerical experiments.

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A non-conforming domain decomposition approximation for the Helmholtz screen problem with hypersingular operator

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Keywords: Helmholtz problem, hypersingular operator, boundary element method, domain decomposition, Nitsche method

We present and analyze a non-conforming domain decomposition approximation for a hypersingular operator (thus giving rise to an integral equation of the first kind) governed by the Helmholtz equation in three dimensions. This operator appears when considering the corresponding Neumann problem in unbounded domains exterior to open surfaces. Here, for simplicity we consider piecewise plain orientable Lipschitz surfaces. Wave numbers are assumed to be small and we use low-order approximations with Nitsche coupling across interfaces. Our results are based on [1,2] which analyze the case of the Laplacian, with Nitsche and mortar couplings, respectively.

Under appropriate assumptions on mapping properties of the weakly singular and hypersingular operators with Helmholtz kernel, we prove that our method converges almost quasi-optimally. Specifically, up to a perturbation of the type $h^{-\epsilon}$ with mesh-size h and arbitrary $\epsilon > 0$, the method has the same convergence rate as a conforming variant with low-order basis functions. Numerical experiments confirm our error estimate.

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Scattering by Fractal Screens and Apertures: I - Functional Analysis

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Keywords: Helmholtz equation, fractal screens, Sobolev spaces

We consider time-harmonic acoustic scattering in $\mathbb{R}^{n+1}$, $n = 1, 2$, by planar screens and apertures. When the screen/aperture $\Gamma \subset \mathbb{R}^n$ is a Lipschitz open set, the associated boundary integral equation formulations are classical: the boundary integral operator for the Dirichlet screen problem is the single layer operator $S : \tilde{H}^{-1/2}(\Gamma) \to H^{1/2}(\Gamma)$, and for the Neumann screen problem it is the hypersingular operator $T : \tilde{H}^{1/2}(\Gamma) \to H^{-1/2}(\Gamma)$. The resulting variational formulations are known to be coercive (strongly elliptic), which permits rather explicit error analysis of Galerkin BEM. (See e.g. [4], which builds on earlier work by T. Ha Duong.)

However, when Γ is open but non-Lipschitz (e.g. with a fractal boundary, like the Koch snowflake) the classical formulations may fail to be well posed. And when Γ is a closed set with empty interior (e.g. a fractal set such as a Cantor dust or Sierpinski triangle) then it is not obvious how one should even impose boundary conditions, let alone formulate the integral equations.

It turns out that for an arbitrary $\Gamma \subset \mathbb{R}^n$ there are in general an uncountably infinite number of possible formulations, all producing distinct scattered fields. The physically correct choice can be determined by a limiting geometry principle, viewing the rough set Γ as a suitable limit of a sequence of smoother sets Γ_j, e.g. prefractal approximation of a fractal set.

In this talk I will describe some of our functional analytic contributions published in [1–3], as well as more recent results. We consider a number of intriguing and nontrivial questions, including:

- Given an arbitrary screen $\Gamma \subset \mathbb{R}^n$, what is the correct boundary function space setting generalising the $\tilde{H}^{\pm 1/2}(\Gamma) \leftrightarrow H^{\mp 1/2}(\Gamma)$ duality that holds when Γ is open?
- For Γ closed with empty interior, when is $H^{\pm 1/2}_{\Gamma} = \{u \in H^{\pm 1/2}(\mathbb{R}^n) : \operatorname{supp} u \subset \Gamma\}$ non-trivial?
- For Γ open, when is $\tilde{H}^{\pm 1/2}(\Gamma) = \overline{C_0^\infty(\Gamma)}^{H^s(\mathbb{R}^n)}$ equal to $H^{\pm 1/2}_{\overline{\Gamma}} = \{u \in H^{\pm 1/2}(\mathbb{R}^n) : \operatorname{supp} u \subset \overline{\Gamma}\}$?
- How do function spaces on a sequence of prefractal approximations Γ_j relate to function spaces on the limiting set Γ?

The results we obtain may be surprising to those familiar with Sobolev spaces on smooth domains! In a companion talk, S. N. Chandler-Wilde will present numerical simulations of (pre-)fractal screen scattering problems using BEM, as well as discussing some of the challenging issues arising in the associated numerical analysis.

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First-Kind Galerkin Boundary Element Methods for the Hodge-Laplacian

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Keywords: Hodge-Laplacian, representation formula, Calderón indentities, first-kind boundary integral equations, saddle-point problems, boundary element method (BEM)

Boundary value problems for the Euclidean Hodge-Laplacian in three dimensions,

$$-\boldsymbol{\Delta}_{\mathrm{HL}} := \mathbf{curl}\,\mathbf{curl} - \mathbf{grad}\,\mathrm{div}\ ,$$

lead to variational formulations set in subspaces of $\boldsymbol{H}(\mathbf{curl}, \Omega) \cap \boldsymbol{H}(\mathrm{div}, \Omega)$, $\Omega \subset \mathbb{R}^3$ a bounded Lipschitz domain. Via a representation formula and Calderón identities we derive corresponding first-kind boundary integral equations set in traces spaces of $H^1(\Omega)$, $\boldsymbol{H}(\mathbf{curl}, \Omega)$, and $\boldsymbol{H}(\mathrm{div}, \Omega)$. They give rise to saddle-point variational formulations and feature kernels whose dimensions are linked to fundamental topological invariants of Ω.

Kernels of the same dimensions also arise for the linear systems generated by low-order conforming Galerkin boundary element (BE) discretization. On their complements, we can prove stability of the discretized problems, nevertheless. A particular challenge is that the "discrete" kernels do not have a simple representation and that the BE linear systems generally fail to possess a consistent right-hand side. It takes special Krylov subspace iterative solvers to compute meaningful solutions.

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An isogeometric BEM for a 3D doubly-periodic PEC surface in electromagnetism

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Keywords: isogeometric analysis, IGBEM, electromagnetic scattering

Recently, the role of periodicity is getting important in applied physics, especially photonics and plasmonics. In this study, we consider a perfectly electrical conducting (PEC) surface that has the periodicity in two directions and is irradiated by an electromagnetic incident planewave. In order to analyze and design the doubly-periodic PEC surface, an accurate numerical method to solve the Maxwell equations is necessary.

To meet this demand, we propose a Galerkin boundary element method (BEM) incorporating the concept of the isogeometric analysis (IGA) [1, 2]. Specifically, following the investigation for the non-periodic case by Simpson et al [3], we discretize the boundary integral equation with using the compatible B-spline [4] and NURBS functions as the interpolation and shape functions, respectively. To take account of the quasi-periodicity into the interpolation and weight functions, we enhance the technique established for the RWG basis [5] to the underlying compatible B-spline bases. The high-order discretization thus constructed can bring a high-accurate solution in comparison with the conventional low-order discretization based on the RWG and roof-top bases.

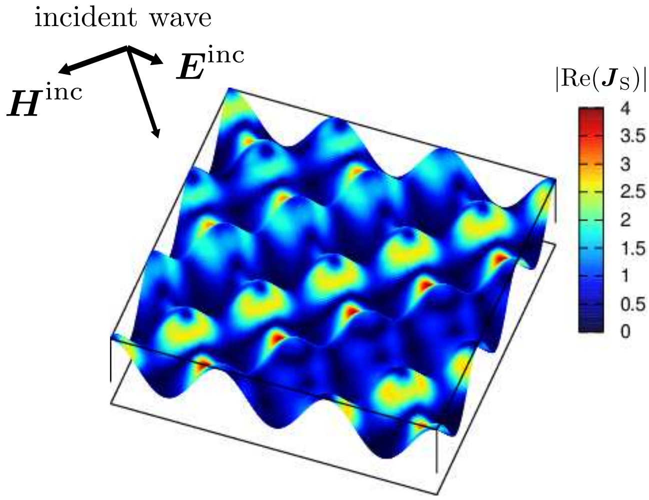

Figure 4: *Magnitude of the surface current density $\boldsymbol{J}_S$ on a 3×3 unit cells of the sinusoidal PEC surface.*

In this talk, we will give the formulation of our isogeometric BEM and access it through some numerical examples. Figure 4 shows a numerical result of surface current density $\boldsymbol{J}_S$ on the sinusoidal PEC surface when an oblique planewave of wavenumber 10 is given.

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A topology optimisation for photonic crystals using a fast boundary element method

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Keywords: Topology optimisation, Photonic crystal, $\mathcal{H}$-matrix

It is well known that a periodic structure of dielectric materials called photonic crystal (PC) exhibits extraordinary properties against electromagnetic waves. For example, electromagnetic waves either propagate through the PC or not, depending on their frequency. The frequency range that cannot propagate are called bandgaps. Owing to this distinguished property, it is expected that the PC gives rise to innovative optical devices such as cloaking, low-loss waveguide, broad-area coherent laser and so forth. To realise such attractive optical devices, we need to design PCs with bandgaps as large as possible at the desired frequency. To this end, we propose a topology optimisation method for PCs.

In the proposed method, we resort to the level set method [2, 2] and the topological derivative which characterises the variation of the objective function (the bandgap size which is a function of eigenvalues of periodic boundary value problems) to a small topological change in the PC. We first derive the topological derivative and show that it is associated with the relevant eigenpairs. We then discuss a fast boundary element method combined with an eigensolver which utilises the contour integral [3] for computation of the bandgap and its topological derivative. Since the boundary element matrices involved in the proposed solver have a common sub-matrix, we can construct a fast solver with the help of the Woodbury formula. In the proposed solver, algebraic operations in the Woodbury formula are further accelerated by $\mathcal{H}$-matrix method.

In this abstract, we show a typical numerical result in Figure 1. In this example, we explored the shape of a perfectly electric conductor (PEC) in a dielectric matrix which maximises the bandgap between 2nd and 3rd bands. In the oral presentation, we show some more numerical examples as well as the detailed formulation of the proposed method.

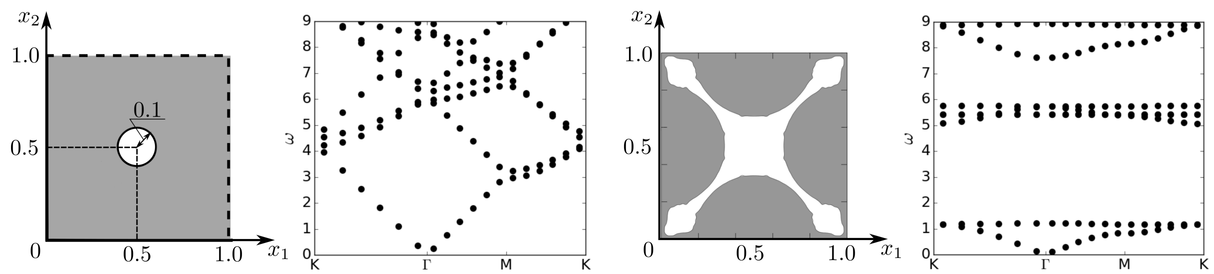

***Figure 5:** Unit cell of PC and its band structure for initial (left) and optimal designs (right).*

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Boundary Element Method for Conductive Thin Layer in 3D Eddy Current Problems

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Keywords: Eddy-Current Problems, Thin Conducting Layers, Transmission Conditions, Boundary Element Method

Conductive thin layers for shielding purpose are considered in a wide range of applications. Modelling such conductive regions requires fine volume discretisation due to the rapid decay of fields through the surface for high frequencies. This leads to the solution of large system of equations that can be highly time consuming. To avoid this difficulty, we derive an equivalent model for 3D Eddy Current problem with a conductive thin layer of small thickness ϵ , where the conductive sheet is replaced by its mid-surface, and its shielding behavior is satisfied by an equivalent transmission condition which connect the electric and magnetic field around the surface.

In [1] equivalent transmission conditions for the full time-harmonic Maxwell equations are derived in 3D. Curved thin sheets are considered, where the material constants can take different values inside and outside the sheet. In this work, we derive equivalent transmission conditions for Eddy-Current problems in 3D, for curved thin sheets, where the materials inside and outside the sheet are non-conductive. Note that these transmission conditions are derived asymptotically for vanishing sheet thickness ϵ, assuming that we have asymptotically constant ratio between skin depth $d_{skin} = \sqrt{2/(\omega\mu_0\sigma_0)}$ and thickness ϵ.

As out of the layer we mainly consider a non-conductive linear homogeneous domain and an open boundary problem, we can easily avoid the volume mesh required in the Finite Element Method (FEM) by using the Boundary Element Method (BEM) that uses only 2D elements on the surfaces. Moreover, BEM is adapted to general field problems with unbounded structures because no artificial boundaries are needed, this is not the case for FEM.

We validate the results by comparing to the formulation given in [2], and to the main problem simulated in COMSOL and solved numerically using the Finite Element Method with very fine mesh. The results show a good agreement between methods. Complementary tests that study the robustness with respect to the sheet conductivity and the convergence of the modelling error will be given.

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Inverse Fast Multipole Method Applied to the Galerkin Boundary Element Method

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Keywords: Galerkin boundary element method, fast multipole method, inverse fast multipole method, iterative solution schemes, preconditioner, Helmholtz problem

Fast solution methods for boundary integral equations

Boundary integral equations accurately describe numerous physical effects in the fields of fluid mechanics, acoustics and electromagnetics. Often, the boundary element method (BEM) is applied for discretization in conjunction with a collocation or Galerkin projection. Due to the nature of the involved integral operators, the obtained system of equations contains dense matrices for which conventional solution schemes imply a cubic complexity with respect to the number of degrees of freedom. Efficient strategies for addressing this issue have been an active area of research in the last few decades. Today, fast solution methods, such as the fast multipole method or $\mathcal{H}^2$-matrix schemes, are available. Incorporated with preconditioned iterative solvers, quasi-linear or even linear complexity can be achieved.

Efficient preconditioning

With fast boundary element methods and well-known iterative solution schemes at hand, the solution process can be further accelerated by choosing an appropriate preconditioner. In this regard, general and easy-to-implement approaches like block diagonal preconditioners or incomplete LU factorizations are often used. At the cost of general applicability, more advanced approaches such as analytical preconditioning techniques or methods based on the sparse approximate inverse are beneficial for specific problems. In this paper, the recently introduced inverse fast multipole method [2] is applied as a preconditioner to the Galerkin BEM. Its performance is compared to common alternatives and to results obtained by other researchers using a collocation discretization. Moreover, a comparison with an analytical preconditioner based on the pseudo-inverse of the hypersingular operator [1] is conducted. Most simulations are performed using the Galerkin boundary element library Bem++ [3].

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Boundary Integral Formulation for Helmholtz and Laplace Dirichlet Problems On Multiple Open Arcs.

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Keywords: Integral Equations, non-Lipschitz domain, disjoint domains, matrix compression

We present a spectral numerical scheme for solving Helmholtz and Laplace problems with Dirichlet boundary conditions on a finite collection of open arcs in $\mathbb{R}^2$. An indirect boundary integral method is employed, giving rise to a first kind formulation whose variational form is discretized using weighted Chebyshev polynomials. Well-posedness of both continuous and discrete problems is established as well as spectral convergence rates under the existence of analytic maps to describe the arcs. In order to reduce computation times, a simple matrix compression technique based on sparse kernel approximations is developed. Numerical results provided validate our claims.

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Multiple Traces Formulation and Semi-Implicit Scheme for Modeling Biological Cells under Electrical Stimulation

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Keywords: Multiple traces formulation, semi-implicit time stepping, biological cells, spectral methods

We model the electrical behavior of several biological cells under external stimuli by extending and computationally improving the semi-implicit multiple traces formulation presented in [1]. Therein, the electric potential and current for a single cell are retrieved through the coupling of boundary integral operators and non-linear ordinary differential systems of equations. Yet, the low-order discretization scheme presented becomes impractical when accounting for interactions among multiple cells. In this note, we consider multi-cellular systems and show existence and uniqueness of the resulting non-linear evolution problem in finite time. Our main tools are analytic semigroup theory along with mapping properties of boundary integral operators in Sobolev spaces. Thanks to the smoothness of cellular shapes, solutions are highly regular at a given time. Hence, spectral spatial discretization can be employed, thereby largely reducing the number of unknowns. Time-space coupling is achieved via a semi-implicit time-stepping scheme shown to be stable and convergent. Numerical results in two dimensions validate our claims and match observed biological behavior for the Hodgkin-Huxley dynamical model.

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Boundary elements for surfaces in contact in three dimensions

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Keywords: BEM, contact elements, fracture mechanics

Contact and fracture mechanics are fields of engineering analysis for which finite elements perform poorly and boundary elements show great promise. Surfaces, including crack faces, may be in contact without sliding, in contact with sliding, or not in contact at a point. Which of these three conditions exists depends upon a coefficient of friction, and in the case of surfaces with asperities an angle of dilation which tends to zero with increasing relative shear displacement.

The work presented here is a continuation of that presented earlier by the author [1]. Crack faces and other surfaces which are potentially in contact are modelled by pairs of boundary elements which share common nodes [2]. Geometry is defined by conforming quadratic and hybrid quadratic-Hermitian cubic shape functions. Displacements are interpolated by the same functions, enriched by functions which exhibit the same singular behaviour at crack roots as the first three eigenvalues of the Williams expansion and its equivalent for antiplane strain [3], whereas tractions are interpolated by quadratic shape functions only. In the interpolation for displacement, over elements adjacent to a crack root the singular shape functions are multiplied by stress intensity factors which are taken to vary quadratically over each element side on the root, and in general the singular functions extend for more than one element away from the root. At each node of an element pair, the unknowns are average displacements of the elements, and either three tractions (contact, no sliding), one traction and two relative displacements (contact, sliding), or three relative displacements (no contact).

An augmented system of equations is constructed in which at nodes of element pairs both the traction and relative displacement vectors are retained as unknowns, in addition to the vector of average displacements [4]. In a first stage of solution by Gaussian elimination, all degrees of freedom associated with nodes at which there is no potential contact between surfaces are eliminated, to form an augmented Schur complement. In a second stage, at each increment of applied load the extent of areas of contact between surfaces is determined iteratively by computations involving only the augmented Schur complement (from which successive actual Schur complements are constructed), and the load vector. The iteration is found to converge reliably and, for typical cases in which at most of the nodes there is no potential contact, the software execution time is very reasonable. As a demonstration of the proposed technique, analyses are shown of a uniaxial compression test specimen of rock in which there are pre-existing buried cracks, the aim of the analyses being to better understand the observed reduction of strength with increasing specimen size which currently is estimated by empirical formulae.

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The inverse of a finite element discretization of the fractional Laplacian can be approximated by $\mathcal{H}$-matrices

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Keywords: Fractional PDE, Integral operators, Hierarchical matrices

Since the pioneering work [2] of Caffarelli and Silvestre, it is known that the fractional Laplace operator $(-\Delta)^s$ for $s \in (0,1)$ can be represented as the Dirichlet-to-Neumann map of a degenerate PDE on an unbounded domain. Indeed, the fractional Laplacian is a non-local operator, and it can be represented as an integral operator with a singular kernel [1]. Hence, we can expect that techniques used in boundary element methods can be applied also in this case. For example, it is known that matrices arising in discretizations of Galerkin boundary element methods can be approximated by Hierarchical matrices [4]. Even more, the same is true for their inverses [3]. In this talk, we show how these results and techniques carry over to the case of the fractional Laplace operator.

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Electromagnetic scattering by ice crystals and implementation using Bempp

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Keywords: boundary element method, electromagnetic scattering, dielectric objects, preconditioning, ice crystals, Bempp

In recent years Calderón preconditioning [1] and appropriate use of basis functions [2] have become a popular strategy to speed up the iterative solution of electromagnetic scattering problems. In this talk we will discuss recent developments in the solution of dielectric scattering problems, extend the ideas of Calderón preconditioning in the case of scattering by multiple dielectric objects and discuss their implementation in the boundary element library Bempp [3].

Of particular interest are cases of light scattering by ice crystals found in cirrus clouds [4, 5]. We will demonstrate how one can use the above theory and the Bempp library to efficiently solve examples of light scattering by single and multiple ice crystals.

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Efficient parallel implementation of $\mathcal{H}$-matrix based solvers for 3D Helmholtz and elastodynamic oscillatory kernels

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Keywords: $\mathcal{H}$-matrices, Low Rank Approximation, Fast BEMs, parallelization, Acoustics, Elastodynamics

We are concerned in this contribution with the improvement of the efficiency of Boundary Element Methods (BEMs) for 3D frequency domain acoustic and elastodynamic problems. The need of efficient tools is crucial for the simulation of many real-life problems such as soil-structure interaction, site-effects phenomenon, non-destructive controle of structure (e.g. in nuclear area), and for the modelling and design of anti-noise walls. On the one hand, BEMs are based on the discretization of Boundary Integral Equations [4] such that only the domain boundary is meshed. On the other hand, they lead to a linear system with a fully-populated influence matrix, conversely to standard volume methods such as finite elements. Hence standard BEM solvers lead to high computational costs both in terms of time and memory requirements. This drawback prevents to treat large scale three-dimensional problems.
Over the last decades, various solutions have been proposed in order to circumvent the full assembly and storage of the matrix. The most popular is probably the Fast Multipole Method [5] to compute the application of the integral operators, i.e the matrix-vector product which is, indeed, the essential operation for an iterative solver. The other solution is based on the use of hierarchical matrices ($\mathcal{H}$-matrices) [6]. $\mathcal{H}$-matrices are commonly used, in conjunction with an efficient rank revealing method such as the Adaptative Cross Approximation (ACA), to lower the memory requirements and computational times. The principle of $\mathcal{H}$-matrices is to approximate the global matrix by finding sub-blocks which can be compressed. The approach, shown to be very efficient for asymptotically smooth kernels, is not optimal for Helmholtz and elastodynamic oscillatory kernels [7]. For a given sub-block, this yields an increase of the rank as the frequency increases. An alternative is the use of $\mathcal{H}^2$-matrix [4], a multigrid-like specialization of $\mathcal{H}$-matrices, at the cost of important implementation efforts. However, encourageous results have been obtained in [5] where the capabilities of $\mathcal{H}$-matrices for oscillatory kernels are illustrated on numerical examples. These results highlight the existence of a preasymptotic regime for which the $\mathcal{H}$-matrix based solvers behave well (for some frequency range) and so they can be seen as a viable alternative to speed-up BEMs.
Following this encouraging work, we are interested in the optimization of the $\mathcal{H}$-matrix based solvers for oscillatory kernels when the standard admissibility condition of asymptotically smooth kernels is used. We study the repartition of the blocks with the highest ranks and derive an efficient set up of the $\mathcal{H}$-matrix representation. Then, we study the complexity of the different parts of the direct and iterative solvers, and their parallelization. The efficiency of the solvers is illustrated on acoustic and elastodynamic large-scale problems.

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The Hierarchy of Numerical Schemes for Boundary Integral Equation Solution in 2D Vortex Methods at Airfoil Polygonal Approximation

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Keywords: vortex method, boundary integral equation, Galerkin approach

The key questions in implementations of Lagrangian vortex methods [1] for viscous incompressible flow simulation, is the choice of numerical approach for no-slip boundary condition satisfaction. The airfoil in the flow can be replaced with vortex sheet of unknown intensity.

There are two ways to no-slip boundary condition satisfaction in the framework of vortex methods. The first one, which follows from the Neumann problem in the potential theory, leads to singular integral equation with Hilbert-type kernel. There are special requirements for the airfoil surface line discretization for its numerical solution; specific quadrature formulae should be used for the principal value extraction. The alternative approach leads to the 2-nd kind [2] boundary integral equation of the Fredholm-type and it is free from the mentioned restrictions.

For numerical solution of the boundary integral equation with respect to unknown vortex sheet intensity, the airfoil is approximated by polygon consists of rectilinear segments. The ideas of the Galerkin method are implemented and the hierarchy of numerical schemes is developed. Vortex sheet intensity distribution is assumed to be linear combination of basis functions, and the unknown coefficients can be found from the orthogonality condition of the residual to projection functions.

Dirac Delta-functions, constant and linear functions can be used as basis and projection functions (in different combinations), the resulting numerical schemes have different accuracy and computational complexity [4]. The piecewise-linear solution can be continuous or discontinuous, or continuous everywhere except some specified points correspond to sharp edges and angle points of the airfoil.

The developed numerical schemes provide the 1-st and the 2-nd order of accuracy with respect to average value of vortex sheet intensity over the panels and in L_1 norm. Expressions for the linear system (which approximates the integral equation) coefficients are obtained. For the corresponding integrals, some of which are improper (but not singular), the exact analytical formulae are derived by authors [3]. Exact analytical expressions are obtained also for the quadratures in the Rhs of the equation (for vortex wake influence), which are consistent with Lhs approximation.

All the developed schemes are generalized for the FSI problems with movable airfoils: attached vortex and source sheets are introduced, their intensities are approximated similarly to free vortex sheet — with delta-functions, piecewise-constant or piecewise-linear distributions. Note, that it does not require significant algorithm modifications. Moreover, it is easy to implement strong coupling in FSI problems and construct fully implicit numerical scheme which is stable at arbitrary time step.

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Hybrid numerical-asymptotic boundary element methods for high frequency scattering by penetrable convex polygons

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Keywords: boundary element method, high frequency scattering, penetrable obstacles

Linear wave scattering problems (e.g. for acoustic, electromagnetic and elastic waves) are ubiquitous in science and engineering applications. However, conventional numerical methods for such problems (e.g. FEM or BEM with piecewise polynomial basis functions) are prohibitively expensive when the wavelength of the scattered wave is small compared to typical lengthscales of the scatterer (the so-called "high frequency" regime). This is because the solution possesses rapid oscillations which are expensive to capture using conventional approximation spaces.

Recently, there has been much interest in the development of "hybrid numerical-asymptotic" methods. These methods use approximation spaces containing oscillatory basis functions, carefully chosen to capture the high frequency asymptotic behaviour, leading to a significant reduction in computational cost. These ideas have been applied to a wide range of scattering problems (see, e.g., [1]), but progress to date has largely been confined to problems of scattering by impenetrable obstacles.

In this talk, we describe what we believe to be the first hybrid numerical-asymptotic method for any problem involving a penetrable scatterer. We consider the problem of scattering by penetrable convex polygons. We reformulate the associated transmission boundary value problem as a direct boundary integral equation for the unknown Cauchy data, which is then represented as a sum of the classical geometrical optics approximation, computed by a beam tracing algorithm, plus a contribution due to diffraction computed by a Galerkin boundary element method using oscillatory basis functions chosen according to the principles of the Geometrical Theory of Diffraction.

Our boundary element method, described in detail in [2], can achieve a fixed accuracy of approximation using only a relatively small, frequency-independent number of degrees of freedom. Moreover, the inclusion of the diffraction term provides an order of magnitude improvement in accuracy over the geometrical optics approximation alone.

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Combination of the CHIEF and the self-regularization technique for solving 2D exterior Helmholtz equations with fictitious frequencies in the indirect BEM and MFS

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Keywords: indirect BEM, MFS, fictitious frequency, CHIEF, self-regularization, degenerate kernel

Regarding the treatment for the fictitious frequency as well as spurious resonance in using the indirect boundary element method (BEM) and the method of fundamental solutions (MFS), we propose an alternative approach in this talk. The present approach is different from the mixed potential approach in the indirect method as well as the Burton and Miller approach in the direct BEM. In the proposed approach, we add some fundamental solutions with unknown source strength in the representation of the field to complete the base of solution space. From the viewpoint of adding point, the present idea is similar to the combined Helmholtz interior integral equation formulation (CHIEF) method in the direct BEM. The difference between the added source points and the CHIEF points is their role. The added source points supply the deficient base due to the fictitious frequency while the CHIEF points provide the extra constraint equations. Therefore, we examine the CHIEF constraint by employing the self-regularization technique for the influence matrix in the direct BEM. Based on this idea, the constraint equation in the present approach may be found by adding the right unitary vectors of zero singular value. Then, a square bordered matrix is obtained. The bordered matrix is invertible for the fictitious frequency if the extra source points do not locate at the failure positions. This is the reason why the property is analogous to the idea of the CHIEF method in the direct BEM. Therefore, the proposed approach can fill in the gap that there is no CHIEF method in the indirect BEM and MFS. Since the proposed approach only need using the single-layer potential, it has an advantage over the existing formulations. To demonstrate the validity of the present idea, the problem of an infinite plane containing a circular radiator or scatter is considered. In the real implementation, all fictitious frequencies in the certain range of the wavenumber are found first by the direct searching algorithm. Both the Dirichlet and Neumman boundary conditions are also considered. Finally, we also analytically derive the locations of possible failure points by using the degenerate kernel.

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The Sparse Cardinal Sine Decomposition (SCSD) and its application to the simulation of suspensions.

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Keywords: Boundary-integral equations, SCSD, Stokes equations, Suspensions.

For many applications (settling, transport...), it is necessary to compute the flow of either dilute or concentrated unbounded suspensions made of solid particles immersed in a Newtonian liquid. For small enough particles, the flow Reynolds number vanishes and the task fortunately reduces to the treatment of the linear Stokes equations [1]. This can be efficiently achieved by solving boundary-integral equations [2].

However, as usual in such formulations, the discretization of the problem leads to dense and non-symmetric linear systems whose size grows as the square of the number of particles. Acceleration techniques are therefore usually employed. Most of them are based on the compression of the underlying matrix in order to obtain efficient matrix vector products (see e.g. Fast Multipole Method, $\mathcal{H}$-matrices, etc.). In this direction, the new Sparse Cardinal Sine Decomposition method (SCSD) was recently developed for the scalar kernel encountered in acoustics [3]. The main idea consists in expanding the kernel in the Fourier space as a finite sum of Cardinal Sine functions. The method has been further extended to the vectorial Stokes kernel in [4] where it has actually been implemented and tested for a single solid particle.

After presenting the SCSD solver for the Stokes kernel, this work investigates its ability to efficiently cope with N-particle clusters immersed in a Newtonian liquid. Cases of large N will be investigated. Both distant or close (packed) particles will be considered.

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Uncoupled Thermoelastic Boundary Element Formulation with Variable Time Step Size

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Keywords: Thermoelasticity, convolution quadrature

To model the deformation of hot forming tools a thermoelastic model is necessary. However, only the effect of the temperature on the mechanical behavior is essential and not vice versa. Hence, a one sided coupled approach is sufficient. Further, the mechanical inertia effects are neglected, which results in the so-called uncoupled quasistatic thermoelasticity. The governing equations consist of the parabolic heat equation and the elastostatic equations with a thermal load due to the coupling with the heat equation.

Here, a boundary element formulation for this one sided coupled problem is proposed. In the heat equation and as well in the right hand side of the mechanical equation convolution integrals in time have to be solved. Several approaches exist to discretize these time integrals (e.g. [1]). Here, the convolution quadrature method (CQM) is applied. The original CQM has been developed by Lubich [1] and was restricted to constant time step sizes. However, in the quasistatic thermoelasticity very often processes are modelled, which develop fast in the beginning and tend to a steady state after some time. For such processes a constant time step size is sub-optimal and a variable step size is preferable. The generalization of the CQM to variable step sizes has been developed by Lopez-Fernandez and Sauter [3, 4]. This methodology is applied here. Applications and performance are presented via numerical results.

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Material derivatives of boundary integral operators in electromagnetism and applications

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Keywords: Maxwell equation, boundary integral operators, material derivatives, inverse problem

This talk deals with the Fréchet differentiability analysis with respect to the boundary parametrisations of the boundary integral operators arising in the potential theory of time-harmonic electromagnetic waves [1]. The material derivatives of integral operators were investigated in the framework of Hölder continuous and differentiable function spaces in the pioneer works of R. Potthast [6] and more recently in the framework of Sobolev spaces by M. Costabel and F. Le Louër [2]. We present novel results [5] using the Piola transform of the boundary parametrisation to transport the integral operators on a fixed reference boundary. This approach spare us the use of technical tools such as the Hodge decomposition of $\boldsymbol{H}^{-\frac{1}{2}}(\Gamma, \mathrm{div})$ [2]. The transported integral operators are infinitely differentiable with respect to the parametrisations and simplified expressions of the material derivatives are obtained. Moreover, the Piola transform allows us to state the differentiability properties of the electromagnetic hypersingular operator without involving a regularization procedure. The material derivative formulas are found explicitly in the form of linear boundary integral operators and are suited for numerical implementations.

Using these results, we extend a nonlinear integral equations approach developed for solving acoustic inverse obstacle scattering problems [4] to electromagnetism [5]. The algorithm has the interesting feature that it avoids the numerous numerical solution of boundary value problems at each Gauss-Newton iteration step [3], that are replaced by single matrix vector products. The effectiveness of the method is highlighted by numerical experiments.

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Complex variable BEM for a Gurtin-Murdoch material surface in the form of a circular arc in an elastic plane under far-field loads

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Keywords: surface elasticity, single layer potential, circular complex variable boundary elements

Recently, several theoretical models have been developed to include surface effects, such as surface elasticity and surface stresses. These effects are relevant to the design and analysis of micro- and nano-structures, e.g. micro- and nanomechanical devices, micro- and nano-electromechanical systems (MEMS and NEMS), etc. One most popular model is the Gurtin-Murdoch model of elastic material surfaces [1–3]. In this model, in addition to a standard elastic model for the bulk material, the Boundary Value Problem also includes special and somewhat intricated boundary conditions in terms of jumps in displacements and/or tractions expressed via a new set of surface variables.

Elastic potentials introduced by Kupradze [4] provide a suitable mathematical tool to represent these jumps. In the present work, a single layer potential is applied to solve the problem of a Gurtin-Murdoch material surface in the form of a circular arc in an elastic plane under biaxial far-field loads. The density of this single layer potential has a physical meaning of the traction jump. This traction jump is expressed in terms of the surface tension (a kind of prestress in the material surface) and the first and second order tangential derivatives of displacements. Then, a Boundary Integral Equation (BIE) for the Gurtin-Murdoch circular arc is deduced in terms of the tangential derivatives of displacements.

To numerically solve this BIE, a suitable complex variable representation for displacements along the circular arc is developed by introducing special complex-variable quadratic circular boundary elements [5]. Additionally, suitable boundary conditions are imposed at the end-points of the circular arc. The BIE discretized in this way leads to a system of linear algebraic equations. Several problems for different circular arc angles, values of the surface tension and far-field loads are solved, and the results for surface stress tensor, surface strain tensor, traction jumps and displacement along the circular are presented. Convergence behaviour of the numerical solutions obtained for an h-refinement of boundary meshes is studied. A special attention is paid to the analysis of the asymptotic behaviour of the solution near end-points of the circular arc.

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Two-level preconditioning for BEM with GenEO

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Keywords: DDM, BEM, Preconditioning

Domain Decomposition Methods (DDM), such as Additive Schwarz (AS), can be used to precondition linear systems arising from Boundary Integral Equations (BIE). Introduced in [1], this approach was widely studied since then and extended in various directions, see e.g. [2, 3]. The basic idea is to adapt the classical FEM-based AS (such as presented in [4]) to the BIE context: this includes a two-level preconditioner relying on a coarse space, which leads to theoretical bounds on the condition number.

Regarding the choice of relevant coarse spaces, important progress has been achieved in recent years for the FEM context. For the construction of coarse spaces, the *Generalized Eigenproblems in the Overlaps* (GenEO) has emerged as one of the most promising approach for symmetric positive definite problems, see [5]. Instead of solving a coarse problem on a coarse mesh, GenEO takes eigenvectors of well chosen local eigenproblems as a basis for the coarse space. As one of its interesting features, GenEO is only based on the knowledge of the stiffness matrix elements and discretization agnostic, left apart a few reasonable assumptions.

In this talk, we will present recent theoretical and numerical results in 2D and 3D aiming at adapting GenEO to the BIE context for symmetric positive definite problems on closed and open surface. Examples of applications are Laplace problems on screens or dissipative Helmholtz problems.

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Accelerated Direct Solvers for Boundary Integral Equations

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Keywords: Direct solver, fast algorithm, structured matrix computations, high order discretization.

The development of linear complexity solvers for the dense linear equations that arise upon the discretization of a boundary integral equation (BIE) has remained an area of active research for several decades. A well established paradigm is to discretize the BIE using a method such as BEM or Nyström, and then to solve the resulting system using an iterative method, where each matrix-vector multiplication is accelerated by the Fast Multipole Method, or some similar fast summation technique. In many environments, the iteration converges rapidly, and the state-of-the-art is very satisfactory.

However, there remain areas where iterative methods are not ideal. One important environment concerns BIEs associated with the Helmholtz equation or the time-harmonic Maxwell equations where the solutions are oscillatory, and the underlying physics of the problem is often highly ill-conditioned. In the presence of multiple reflections, cavities, resonances, etc., iterative solvers often struggle to converge, and finding effective pre-conditioners remains challenging.

Interestingly, it has over the last several years been demonstrated that it is often possible to directly compute an approximate inverse to the coefficient matrix in linear (or close to linear) time. The underlying idea is exploit the fact that interactions between different subdomains tend to have low numerical rank, and to use this observation to store all large matrices involved in so called "data-sparse" formats. The talk will describe some recent work in this field, and will argue that the resulting direct solvers allow us to completely sidestep the difficulties of slow convergence of iterative solvers. Moreover, the direct solvers being proposed have low communication requirements, and appear to be very well suited to parallel implementations.

One drawback of direct solvers is that methods that are currently known tend to require substantially more storage per degree of freedom than competing iterative methods. The talk will discuss how this makes direct solvers particularly effective when they are combined with high order discretizations.

Application of numerical continuation method to BIE for steady-state wave scattering by a crack with contact acoustic nonlinearity

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Keywords: contact acoustic nonlinearity, wave scattering, numerical continuation method, harmonic balance method

Nonlinear ultrasonic testing (NLUT) based on contact acoustic nonlinearity (CAN) has been developed for inspection of closed cracks [1]. Accurate NLUT requires an understanding of the behavior of higher- and sub-harmonic waves, which are used for the defect evaluation. However, the theoretical explanation of the nonlinear scattering phenomena with CAN is not sufficient at present. In particular, there remains investigation of nonlinear resonance due to the interaction among incident frequency and amplitude, size of crack, and CAN.

In order to investigate the behavior of nonlinear resonance with higher- and sub-harmonic generation, the present study deals with the steady-state analysis of in-plane wave scattering by a crack with contact boundary conditions. The system is composed of an unbounded elastic solid which includes a crack under pre-opening displacement or static compressive stress. A time-harmonic P or SV wave is incident, and clapping motion and dynamic friction are induced as a nonlinear phenomenon.

As is well known, transient wave scattering by a crack with nonlinear boundary conditions can be described in a retarded potential boundary integral equation (BIE). On the other hand, the authors proposed the steady-state BIE by means of a harmonic balance method as an asymptotic expression of the vibration of crack faces after a sufficient elapsed time [2], and the validity of the steady-state BIE has been confirmed numerically. However, the steady-state BIE has multiple solutions under the condition with respect to the sub-harmonic resonance reported in [3]. Therefore, it is necessary for comprehension of the nonlinear resonance to investigate the structure of steady-state solution space.

For the above purpose, a numerical continuation method [4] is applied to the steady-stae BIE for tracking the solution path, and a bordered method [5] is used for accurate detection of bifurcation points. Through the numerical results, the relation between the bifurcation of steady-state solution and the nonlinear resonance is investigated.

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A Fast Direct Solver for the One-Periodic Transmission Problems Formulated with the Multi-Trace Boundary Integral Equation

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Keywords: transmission problems, fast direct solver, interpolative decomposition

Wave scattering problems for domains having periodic structures are of interest since they have many applications in engineering and science, e.g., metamaterials with special properties such as negative refractive index. Therefore, there exist increasing demands for numerical methods of analysis for one-periodic transmission problems for wave scattering. Fast direct solvers for the periodic wave scattering problems have already been proposed by several research groups [1,2]. The fast direct solvers by Gillman et al. [1] and Greengard et al. [2] are based on the interpolative decomposition approaches. In interpolative decomposition approaches, it is important to speed up the algorithm by using a technique called proxy which replaces the evaluation of the influence from far boundary by that from a virtual local boundary. In the transmission problems, integral equations for more than two domains are coupled, thus making the use of proxy more delicate than in single domain problems. However, the use of proxy in multi-domain problems have not been discussed very much so far.

Therefore in this presentation, we apply the Martinsson-Rokhlin type fast direct solver [3] to one-periodic transmission problems using multi-trace boundary integral equation formulation [4]. We point out that the numerical accuracy of the well-known PMCHWT formulation decreases when the contrast between the interior and the exterior dielectric constants is small. As in the non-periodic case [5], we show that the deterioration of the numerical accuracy can be avoided with the help of the multi-trace boundary integral equations. We also propose a method to improve the numerical accuracy using a new construction of the proxy. Finally, we demonstrate the validity of the proposed method with several numerical examples and compare its performance with that of the periodic fast multipole boundary element method based on the PMCHWT formulation and GMRES [6].

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A topology optimisation of elastic wave absorber with the BEM and $\mathcal{H}$-matrix method

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Keywords: topology optimisation, elastic wave, wave absorber, $\mathcal{H}$-matrix method

1 Introduction

Vibration of a structure is one of the most important issues in engineering, and a lot of works have been devoted to the realisation of vibration reduction devices. Recent studies have applied the topology optimisation to design structures with desirable vibration properties and showed its effectiveness, e.g., [1]. In those optimisations, the finite element method (FEM) is commonly used to compute design sensitivity. However, when the design object is large enough that its size is assumed to be infinite, the boundary element method (BEM) is preferred to the FEM because the BEM can deal with the unbounded domain accurately with only a boundary mesh.

In this study, we present a numerical method for designing elastic wave absorber composed of an elastic matrix and viscoelastic inclusions using a topology optimisation. We mainly discuss the topological derivative for such a problem and its numerical evaluation. Since the topological derivative consists of solutions of two boundary value problems that differ only in boundary conditions, it is preferable to solve them using the (fast) direct boundary element method, rather than the iterative one. For this reason, we use LU decomposition based on $\mathcal{H}$-matrix method to compute the topological derivative. With the computed topological derivative, the topology optimisation is done with a level-set-based solver [2].

2 Numerical example

We present a numerical example of the topology optimisation to find a configuration which maximises the absorbed energy per unit time J in the fixed design domain $[0\,\mathrm{m}, 1\,\mathrm{m}] \times [0\,\mathrm{m}, 1\,\mathrm{m}]$. The elastic matrix $\Omega^{(1)}$ and viscoelastic inclusion $\Omega^{(2)}$ are respectively assumed to be steel (mass density $\rho = 7.8 \times 10^3\,\mathrm{kg}$, Young's modulus $E = 205\,\mathrm{GPa}$, Poisson's ratio $\nu = 0.3$) and epoxy resin ($\rho = 1.85 \times 10^3\,\mathrm{kg}$, $E = 3\,\mathrm{GPa}$, $\nu = 0.34$, loss factor $\tan(20/180)\pi$) under plane strain conditions. The incident wave is set to be a plane P-wave propagating in x_1 direction with the frequency $5\,\mathrm{kHz}$ and amplitude $1\,\mu\mathrm{m}$. Figure 6 shows the optimisation result, from which we confirm the effectiveness of the proposed method.

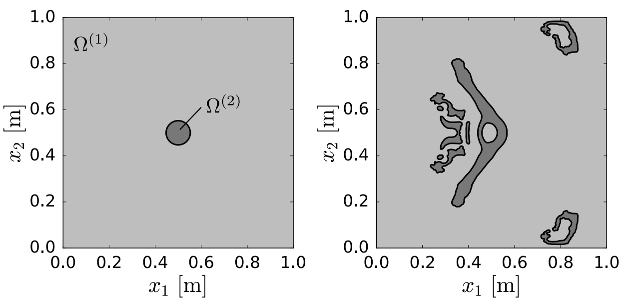

Figure 6: *Optimisation result. The left and right figures show the initial configuration ($J = 0.106\,\mathrm{kW/m}$) and optimal one ($J = 17.9\,\mathrm{kW/m}$), respectively.*

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A Padé-localized absorbing boundary condition for 2D time-harmonic elastodynamic scattering problems

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We focus on the construction of an absorbing boundary condition (ABC) for 2D isotropic elastic waves in high frequency regime.
Keywords: scattering, elastic waves ABC, approximate Dirichlet-to-Neumann map

Problem statement

Let us consider $\Omega^- := \{\mathbf{x} \in \mathbb{R}^2 : |\mathbf{x}| \leq r_{\text{int}}\}$ with boundary Γ and its complementary $\Omega^+ := \mathbb{R}^2 \backslash \Omega^-$. The Lamé parameters, μ and λ; the wavenumbers κ_p and κ_s (associated with the pressure wave, respectively the shear wave) and the density ρ are positive constants. Considering a time-harmonic incident wave $\mathbf{u}^{\text{inc}}$, the scattering problem is formulated as follows: find the displacement $\mathbf{u}$ in Ω^+ solution to the Navier equation with a Dirichlet boundary condition $\mathbf{u} = -\mathbf{u}^{\text{inc}}$ on Γ and satisfying the Kupradze radiation conditions at infinity. The stress tensor σ is isotropic. In view of a finite element discretization, Ω^+ is truncated by an artificial boundary Γ^∞, which delimits the bounded domain Ω under study. Denoting $\mathbf{n}$ the outgoing normal vector to Γ^∞, $\mathcal{T} := 2\mu\partial_{\mathbf{n}} + \lambda\mathbf{n}\text{div} + \mu\,\mathbf{n} \times \text{curl}$ the traction operator and $\mathcal{B}$ the absorbing operator describing boundary conditions at infinity, the solution satisfy on the fictitious boundary Γ^∞: $\mathcal{T}\mathbf{u} = \mathcal{B}\mathbf{u}$.

Analytical and numerical comparisons of two ABCs

Multiple choices are possible for the absorbing operator $\mathcal{B}$, the optimal operator being the exact exterior Dirichlet-to-Neumann map. We investigate two approximations:

○ The Lysmer and Kuhlemeyer condition: $\mathcal{B}\mathbf{u} = \text{i}[(\lambda + 2\mu)\kappa_p(\mathbf{n}\cdot\mathbf{u}) + \mu\kappa_s(\boldsymbol{\tau}\cdot\mathbf{u})]$, with $\boldsymbol{\tau}$ the tangential vector to Γ^∞. We point out the limitations of this low-order condition in high-frequency regime and/or with incident S-waves. It motivates the investigation of a high-order condition.

○ A Padé-localized condition: $\mathcal{B}\mathbf{u} = (I + \Lambda_2)^{-1}\Lambda_1\mathbf{u} + 2\mu\mathcal{M}\mathbf{u}$, with $\mathcal{M}$ the tangential Günter derivative [2]; $\Lambda_1 = i\rho\omega^2\left[\mathbf{n}(\partial_s^2 + \kappa_{p,\epsilon}^2 I)^{-1/2}\mathbf{n}.I_n + \boldsymbol{\tau}(\partial_s^2 + \kappa_{s,\epsilon}^2 I)^{-1/2}\boldsymbol{\tau}.I_t\right]$; $I_n = \mathbf{n} \otimes \mathbf{n}$, $I_t = I - I_n$; ∂_s the curvilinear derivative; $\Lambda_2 = -i\left[\boldsymbol{\tau}\left(\partial_s(\partial_s^2 + \kappa_{s,\epsilon}^2 I)^{-1/2}\mathbf{n}.I_n\right) - \mathbf{n}\left(\partial_s(\partial_s^2 + \kappa_{p,\epsilon}^2 I)^{-1/2}\boldsymbol{\tau}.I_t\right)\right]$ and $\kappa_{p/s,\epsilon} := \kappa_{p/s} + i\epsilon_{p/s}$, with $\epsilon_{p/s} > 0$ damping parameters. Local representation of the inverse of the square-root operators are obtained using complex Padé approximants with rotating branch-cut [1, 3]. We detail the choice of the different parameters and the construction of this high-order condition which is an adaptation of Chaillat et al. [2] to the 2D case. Numerical simulations in low and high frequency regime with incident P-waves or S-waves attest the efficiency of this ABC.

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Boundary-Domain Integral Equations for Stokes and Brinkman Systems with Variable Viscosity in Lp-based spaces on Lipschitz Domains

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Keywords: Variable-coefficient Brinkman system, Robin problem, Lipschitz domains, Boundary-domain integral equations, Sobolev spaces

The purpose of this presentation is to obtain well-posedness results in L_p-based Sobolev spaces for boundary value problems of Robin type for the Stokes and Brinkman systems in a bounded Lipschitz domain in $\mathbb{R}^3$ with the variable viscosity coefficient and data in L_p-based Sobolev and Besov spaces. First, we introduce a parametrix and construct the corresponding parametrix-based variable-coefficient Stokes Newtonian and layer integral potential operators with densities and the viscosity coefficient in L_p-based Sobolev or Besov spaces. Then we generalize various properties of these potentials, known for the Stokes system with constant coefficients, to the case of the Stokes system with variable coefficients. Next, we show that the solvability of our Robin boundary value problem for the Stokes system with variable coefficients can be reduced to that of a system of segregated Boundary-Domain Integral Equations (BDIEs). Then we prove that solvability of the variable coefficient system of BDIEs can be reduced to the solvability of a corresponding problem with constant coefficients in L_p-based Sobolev and Besov spaces, which we show to have a unique solution by exploiting known results for the Robin boundary value problem associated to the Stokes system. Finally, the well-posedness for the Stokes system is used to reduce the Robin problem for the variable coefficient Brinkman system to an equivalent Fredholm equation which is uniquely solvable in L_p-based Sobolev and Besov spaces. The presentation outlines some further development of [1–3].

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Converging expansions for Lipschitz self-similar perforations of a plane sector

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Keywords: Dirichlet problem, corner singularities, perforated domain

In contrast with the well-known methods of matching asymptotics and multiscale (or compound) asymptotics, the "functional analytic approach" proposed by Lanza de Cristoforis [5] allows to prove convergence of expansions around interior small holes of size ε for solutions of elliptic boundary value problems. Using the method of layer potentials, the asymptotic behavior of the solution as ε tends to zero is described not only by asymptotic series in powers of ε, but by convergent power series. In this talk we present the result of [2], where we use this method to investigate the Dirichlet problem for the Laplace operator where holes are collapsing at a polygonal corner of opening ω. The strategy relies on a combination of odd reflections and conformal mappings so that the original problem is transformed into a similar problem where the perforations are near the center of a disc, on potential theory on Lipschitz domains (see [3]), and on a detailed analysis of the solution of the limiting problem in proximity of the corner (cf. [1]). We show that in addition to the scale ε there appears the scale $\eta = \varepsilon^{\pi/\omega}$ when π/ω is irrational, the solution of the Dirichlet problem is given by convergent series in powers of these two small parameters. The final outcome can be compared with the multi-scale expansions [4] for which convergence does not hold in general.

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A shape and topology optimisation using the BEM and an explicit boundary expression with the level set method

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Keywords: Shape and topology optimisation, Level set method, Boundary element method

In the past three decades, topology optimisation has been extensively researched as optimal design methods in material mechanics and is now recognised as the most promising one because of its design flexibility. Now, some researchers have started to apply the topology optimisations to wide varieties of physical problems such as fluid mechanics, thermal engineering, acoustics, elastodynamics, electromagnetism and so forth. In the typical topology optimisation, the geometrical information on the design object is expressed as a distribution of a function. The level set method is one of such geometry expression methods [1]. In the level set method, the structural boundary is implicitly defined as a zero-iso-contour of a scalar function which is often called as the level set function (LSF). By its definition, the inner/outer regions of the boundary are recognised by the sign of the LSF. With the level set method, we can naturally express the topological change in design objects i.e. merging materials/split of a material. Most of the existing level-set-based topology optimisation, however, use a fixed finite element mesh for the sensitivity analysis to reduce the meshing cost in optimisation steps, and material properties of each finite element are given in proportional to the LSF, which may lead low accuracy in the sensitivity analysis [2].

To reconcile low meshing cost and high accuracy, we have developed a topology optimisation method using the BEM [3]. Since the BEM requires the meshing only on the boundary, numerical efforts for the mesh generation is very cheap compared to that for the FEM. This is especially true in the context of wave scattering problems for which the FEM requires large computational costs to generate the mesh. Thus, the infinite domain needs to be approximated by a huge finite domain in order to obtain accurate results. Hence, for the topology optimisation especially in the wave scattering problems, the BEM is far more suitable than the FEM. In the previous researches including ours, however, configurations of design objects are updated by using only the topological derivative, which causes a slow convergence of the optimisation. Furthermore, it is not clear if it is mathematically valid to deform the existing boundaries by the topological derivative.

In this study, we propose a novel topology optimisation which uses both shape and topological derivatives. We consider associating the variation of the objective functional by a shape deformation and topological changes with an infinitesimal increase of the LSF, which makes it possible to naturally treat shape and topological derivatives simultaneously. Additional computational efforts to compute the two derivatives are quite cheap since these derivatives can be expressed by the solution of the same boundary value problems, but the convergence of the proposed optimisation is improved nonetheless.

In the oral presentation, we show the detailed formulation and some numerical examples in two-dimensional electromagnetic fields which indicate the effectiveness of the proposed method.

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Computation of layer potentials in the BEM with the space-time method for the heat equation in 2D.

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Keywords: space-time method, time-domain boundary element method, heat equation

A discretisation method based on the space-time method [1] for a boundary integral equation is discussed. The space-time method is a discretisation method, which treats the time direction as an additional spatial coordinate and discretises boundary integral equations in the space-time domain. In this talk, the time-domain boundary integral equation for the heat equation in 2D [2] is discretised with the space-time method. In a standard time-domain boundary element method, integrals in the layer potentials consist of integrals with respect to space and time. The time integral is known to be estimated explicitly and the space integral is usually calculated with an appropriate quadrature rule [3]. We will show that a similar approach for computations of the integrals in the space-time method requires strong restriction for discretisation mesh. In order to avoid this restriction, we propose another way of estimating those integrals. The computational time and accuracy of this method are verified thorough some numerical examples.

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Optimisation of Electromagnetic Metamaterials Using Periodic FMM and Cylindrical-Hole Topological Derivatives

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Keywords: Periodic FMM, Maxwell's equation, topological derivative, metamaterial, optimisation

In the proposed presentation, we discuss an application of the periodic fast boundary integral method for Maxwell's equations in topological optimisation problems for electromagnetic metamaterials.

Electromagnetic metamaterials are micro-structured artificial 'materials' designed to exhibit macro-scale behaviours which no natural materials posses. NIMs (negative-index materials), having apparent negative refractive indices, are examples of electromagnetic metamaterials which are typically realised with fishnet structures [1], i.e., dielectric films sandwiched between thin metallic layers having periodical holes. One may want to optimise the micro-structure of metamaterials, e.g., the shape, number and arrangement of holes in the fishnet structure case, in order to obtain enhanced performances. One may therefore be interested in the topological optimisation of metamaterials, in which the topological derivatives of objective functions play an important role. In optical metamaterials, however, the size of the micro-structure is of the order of the wavelength of light, thus restricting the degrees of freedom for fabrication. Therefore, possible structures of optical metamaterials are often two dimensional although the underlying electromagnetic phenomena are three dimensional. In such cases, one is more interested in topological derivatives associated with circular cylindrical holes than the conventional ones associated with spherical holes.

For example, we consider domains which are periodic in $x_{1,2}$ directions but extend to $\pm\infty$ in x_3 direction as shown in Figure 1. The domain Ω_- (Ω_+) is filled with air (dielectric material) and one is interested in obtaining time harmonic electromagnetic fields in these domains. In the domain Ω_+ one may consider a cylindrical hole having a radius of ρ which connects the air regions. The cylindrical-hole topological derivative (CHTD) $\check{\boldsymbol{E}}$ of the electric field $\boldsymbol{E}$ is defined by $\check{\boldsymbol{E}} = \lim_{\rho\downarrow 0}(\boldsymbol{E}(\Omega\setminus\Omega_\rho) - \boldsymbol{E}(\Omega))/c\rho^2$ where $\boldsymbol{E}(\Omega\setminus\Omega_\rho)$ ($\boldsymbol{E}(\Omega)$) stands for $\boldsymbol{E}$ with (without) cylindrical hole and c is a constant. This limit exists in the present context.

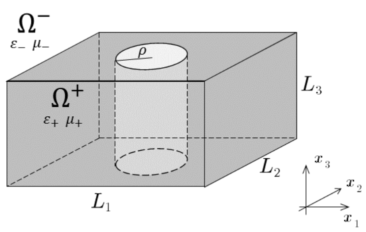

Figure 7: *Domains and cylindrical hole*

In the proposed presentation we consider a thin metallic film having 2 dimensional periodic array of holes. The structure is subject to an incident electromagnetic wave (light). We derive formulae for the CHTDs of both electromagnetic fields and the magnitudes of the transmitted waves. We then use these CHTDs together with Yamada's level set method [2] to formulate a topological optimisation method. No penalty for the number of holes is needed since this approach automatically excludes too complicated hole geometries. The required computation of the periodic electromagnetic fields is carried out efficiently with the help of the periodic FMM for Maxwell's equations [3]. We demonstrate the usefulness of the proposed optimisation method by numerical examples in which the magnitude of the transmitted electric field is maximised subject to the condition that the total area of holes is less than a given limit.

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Some boundary element methods for multiply-connected domains

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Keywords: boundary element method, multiply-connected domains

In case of multiply-connected domains, some properties of the boundary integral operators differ from the setting of a simply-connected domain. In particular, the kernel and the ellipticity property of the hypersingular operator are different. As a consequence, some boundary integral formulations, like the symmetric formulation of mixed boundary value problems, may have a larger kernel than the considered problem itself.

We will discuss some details of the changes in the analysis of the boundary integral operators and of the considered boundary integral formulations. We will show some examples of failures of specific formulations and how to fix these by appropriate modifications.

A Fast Boundary Integral Method for Generating High-order Surface Meshes

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Keywords: surface mesh, high-order, fast multipole method, integral equations, fast Gauss transform

In order to develop truly high-order integral equation-based solvers for boundary value problems in three dimensions, all aspects of the solver must be high-order: discretization of the unknown, quadratures for singular Green's functions, and most importantly, the description of the underlying geometry (surface meshes in this case). Since the development of fast high-precision algorithms such as the fast multipole method 30 years ago, there have been amazing advances in the areas of discretization, singular quadrature [2], and fast direct solvers [3, 6]. Unfortunately, due to the lack of robust schemes for generating high-quality high-order triangulated surfaces, most integral equation-based simulations have been limited to simple analytically defined geometries (despite the available high-order tools), and real-world complex engineering geometries can only be described by flat-triangulations.

In two dimensions, high-order algorithms exist for generating periodic curves from a collection data points [1], as well as for constructing high-order rounded geometries from polygons [5]. The corresponding numerical codes are relatively short and straightforward. However, in three dimensions, obtaining a smooth surface from an existing flat triangulation has proven to be more complicated. Mesh-repair algorithms [4], while powerful, are generally limited to re-generating flat triangulations.

In this talk we will describe a recently developed algorithm for transforming a flat triangulation (i.e. skeleton) of a smooth boundary Γ of a domain Ω into a high-order curvilinear triangulation that can then be coupled with high-order integral equation methods. The algorithm is based on the fact that convolution of the indicator function of the domain, 1_Ω , with a Gaussian G_σ that has width σ results in an infinitely differentiable level-set function Φ in the volume. The $\Phi = 1/2$ level-set can then be meshed to high-order, and coincides closely with the original skeleton of Γ. Furthermore, this computation can be reformulated as a boundary integral, accelerated via an FMM, and σ can be constructed to depend on the local mesh size of the skeleton. Various numerical examples will be shown, including high-order results from acoustic and electromagnetic scattering problems.

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A Hybrid Numerical-Asymptotic Collocation BEM for High-Frequency Scattering by 2D Planar Screens

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Keywords: Boundary Element Method, High-Frequency Scattering, Oscillatory quadrature

Conventional BEMs based on piecewise polynomial approximation spaces are computationally expensive for high frequency scattering problems, because one needs at least a fixed number of degrees of freedom per wavelength in order to capture the oscillatory solution. The Hybrid Numerical-Asymptotic (HNA) approach attempts to overcome this by introducing partial knowledge of the high-frequency asymptotic solution behaviour into the numerical approximation space, representing the BEM solution as a sum of prescribed oscillatory functions multiplied by piecewise polynomial amplitudes on coarse (essentially frequency-independent) meshes. Such approximation strategies have been shown, for a range of scattering problems, to dramatically reduce the number of degrees of freedom required to accurately represent the oscillatory solution at high frequencies [2].

The price one pays for this reduced number of degrees of freedom is that calculating the entries of the (small) BEM matrix now involves the evaluation of oscillatory integrals. Hence an efficient HNA BEM implementation needs fast oscillatory quadrature methods. Most HNA methods analysed in the literature to date involve Galerkin (variational) formulations. Here the oscillatory quadrature is particularly challenging because of the high dimensionality of the integrals involved (double/quadruple integrals for 2D/3D problems, respectively). In spite of this, efficient implementations are possible, and for the particular case of scattering by arbitrary 2D planar screens the Galerkin HNA BEM presented in [3] achieves fixed accuracy of approximation with a frequency-independent computational cost. Collocation formulations offer a simpler alternative, since they involve lower-dimensional integrals. However, they have not been investigated as widely as Galerkin implementations, with the low-order method in [1] being a notable exception.

In this talk we present a collocation BEM implementation of the high-order exponentially accurate *hp* HNA approximation space used in [3]. The (one-dimensional) oscillatory integrals in the BEM matrix are computed with frequency-independent computational cost using Filon quadrature, and our Python/C++ implementation can compute arbitrarily high frequency solutions in just a few seconds on a laptop. We report results from our numerical investigations into how the design of the HNA approximation space and the choice of collocation points affect accuracy and conditioning. We found that when compared to the Galerkin scheme, similar convergence rates can be achieved, provided that we "oversample" slightly, using a number of collocation points somewhat larger than the number of degrees of freedom.

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Radial Integration BEM for Solving Convection-Conduction Problems

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Keywords: convection-conduction problem, BEM, radial integration method, domain integral

In this paper, the radial integration boundary element method without internal cells is presented for solving the steady/unsteady convection-conduction problems with spatially variable velocity and coefficient. The temperature boundary integral equation is derived by employing the fundamental solution for the potential problems (Green function) as well as the normalized temperature and thermal conductivity [1], which result in the appearance of domain integrals including the unknown quantities. To avoid evaluating the domain integrals with internal cells, the transformation of domain integrals to boundary integrals is carried out by employing the radial integration method [2] and approximating the unknown quantities with the use of the compactly supported fourth-order spline radial basis functions combined with polynomials in global coordinates [3]. Based on the central finite difference technique, an implicit time marching solution scheme is developed for solving the time dependent system of equations. Finally, a boundary element method without internal cells is established for the analysis of steady/unsteady convection-conduction problems with variable velocity and coefficients. Several numerical examples are given to demonstrate the validity and effectiveness of the proposed method.

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Radial integration BEM for nonlinear heat conduction problems with temperature-dependent conductivity

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Keywords: Nonlinear heat conduction problem, BEM, Temperature dependent conductivity, Radial integration method

In this paper, a new and simple boundary-domain integral equation is presented to solve nonlinear heat conduction problems with temperature-dependent conductivity of materials. The boundary-domain integral equation is formulated for nonlinear heat conduction problems by using the fundamental solutions for the corresponding linear heat conduction problems [1, 2], which results in the appearance of a domain integral due to the variation of the heat conductivity with temperature. The arising domain integral is converted into an equivalent boundary integral using the radial integration method (RIM) [3] by expressing the temperature as a series of basis functions. This treatment results in a pure boundary element algorithm and requires no internal cells to evaluate the domain integral. To solve the final system of algebraic equations formed by discretizing the boundary of the problem into boundary elements, the Newton-Raphson iterative method is applied. Numerical examples are presented to demonstrate the accuracy and efficiency of the present method.

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Boundary Integral Equations and Hierarchical Matrices for a Waveguide Mode Solver

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Keywords: Photonic Crystal fiber, Boundary Element Method, H-matrices

Propagation of light in Photonic Crystal Fibers (PCF) depends on several geometrical parameters and material characteristics, therefore numerical methods are needed to accurately study various PCF configurations. More precisely, PCF cross sections consist of homogeneous inclusions with refractive index n_{i} surrounded by a medium with refractive index n_{e} (see Fig. 8); the total effective refraction index n_{eff} is the quantity of interest. In recent works [1], a waveguide mode solver based on the Boundary Element Method (BEM) was proposed to determine n_{eff}. Our aim is to improve the performances of this solver by using compression techniques and hierarchical matrices for the BEM matrices [2, 3].

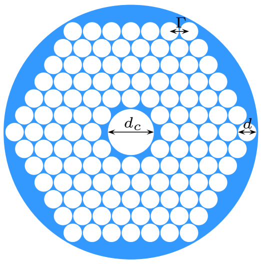

Figure 8: *Schematic of a PCF cross section*

The mode solver requires to solve many instances of a direct problem. The use of compression techniques for the direct problem leads to many difficulties because the involved matrix is not a classic BEM matrix but a matrix with sparse and full blocks and is ill-conditioned. An iterative solution with an approximate hierarchical LU preconditioning enables to enhance the convergence rate. For this preconditioner, as reported in Table 1, the accuracy criterion ϵ_{LU} has to be smaller than the other criteria ϵ_{ACA}=ϵ_{GMRES}=10^{-5} to be efficient which is unusual. Note also that the coarsening performed to simplify the hierarchical structure can enhance the performances for the smallest values of ϵ_{LU}. The results and the corresponding explanations will be reported at the conference.

ϵ_{LU}	without coarsening	with coarsening
10^{-4}	1569	1228
10^{-5}	622	271
10^{-6}	92	38

Table 1: *Number of iterations for a case with 4840 unknowns and* ϵ_{GMRES}=10^{-5}.

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Collocation Methods for Retarded Potential Boundary Integral Equations with Space-Time Trial Spaces

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Keywords: time domain, 3D wave equation, space-time mesh

Most popular approximation methods for time domain boundary integral equations have in common that they discretize space and time separately. In particular, well-established boundary element methods for stationary integral equations are combined with suitable time discretization schemes [1]. On the other hand, space-time discretizations treat the time variable like an additional spatial coordinate, facilitating the application of traditional finite element technology to the entire initial boundary value problem. While space-time finite element methods have already reached a certain degree of maturity, the development of such boundary element methods is lacking.

In this talk, we discuss collocation schemes for retarded potential boundary integral equations with space-time basis functions. The lateral boundary of the space-time cylinder is described by an unstructured tetrahedral mesh. On this mesh standard finite element spaces are constructed and used to approximate the densities. The central challenge is the robust numerical integration over the intersection of the space-time mesh and the surface of the backward light cone. We provide a first concept of such a cubature method based on popular techniques utilized in the context of implicitly defined surfaces [2]. Several numerical examples with encouraging results are examined.

The talk concludes by addressing critical issues encountered in this early stage of development, the limitations of the presented approach as well as its competitiveness.

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A Boundary Integral Equation for the Clamped Bi-Laplacian Eigenvalue Problem

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Keywords: eigenvalues, biharmonic equation, clamped points, mode elimination, vibration confinement

3 Problem Statement

We consider small amplitude out-of-plane vibrations of thin elastic plates with defects. This is modelled as an eigenvalue problem of the bi-Laplacian with a discrete set of clamped points. A boundary integral equation (BIE) is used to compute the eigenvalues for a wide range of two-dimensional geometries. Two key phenomena of the eigenfunctions will be discussed. First, careful placement of clamping points can entirely eliminate particular eigenvalues and suggests a strategy for manipulating the vibrational characteristics of rigid bodies so that undesirable frequencies are removed. Second, clamping can result in partitioning of the domain so that vibrational modes are largely confined to certain spatial regions. Future work includes solving similar eigenvalue problems to simulate porous media flow with small obstacles.

4 Boundary Integral Equation Formulation

We are interested in solutions of the eigenvalue problem $\Delta^2 u = \lambda u$, $\mathbf{x} \in \Omega \subset \mathbb{R}^2$, with the clamping constraint $u(\mathbf{x}_k) = 0$, $k = 1, \ldots, M$ [2]. We use the clamped boundary condition $u = \partial_{\mathbf{n}} u = 0$, $\mathbf{x} \in \partial\Omega$. Letting G be the fundamental solution of the eigenvalue operator $\Delta^2 - \lambda$, the local behavior of the eigenfunction at the clamped points $\mathbf{x}_k$ satisfies $u(\mathbf{x}) \sim G(\mathbf{x} - \mathbf{x}_k)$, $\mathbf{x} \to \mathbf{x}_k$. Then, the eigenfunction can be decomposed into a singular and regular part $u(\mathbf{x}) = u_S(\mathbf{x}) + u_R(\mathbf{x})$, where

$$u_S(\mathbf{x}) = \sum_{k=1}^{M} \alpha_k G(\mathbf{x} - \mathbf{x}_k),$$

and u_R satisfies the homogeneous PDE

$$\begin{aligned} \Delta^2 u_R - \lambda u_R &= 0, && \mathbf{x} \in \Omega, \\ u_R = -u_S,\ \partial_{\mathbf{n}} u_R &= -\partial_{\mathbf{n}} u_S, && \mathbf{x} \in \partial\Omega. \end{aligned}$$

Using techniques for BIEs of fourth-order PDEs [1], the PDE for u_R is recast as a second-kind system of BIEs. The singularity strengths, α_k, and eigenvalues, λ, are found by solving the non-linear equation

$$\mathbf{F}(\alpha_1, \ldots, \alpha_M, \lambda) = [u_S(\mathbf{x}_1) + u_R(\mathbf{x}_1), \ldots, u_S(\mathbf{x}_M) + u_R(\mathbf{x}_M)] = 0,$$

with the normalization condition $\alpha_1^2 + \cdots \alpha_M^2 = 1$. By applying high-order quadrature to the integral equation and an appropriate non-linear solver to F, eigenvalues and eigenfunctions can be reliably computed with high accuracy in complex two-dimensional domains.

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Solving inverse multiple scattering problems in three-dimensional electromagnetism by topological gradient methods

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Keywords: Electromagnetism, shape optimization and inverse problems, topological derivative

Abstract

Solving inverse scattering problems related with the recovery of the shape of multiple obstacles when no a priori information about their number, size or location is provided, is an important field in Applied Mathematics and Physics that arises in a large number of different industrial and engineering applications such as non-destructive testing, geophysical exploration, biomedicine, radar imaging and antenna design.

In this work we study an iterative method based on the computation of iterated topological derivatives for the detection and shape identification of multiple electromagnetic scatterers characterized by various kind of boundary conditions [1, 2]. The topological derivative of a shape functional measures the sensitivity of such functional to having an infinitesimal scatterer at each point of the region of interest. It can be used as an indicator function that classifies each point as belonging either to one scatterer or to the background media. In this work we obtain closed-form formulae for the detection of electromagnetic defects. The formulae rely on the computation of shape derivatives followed by asymptotic expansions using Mie series derived from a boundary integral formulation of the involved forward problem.

The iterative inverse algorithm, that requires to solve the forward problem at each iteration step, can be coupled to any boundary integral equation solver. We use the fast spectral algorithm developed in [3–5]. Numerical experiments illustrating the ability of the method to find shapes accurately without a priori information in a rather small number of iterations will be shown.

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Boundary element based solution of Navier-Stokes equations with variable material properties

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Keywords: variable material parameters, boundary-domain integral formulation, multiphase flow, nanofluid, domain-decomposition, adaptive cross approximation

Many natural phenomena are governed by diffusion and convection transport processes. The transport phenomena usually occur in environments where the velocity of the fluid changes within the domain in question. In some cases, for example, in turbulence modelling with turbulent viscosity hypothesis, or in cases of temperature dependent material properties, the diffusion coefficient also changes within the domain.

The solution of the unsteady convection-diffusion partial differential equation is a challenging task, for which many numerical algorithms have been proposed. If the fundamental solution of the convection-diffusion equation is adopted, the problem can be, in the case of a constant velocity field and constant coefficients, described by a pure boundary integral equation. Variable velocity and coefficients lead to domain integrals in the integral formulation. A decomposition of the velocity field into a constant and a variable part has been proposed in the past [1]. Such decomposition leads to a domain integral involving the variable part of the velocity and the unknown field function.

In this work, we present an alternative formulation [2, 3] which leads to an integral formulation where the gradient of the field function is not needed but the gradient of the diffusion coefficient is needed instead. Thus, the final integral equation includes only the unknown function on the boundary and in the domain and its derivative on the boundary.

We present boundary-domain integral formulations of the vorticity transport equation and the energy transport equation, where material properties and the velocity field are spatially and temporarily variable. When coupled with the solution of the kinematics equation, we are able to solve flow and heat transport problems. Discretization has been done using a collocation scheme. Second order accuracy of the proposed scheme has been shown. The domain integral contribution has been treated via a combination of a domain decomposition approach and matrix approximation using the Adaptive Cross Approximation method.

The developed boundary element based numerical algorithm has been used to perform simulations of flow and heat transfer of water-based suspensions of nanoparticles - nanofluids, which exhibit temperature dependent material parameters. The results show good agreement with benchmark test cases and experimental measurements and verify the validity of the proposed simulation algorithm.

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On increasing efficiency of kernel-independent fast multipole method in 2D and 3D problems

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Keywords: kernel-independent fast multipole method, boundary element method, congruent spherical elements

Numerical simulation of engineering problems with a large number of DOFs usually entails an excessive time and memory expense. Using the fast methods, which key idea is to separate the mutual node-to-node interactions into short and long-range interactions, is a promising alternative. The two commonly used fast multipole methods (FMM) are: the analytical FMM (A-FMM, [1]) and the kernel-independent FMM (KI-FMM, [2], [3]). The first one, requires involved preliminary work to obtain analytic expansions, and thus may have notable restraints. For the second, the only analytical work required is that performed for conventional (not fast) boundary element method (BEM); it concerns with accounting for *short-range* interactions between neighboring boundary elements.

The KI-FMM consists of representing *long-range* interactions by the influence of equivalent densities distributed on equivalent surfaces. The complexity of the method is proportional to p^2, where p is the number of nodes on a surface. Therefore, a proper choice of the shape of a surface, which enables reduction of a number p of nodes, maintaining the required accuracy, is of prime significance to speed up calculations and is the main goal of our investigations. For 2D problems, as we have shown in [3], the circular surfaces, being smooth, provide computational advantages over the square surfaces. This suggests using for 3D problems smooth spherical equivalent surfaces, rather than cubic surfaces. However, for spherical surfaces the problem of appropriate approximation of the surface and the density and corresponding quadrature rules arises. For solving this problem, we propose using *congruent* spherical elements, defined in a special system of angular coordinates. The special spherical elements provide an exact representation of the spherical surface and enable significant decrease of the number p of nodes. (In practice, to approximate the whole sphere, we use 6 congruent elements with the total number of 26 nodes only). Efficient integration of densities over these elements is suggested and implemented into a procedure of general use.

For illustration, 2D potential and 3D elasticity problems are considered. The key parameters of the suggested KI-FMM [3] are established in numerical experiments. The results of preliminary tests for systems with a several thousand of unknowns, confirm efficiency of the modified KI-FMM developed: (i) the accuracy of the method is at the level of the analytical FMM (in 2D case) and the conventional BEM; (ii) for 3D problems the KI-FMM with spherical surfaces is 2-3-fold faster, more accurate and stable, than when using cubic surfaces.

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Matrix-valued radial basis functions for the BEM treatment of the Lamé system

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Keywords: matrix-valued RBF; Lamé system; particular solutions

The problem of finding an approximate particular solution of an elliptic system of partial differential equations is considered. To construct the approximation, the differential operator is applied to a vector of radial basis functions, [1]. The resulting vectors are linearly combined to interpolate the vector-valued function on the right hand side. For the conservative body forces, the problem can be reduced to the scalar case, [2]. The solvability of the interpolation problem is established. Additionally, stability and accuracy estimates for the method are given. These theoretical results are illustrated on several numerical examples related to the Lamé system and its numerical solution by the use of the BEM.

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Time Domain Boundary Integral Equations for scattering by obstacles with locally homogeneous material properties

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Keywords: Time Domain Boundary Integral Equations, Galerkin methods, Convolution Quadrature

We will first discuss a single trace boundary integral formulation of a problem of transient scattering of waves by obstacles with locally homogeneous material properties. Among the aspects that will be handled are: mapping properties of the solution operator, the effect of Galerkin semidiscretization in space, and the convergence properties of Runge-Kutta Convolution Quadrature applied to the semidiscrete system of retarded equations. The exposition will be done for acoustic waves, but all results hold for elastodynamics as well.

Elastodynamic BE formulation with Runge-Kutta based Generalised Convolution Quadrature Method

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Keywords: Convolution Quadrature Method, variable time step size, Runge-Kutta methods

Boundary element formulations in time domain are well established in the engineering and the mathematical literature. In principle three types of formulations can be found:

- Direct in time domain with analytical integration of the time convolution
- Calculation in Laplace or Fourier domain with a subsequent numerical inverse transformation
- Formulations based on the Convolution Quadrature Method (CQM)

The latter formulation goes back to Lubich [1] and can either be formulated as a true time stepping method or as a calculation of decoupled Laplace domain problems with an inverse transformation (see, e.g., [2]). Common to all these approaches is the restriction to a constant time step size.

The generalisation of the CQM to a variable time step size has been done by Lopez-Fernandez and Sauter [3], where the initial works use the implicit Euler as underlying time stepping method. This choice limits the convergence order to one. To obtain higher convergence orders implicit Runge-Kutta methods as underlying time stepping method has been proposed in [5]. These formulations have been presented for the single layer potential in acoustics.

Here, the generalised CQ is applied to elastodynamics, where the single and double layer approach as well as a direct formulation for mixed boundary value problems will be presented. Essentially, the performance of the Runge-Kutta based generalized CQ is studied with respect to its convergence behaviour. As usual, the convergence order of the formulation is restricted by either the order of the Runge-Kutta method or by the spatial convergence order. In the presentation only a low order spatial discretisation is used. Numerical examples show the expected behavior.

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Adaptive BEM with inexact PCG solver yields almost optimal computational costs

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Keywords: adaptive mesh-refinement, additive Schwarz preconditioning, convergence of adaptive BEM, optimal convergence rates, almost optimal computational costs

We consider the preconditioned conjugate gradient method (PCG) with optimal preconditioner in the frame of the boundary element method (BEM) with adaptive mesh-refinement. As model problem serves the weakly-singular integral equation $V\phi = f$ associated with the Laplace operator. Given an initial mesh $\mathcal{T}_0$, adaptivity parameters $0 < \theta \le 1$ and $\lambda > 0$, counter $j = 0$ (for the mesh-sequence $\mathcal{T}_j$) and $k = 0$ (for the PCG steps per mesh $\mathcal{T}_j$), as well as a discrete initial guess $\phi_{00} \approx \phi$ on $\mathcal{T}_0$ (e.g., $\phi_{00} = 0$), our adaptive strategy reads as follows:

(i) Update counter $(j,k) \mapsto (j,k+1)$.
(ii) Do one PCG step to obtain ϕ_{jk} from $\phi_{j(k-1)}$.
(iii) Compute refinement indicators $\eta_j(T,\phi_{jk})$ for all elements $T \in \mathcal{T}_j$.
(iv) If $\lambda^{-1}\,\|\phi_{jk} - \phi_{j(k-1)}\|^2 > \eta_j(\phi_{jk})^2 = \sum_{T\in\mathcal{T}_j} \eta_j(T,\phi_{jk})^2$, continue with (i).
(v) Otherwise determine marked elements $\mathcal{M}_j \subseteq \mathcal{T}_j$ such that $\theta\,\eta_j(\phi_{jk})^2 \le \sum_{T\in\mathcal{M}_j} \eta_j(T,\phi_{jk})^2$.
(vi) Refine all $T \in \mathcal{M}_j$ to obtain the new mesh $\mathcal{T}_{j+1}$.
(vii) Update counter $(j,k) \mapsto (j+1,0)$ and continue with (i).

For *a posteriori* error estimation, we employ the weighted-residual error estimator. If the final ϕ_{jk} on each mesh $\mathcal{T}_j$ is the exact Galerkin solution, then linear convergence of adaptive BEM, even with optimal algebraic rates, has first been proved in [3, 4]. As a novel contribution, we now extend this result to adaptive BEM with inexact PCG solver.

We prove that the proposed adaptive algorithm does not only lead to linear convergence of the error estimator (for arbitrary $0 < \theta \le 1$ and $\lambda > 0$) with optimal algebraic rates (for $0 < \theta, \lambda \ll 1$ sufficiently small), but also to almost optimal computational complexity, if $\mathcal{H}^2$-matrices (resp. the fast multipole method) are employed for the effective treatment of the discrete integral operators. In particular, we provide an additive Schwarz preconditioner which can be computed in linear complexity and which is optimal in the sense that the condition numbers of the preconditioned systems are uniformly bounded (see [2] in the context of the hypersingular integral equation).

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Stable non-symmetric coupling with the boundary element method for a convection-dominated parabolic-elliptic interface problem

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Keywords: finite volume method, upwind stabilisation, boundary element method, SUPG, parabolic-elliptic interface problem, coupling

Many problems in electrical engineering or fluid mechanics can be modelled by parabolic-elliptic interface problems, where the domain of the exterior elliptic problem might be unbounded. This class of problems can be solved by the non-symmetric coupling of finite elements (FEM) and boundary elements (BEM) analysed in [1]. However, if the parabolic equation in the interior domain is convection dominated, this method is not stable anymore, i.e., heavy oscillations can occur in the computed solution.

A possible remedy for this unwanted behaviour is to use an upwind stabilised vertex-centered finite volume method (FVM), thus we propose a (non-symmetric) coupling of FVM and BEM. This method has already been analysed for stationary problems, see [2], but not for time-dependent ones. As an alternative for the stabilised FVM we will also use the Streamline Upwind Petrov-Galerkin method (SUPG), which is based on FEM. Hence, this leads to the (non-symmetric) SUPG-BEM coupling.

In this talk we will present the main ideas of both coupling methods. Furthermore, we will discuss the analysis of the methods concerning convergence and error estimates. Finally, some numerical examples compare the methods and illustrate the theoretical findings.

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Weak imposition of boundary conditions using a penalty method

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Keywords: penalty method, Nitche's method, Robin boundary conditions

In recent years, Nitche's method [1] has become increasingly popular within the finite element community as a method for weakly imposing boundary conditions. Inspired by this, we propose a penalty-based method for weakly imposing boundary conditions within boundary element methods.

We consider boundary element methods where the Calderón projector is used for the system matrix and boundary conditions are weakly imposed using a particular variational boundary operator. Regardless of the boundary conditions, both the primal trace variable and the flux are approximated. We focus on the imposition of Dirichlet, mixed Dirichlet–Neumann, and Robin conditions for Laplace problems.

The theory is illustrated by a series of numerical examples using the software library Bempp [2].

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Fundamental coupled MHD creeping flow and electric potential for a conducting liquid bounded by a plane slip wall

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Keywords: fundamental MHD flow, solid wall, Navier slip condition, 2D Fourier transform

We determine the coupled fundamental MHD [1, 2] velocity $\mathbf{u}$, pressure p and electric potential ϕ produced by a concentrated source point immersed in a conducting Newtonian liquid located above the $z = 0$ plane, solid, motionless and impermeable slip wall. The liquid has uniform viscosity μ and conductivity $\sigma > 0$. It is subject to a prescribed uniform magnetic field $\mathbf{B}$ *normal* to the wall and with magnitude $B > 0$. Moreover, the flow Reynolds number vanishes so that $(\mathbf{u}, p)$ is a creeping flow driven by the non-uniform Lorentz body force $\mathbf{f} = \mathbf{j} \wedge \mathbf{B}$ with $\mathbf{j}$ the current. The vector $\mathbf{j}$ is given by the Ohm's law $\mathbf{j} = \sigma(\mathbf{u} \wedge \mathbf{B} - \nabla\phi)$ and obeys the charge conservation property $\nabla.\mathbf{j} = 0$. This work actually extends recent papers devoted to the unbounded liquid case [3] and to a liquid bounded by a plane no-slip wall [4]. Here, the slip plane wall is either a perfectly conducting or insulating boundary on which the famous Navier [5] slip condition is required for the fundamental flow $(\mathbf{u}, p)$. Using a two-dimensional Fourier transform (for coordinates x and y parallel with the wall) makes it possible to *analytically* obtain the required MHD fields $(\mathbf{u}, p, \phi)$ in terms of one-dimensional integrals, whatever the prescribed source point unit strength $\mathbf{e}$. At point $\mathbf{x}(x, y, z)$ in the liquid the quantities $(\mathbf{u}, p, \phi)$ are found to deeply depend on both $\mathbf{e}$ and $(x - x_0, y - y_0, z, h, d)$ with $\mathbf{x}_0(x_0, y_0, h)$ the source point location and $d = (\sqrt{\mu/\sigma})/B$ the so-called Hartmann layer thickness [6]. This dependence will be illustrated at the oral presentation by exhibiting some fundamental flow patterns computed for different wall properties (insulating or perfectly conducting nature, wall slip length taken in the Navier slip boundary condition) and for different point force orientations (i. e. taking $\mathbf{e}$ either parallel with or normal to the slip wall).

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Particle-particle interactions in axisymmetric MHD creeping flow

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Keywords: axisymmetric MHD flow, particle-particle interactions, boundary-integral equation

For some applications it is relevant to determine the MHD flow about a solid body moving in a quiescent conducting liquid in presence of an ambient magnetic field [1]. However, even for a translating sphere [2], this problem is very involved since one has to simultaneously gain not only the liquid velocity $\mathbf{u}$ and pressure p but also the magnetic field $\mathbf{B}$ and the electric field $\mathbf{E}$ prevailing in the liquid. Indeed, the quantities $(\mathbf{u}, p, \mathbf{B}, \mathbf{E})$ satisfy Navier-Stokes and Maxwell equations which are coupled by the Lorentz body force $\mathbf{f} = \sigma(\mathbf{E}+\mathbf{u}\wedge\mathbf{B})\wedge\mathbf{B}$ and the conservation law $\nabla.(\mathbf{E}+\mathbf{u}\wedge\mathbf{B}) = 0$ where $\sigma > 0$ denotes the liquid uniform conductivity. Under the assumption of low magnetic Reynolds number, $\mathbf{B}$ keeps in the entire liquid its prescribed far-field uniform value $B\mathbf{e}_z$ [2]. In addition, for a solid and insulating sphere, with radius a, translating parallel with $\mathbf{B}$ at the velocity $U\mathbf{e}_z$, the flow $(\mathbf{u}, p)$ is axisymmetric and without swirl so that $\mathbf{E} = \mathbf{0}$ [2]. This flow depends upon two dimensionless numbers: the Reynolds number $\mathrm{Re} = \rho|U|a/\mu$ (here ρ and μ designate the liquid uniform density and viscosity, respectively) and the Harmann number $\mathrm{Ha} = a/d$ comparing the sphere radius with the so-called [3] Hartmann layer thickness $d = (\sqrt{\mu/\sigma})/|B|$. As soon as $\mathrm{Re} \ll 1$ inertial effects are negligible and $(\mathbf{u}, p)$ obeys the linear creeping flow equations with Lorentz body force $\mathbf{f} = \sigma(\mathbf{u}\wedge\mathbf{B})\wedge\mathbf{B}$. This flow has been asymptotically obtained in [4] for $\mathrm{Ha} \ll 1$ and in [5] for $\mathrm{Ha} \gg 1$. Recently, [6] provided the solution whatever $\mathrm{Ha} > 0$ by employing a new boundary approach based on two fundamental axisymmetric creeping MHD flows obtained in [7]. Since particle-particle interactions are also encountered in practice, the present works extends [6] to the case of two axisymmetric spherical particles admitting the same axis of revolution parallel with $\mathbf{B}$ and translating along this axis (at not necessary equal velocities). Not only the computed force experienced by each sphere but also the flow pattern about the 2-sphere cluster will be given for several values of the sphere-sphere gap and of the Harmann number.

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Quantification of the Impact of Small Random Perturbations in Electromagnetic Scattering from Reflective Gratings

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Keywords: gratings, uncertainty quantification, hierarchical basis

Gratings are ubiquitous in optical and electromagnetic systems due to their remarkable properties. In this presentation, we present a novel deterministic method [1] to quantify the effect of stochastic perturbations on perfect electric conductor (PEC) grating surfaces. In our approach, the first two statistical moments—mean and variance—are obtained based on a first-order shape-Taylor approximation [2]. The Helmholtz equation is then solved via the boundary elements method (BEM) with hierarchical bases or Haar wavelets and to find the mean; the second moment is obtained through tensorization of the Helmholtz problem for the shape derivative and solving using a sparse approximation [3]. We compared our method with the Galerkin Monte-Carlo (MC) algorithm to validate the approximation and assess the limitations of the approach. We show that the sparse approximation converges faster than a dense one, with significantly less computational effort than MC based approaches. In addition, we compare our method with the well-known small perturbation method (SPM) [4] to obtain additional insights. Moreover, simulations of grating efficiency reveal the applicability of the approach for problems of practical significance.

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The Helmholtz h-BEM: what can be proved about the pollution effect and the behaviour of GMRES?

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Keywords: Helmholtz equation, high frequency, boundary element method, GMRES, pollution effect

This talk is concerned with the wavenumber-explicit numerical analysis of boundary integral equations for the Helmholtz equation $\Delta u + k^2 u = 0$, with wavenumber $k > 0$, posed in the exterior of a 2- or 3-dimensional bounded obstacle Ω with Dirichlet boundary conditions on $\partial\Omega$. We consider the standard second-kind combined-field integral-equation formulations of this problem

$$A'_{k,\eta} v = f_{k,\eta} \quad \text{and} \quad A_{k,\eta}\phi = g_k, \tag{5}$$

where the integral operators $A'_{k,\eta}$ and $A_{k,\eta}$ are defined by

$$A'_{k,\eta} := \frac{1}{2}I + D'_k - \mathrm{i}\eta S_k, \qquad A_{k,\eta} := \frac{1}{2}I + D_k - \mathrm{i}\eta S_k,$$

$\eta \in \mathbb{R} \setminus \{0\}$ is an arbitrary coupling parameter, S_k is the single-layer operator, D_k is the double-layer operator, and D'_k is the adjoint double-layer operator.

We consider solving the equations in (5) in $L^2(\partial\Omega)$ using the h-BEM; i.e. the Galerkin method where the approximation spaces are piecewise polynomials of fixed degree on shape-regular meshes of diameter h, with h decreasing to zero. To find the Galerkin solution one must solve a linear system of dimension $N \sim h^{-(d-1)}$; this is often done using Krylov-subspace iterative methods such as the generalized minimal residual method (GMRES).

For the numerical analysis of this situation when k is large, there are now, roughly speaking, two main questions:

Q1. How must h decrease with k in order to maintain accuracy of the Galerkin solution as $k \to \infty$?

Q2. How does the number of GMRES iterations required to achieve a prescribed accuracy grow with k?

Regarding Q1: Numerical experiments indicate that, in many cases, the condition $hk \lesssim 1$ is sufficient for the Galerkin method to be quasi-optimal (with the constant of quasi-optimality independent of k); this feature can be described by saying that the h-BEM does not suffer from the pollution effect (in constrast to the h-FEM).

This talk will present rigorous results on Q1 and Q2, recently obtained in [1] (building on, and using, the results of [2] and [3]), and then compare them with the results of numerical experiments.

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On the coupling of space–time finite and boundary element methods

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Keywords: Heat equation, space–time, FEM/BEM coupling

We first review some recent developments in the numerical analysis of space–time finite and boundary element methods for the solution of parabolic evolution equations. These results are then used to discuss the non–symmetric coupling of finite and boundary element methods for the heat equation in free space. Numerical examples are given.

Time Domain BEM for Fluid-Structure Interaction

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Keywords: time domain boundary element method, fluid-structure interaction, a posteriori error estimates

We consider well-posedness, convergence and a posteriori error estimates for fluid-structure interaction in the time domain. For an elastic body immersed in a fluid, a single-layer ansatz reduces the exterior linear wave equation for the fluid to an integral equation on the boundary. The resulting problem is solved using a Galerkin boundary element method in time domain, coupled to a finite element method for the Lamé equation inside the elastic body. Based on ideas from the time–independent coupling formulation, we give a priori and a posteriori error estimates, which demonstrate the convergence and give rise to adaptive mesh refinement procedures.

Wavelength stable field-only boundary regularised integral solution of electromagnetic scattering based on the Helmholtz equation

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Keywords: Electromagnetics, Helmholtz equation, Boundary regularised integral equation, Wavelength stable

Surface integral methods are effective ways to obtain solutions of Maxwell's equations for electromagnetic scattering. At the moment, the most famous surface integral methods in computational electromagnetics (CEM) are based on the Stratton-Chu formulism [1] in which the induced vector surface current densities are found by solving integral equations that contain dyadic Green's functions with hypersingular behaviours. Nonetheless, not only special numerical treatments are needed to deal with the integrals with hypersingular kernels, but also the fields on and close to the surface are difficult to be obtained with high accuracy by post processing. Also, the intrinsic numerical instability of the traditional surface integral methods at long wavelength limit makes them not suitable to solve problems when parts of the boundary are close to each other or multiscale problems. In this talk, we introduce a novel, robust boundary integral method of CEM by solving directly the electric field $\boldsymbol{E}$. This can be achieved because each component of $\boldsymbol{E}$ obeys the Helmholtz equation, and in the Cartesian coordinate system, the continuity condition $\nabla \cdot \boldsymbol{E} = 0$ is equivalent to $\nabla^2(\boldsymbol{r} \cdot \boldsymbol{E}) + k^2(\boldsymbol{r} \cdot \boldsymbol{E}) = 0$ where k is the wavenumber and $\boldsymbol{r}$ is a position vector. As a result, we solve CEM problems with three Helmholtz equations for three Cartesian components of E and another Helmholtz equation for the scalar function $(\boldsymbol{r} \cdot \boldsymbol{E})$ when the usual boundary conditions of $\boldsymbol{E}$ are imposed on the surface [2,3]. By using the recently developed boundary regularized integral solution formulation for the Helmholtz equation in which the singularity associated with the Green's function is removed analytically [4], we are able to solve the problems with long wavelength limit or low frequency limit without any numerical instability issue [2,3]. Also, this new integral solution formulation does not have the term proportional to the solid angle. Together with the regular integrands, we can apply high order surface elements to obtain results with high accuracy [2–4].

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Modeling Multiscale Interface Phenomena Using Nonlinear Transmission Conditions

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Keywords: solvation, electrostatics, Poisson-Boltzmann, nonlinear, SLIC

The atomic-scale structure of fluids at the solid-liquid interface plays central roles in problems ranging from understanding proteins to improving battery technology. This poses serious challenges for quantitative modeling: on one hand, classical continuum models fail to reproduce known facts even qualitatively correctly; on the other hand, for many problems, atomistically detailed models are impractically expensive. Most approaches for addressing this multiscale problem rely either on complicated partial differential equation models, or on coupling atomistic and continuum models. However, existing approaches have failed to capture key nonlinear phenomena in the first layer of fluid molecules. Using the electrostatic response of a liquid surrounding a charged biomolecule as an example, we propose a new approach, which capitalizes on the well-known fact that boundary-integral equations focus attention on the interface itself, and in particular on the transmission conditions [1,2]. We have shown that the transmission condition associated with the classical continuum model, based on macroscopic dielectric theory, is easily corrected with a simple nonlinear term that is a function of the local electric field [1]. We call this the solvation-layer interface condition (SLIC) model. In this talk, we will discuss solution existence and uniqueness, as well as numerical methods. The corrected model is easy to compute numerically on complicated geometries, as it represents merely a diagonal perturbation of the usual BEM problem, combined with a short nonlinear iteration [1,2]. Results illustrate that this remarkably simple correction to a familiar continuum model increases accuracy to the level of fully atomistic calculations thousands of times more expensive, and achieves this accuracy while reducing the number of model parameters by an order of magnitude. A variety of related problems in interfacial response lead to modified transmission conditions, and we suggest that the boundary-element method community has a myriad of opportunities to advance multiscale modeling.

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Fast Galerkin BEM for parabolic moving boundary problems

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Keywords: Heat Equation, Moving Boundary Problem, Shape Optimization, Stefan Problem

Time dependence in boundary integral reformulations of parabolic PDEs is reflected in the fact that the layer potentials involve integrals over time in addition to integrals over the boundary surface.

This implies that in the numerical solution a time step involves the summation over space and the complete time history. Thus the naive approach has order N^2M^2 complexity, where N is the number of unknowns in the spatial discretization and M is the number of time steps. However, with a space-time version of the fast multipole method the complexity can be reduced to nearly NM.

The talk will focus on the application of the methodology to problems with time dependent geometries. Two different situations will be considered: In the first the boundary at time t is a differomorpic image of a fixed reference geometry. Here the discretization consists of simple space-time tensor product finite element spaces. Since this setting does not allow for topology changes in time, we will also consider a full space time discretization to handle more general situations.

The eventual goal of this work is to solve free surface problems, such as the Stefan problem, which describes the evolution of a phase change interface. The talk will conclude with a new approach to solve one dimensional problems with shape optimization techniques.

Single scattering preconditioner applied to boundary integral equations

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Keywords: Boundary Integral Equation, Multiple scattering, Preconditionning, Helmholtz Equation

In a homogeneous medium, when illumated by an incident time-harmonic acoustic wave u^{inc}, the $M > 1$ obstacles Ω_p, $p = 1, \ldots, M$, generate a scattered wave u solution of the Helmholtz equation:

$$\left\{ \begin{array}{rcll} \Delta u + k^2 u & = & 0 & \mathbb{R}^3 \setminus \overline{\cup_{p=1}^{M} \Omega_p} \\ u & = & -u^{inc} & \cup_{p=1}^{M} \Gamma_p \\ & u \text{ is radiating.} & & \end{array} \right.$$

The quantity k is the positive wavenumber, the radiating condition stands for the Sommerfeld one and Γ_p are the boundaries of Ω_p. The boundary condition is here set to Dirichlet but another condition can be imposed.

It is well known that this problem can be rewritten equivalently under the form of a system of boundary integral equations (BIEs) with the densities ρ and λ as unknowns. If $\mathcal{L}$ and $\mathcal{M}$ represent respectively the volume single- and double-layer integral operators, then

$$u(x) = \mathcal{L}\rho(x) + \mathcal{M}\lambda(x), \qquad \forall x \notin \cup_{p=1}^{M} \overline{\Omega_p}.$$

Following [1], the BIE can be classified as direct or indirect, depending on whether or not the unknown dentities ρ and λ are the Cauchy data. Direct BIE here also refers to the *null-field method*.

In multiple scattering context, a natural preconditioner is the one representing single scattering effects. For M obstacles and given the matrix of a discretized integral equation, this preconditioner is composed by the M blocks located on the diagonal of this matrix. Each block represents the scattering problem by one obstacle. This geometric preconditioner is called single scattering preconditioner.

This talk focuses on the effects of this preconditioning on boundary integral equations. The main result is that, after being preconditioned by their single scattering preconditioner, every direct integral equations become exactly the same [1]. This does not depend on the geometry of the obstacles and can moreover be extended in a different form for indirect integral equations such as the one of Brakhage-Werner. These properties imply in particular that the convergence rate of a Krylov subspaces solver will be exactly the same for every preconditioned integral equations. To illustrate this, some numerical simulations are provided using $\mu-$diff, an open-source Matlab toolbox for solving multiple scattering by disks [2].

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Acceleration of the boundary-domain integral representation of the velocity-vorticity form of Navier-Stokes equations

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Keywords: boundary element method, adaptive cross approximation, velocity-vorticity formulation

In this paper we present a method to decrease the computational cost of the BDIM (boundary-domain integral method), which was introduced by Škerget et. al. [1]. The method has been adapted for the solution of three-dimensional flow and heat transfer problems by solving the velocity-vorticity formulation of Navier-Stokes equations. The governing equations form a non-linear system of three partial differential equations - the kinematics equations, which links the velocity and vorticity fields in space and time, the vorticity transport equation, which governs diffusive and advective transport of vorticity and the energy equation, which governs transport of heat.

The BDIM numerical algorithm is based on the integral formulation of the governing equations using the Laplace fundamental solution and a collocation scheme. In order to determine boundary conditions for the vorticity transport equation, the boundary element method is employed to the kinematics equation in order to solve for the unknown vorticity values. The equations are solved in an iterative manner in order to account for the non-linearities. With the aim of reducing computational cost of this procedure, several fast techniques have been introduced in the past (Ravnik et. al [2]), such as fast multipoles, wavelet transform and domain decomposition.

Recently, [4], we focused on the acceleration of the domain integral contribution by employing the adaptive cross approximation method [5] coupled with the $\mathcal{H}$ matrix structure [4]. We study the influence of the problem non-linearity (flow Reynolds and Rayleigh number values) and the problem size (computational grid) on the acceleration rate and accuracy of resulting the flow and temperature fields especially in the areas of where sharp function profiles are expected (flow boundary layers). The results show, that the acceleration strategy should be tuned to the expected flow circumstances.

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Operator Preconditioning for the Electric Field Integral Equation on Screens

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Keywords: open surface problems, electric field integral equations, preconditioning

We consider the electric field integral equation (EFIE) arising from the scattering of time-harmonic electromagnetic waves by a perfectly conducting screen. When discretizing the EFIE by means of Galerkin boundary element methods (BEM), one obtains ill-conditioned systems on fine meshes and iterative solvers perform poorly. In order to reduce the number of iterations needed to find a solution, one uses preconditioning. Finding a suitable preconditioner for the case of screens poses some challenges due to the energy trace spaces at hand. Moreover, since solution of the EFIE on screens feature edge singularities, its amenability to adaptive refinement is desirable.

The standard "Calderón preconditioning" technique is suboptimal when dealing with screens [1]. In addition, it requires a div-conforming dual finite element space such that the curl/div duality pairing matrix is well conditioned. The existing technique resorts to BC functions [2] to fulfill this property on uniform meshes. However, the resulting dual pairing matrix becomes ill-conditioned as the ratio $h_{\max}/h_{\min}$ increases and demands additional manipulations in order to handle non-uniform meshes.

In this presentation, we discuss a new strategy to build a preconditioner for the EFIE on screens using operator preconditioning. For this, we construct a compact equivalent inverse of the EFIE operator on the disk using recently found Calderón-type identities [3]. Furthermore, stable discretization of our preconditioner only requires dual meshes for low-order Lagrangian finite element spaces, which are used to discretize the same energy trace spaces that arise from the Laplacian. As a consequence, our approach allows for non-uniform meshes without additional computational effort. Finally, we present some numerical experiments validating our claims.

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Using boundary element methods to analyse the low-frequency resonance of fish schools

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Keywords: boundary element method, acoustics, resonance, transmission

At specific frequencies, schools of fish can exhibit a high reflectivity of acoustic signals from sonar systems, resulting in a a strong impact on the quality of the sonar signal used for underwater surveillance. This phenomenon happens for fish that have swim bladders filled with air. Because of the high contrast in density between air and water, a strong low-frequency resonance is present. These resonances, also known as Minnaert resonances, have been observed in practice and can be explained theoretically.

Although the resonance frequency of a single air bubble in water can be determined analytically with Mie series, numerical methods need to be used to investigate the impact of the shape as well as the number of bubbles in the system. Specifically, the resonance frequency of a cloud of bubbles depends on the configuration and distances between them. When bubbles are close by each other, high-accuracy numerical methods need to be used to compute the resonance frequency of the coupled system. The boundary element method (BEM) for the multiple traces formulation (MTF) of the Helmholtz transmission problem will be used to accurately analyse the low-frequency resonances. The numerical results will be compared with a method based on transmission matrices. It will be shown that the BEM accurately predicts the pronounced frequency shifts in the resonances of the clouds of bubbles.

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Optimal preconditioning of operators of negative order

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Keywords: opposite order preconditioning, single layer operator, locally refined triangulations, higher order elements, boundary element method

We construct a preconditioner for negative order operators discretized by discontinuous triangular Lagrange elements. The canonical example is the Single Layer operator discretized by piecewise constant functions. We propose a variation of the well-studied dual mesh preconditioning technique [1–3]. The resulting preconditioner yields a uniformly bounded condition number. Our approach easily extends to operators discretized on locally refined triangulations, and higher order discontinuous elements.

Compared to earlier proposals, the preconditioner has the following advantages: It does not require the inverse of a non-diagonal matrix; it applies without any mildly grading assumption on the mesh; and it does not require a barycentric refinement of the mesh underlying the trial space.

The preconditioning strategy requires the application of an opposite order operator, e.g. for preconditioning of the Single Layer operator one can use the Hypersingular operator, or, in any case for uniform triangulations, the multilevel operator from [4]. The total cost of the preconditioner is the sum of the cost of the opposite order operator, which for the multilevel operator is proportional to the number of triangles N, and additional cost that is always proportional to N.

A numerical study of our preconditioner for the Single Layer operator will be presented, which compares various trial spaces, and for the opposite order operator, the Hypersingular operator against the multilevel operator.

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Integral equation methods for electro- and magneto-hydrodynamics of soft particles

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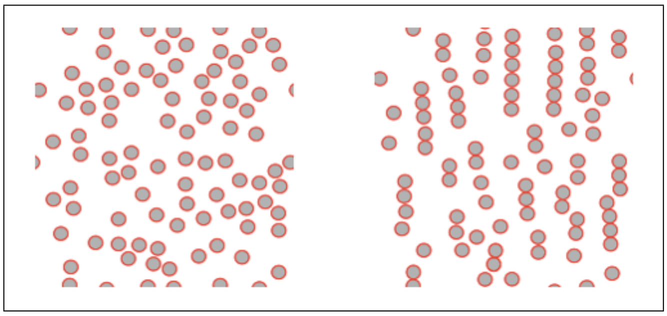
Viscous flow of soft particles coupled to electric, magnetic or other physical variables that evolve simultaneously can be observed in many physical systems. Stability, accuracy and computationally efficiency of numerical solvers become critically important and intricately coupled for such problems. Integral equation methods become particularly attractive for such multi-scale and multi-physics problems owing to their well-known strengths in reducing the dimensionality of the problem and the exact satisfaction of far-field boundary conditions. We describe new second-kind boundary integral equation formulation for a certain class of these problems, an efficient close evaluation scheme [1] and a new periodization scheme [2, 3]. Application of the method to study the electro-hydrodynamics of vesicle suspensions [4, 5] and magneto-hydrodynamics of soft particles in the context of smart material design [6] will be presented.



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Isogeometric Boundary Element Methods for Electromagnetic Problems: Discretisation and Numerical Examples.

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Keywords: Maxwell's equations, higher order methods, isogeometric analysis, B-splines

Since the introduction of Isogeometric Analysis by [5], the methodology has gained attention in various communities. Many applications of boundary element methods within the isogeometric framework have been introduced and investigated; where B-splines related to the regularity of the geometry mappings are used as ansatz- and test functions [3, 6].

As shown by [1], the intrinsic properties of the B-spline bases make them exceptionally well suited for a discretization of Hilbert Complexes, and thus for application in electromagnetic problems, where first implementations have successfully been tested [7].

We will briefly discuss the approximation properties of the spline spaces w.r.t. the trace spaces of the three-dimensional de Rahm Complex, i.e. the spaces $H^{1/2}(\Gamma)$, $\mathbf{H}_\times^{-1/2}(\mathrm{div}_\Gamma, \Gamma)$, and $H^{-1/2}(\Gamma)$, and compare to classical results [2, 8].

We will introduce the concepts mentioned above and discuss numerical results of a fast boundary element method, built on the implementation presented in [3], utilising an interpolation based fast multipole method [4]. We will finally compare to classical Raviart-Thomas elements of higher order, see e.g. [8].

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Reconstruction of surface impedance functions from the acoustic far field pattern

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Keywords: Generalized impedance boundary condition, integral operators, inverse problem

We consider the reconstruction of a surface impedance function of a sound soft obstacle coated with a thin layer of an penetrable material from few far field data at a fixed frequency. For a long time most of the investigations of such problems were made for the first order impedance boundary condition. Only recently Bourgeois and Haddar [4] proved the uniqueness for the solution of 3D problem with generalized impedance boundary condition and proposed a method, [5], for 2D GIBC inverse problems based on the minimization of the cost function for the far field map, which was approximated by finite element method.

The method we use is based on an iteratively regularized Newton-type method, which combines ideas of both iterative and decomposition methods. Employing a boundary integral equation approach an inverse problem is proved to be equivalent to a system of nonlinear integral equations. It was firstly introduced in [3] for 2D electrostatic case and then the method was extended too many different inverse problems. In 2011 Ivanyshyn and Kress [2] applied the method for reconstructing the first order surface impedance for Helmholtz equation in 3D and in 2013 Cakoni and Kress [1] applied the nonlinear integral equation method for 2D electrostatic problem with GIBC.

In this study we present the results on the extention of a nonlinear integral equation approach to the inverse scattering problems for a generalized surface impedance in three dimensions. The method turns out to be efficient, since it avoids solving the direct problem at each iterative step, and the method is stable even for a limited data.

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Space-Time Variational Formulations for the Wave Equation

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Keywords: Wave Equation, Space-Time Methods, Boundary Integral Equations

For the discretisation of time-dependent partial differential equations usually explicit or implicit time stepping schemes are used. An alternative approach is the usage of space-time methods, where the space-time domain is discretised and the resulting global linear system is solved at once. In this talk the model problem is the scalar wave equation. First, a brief overview of known results for the wave equation and its boundary integral equations is presented. Second, a space-time boundary integral equation, motivated by variational formulations in the domain [1], is examined. For this space-time formulation a space-time boundary element method is introduced. Finally, numerical examples for a one-dimensional spatial domain are presented and discussed.

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Vectorized approach to the evaluation of boundary integral operators

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Keywords: boundary element method, quadrature, SIMD, vectorization, parallelization

Efficient evaluation of discretized boundary integral operators is one of the key ingredients of any implementation of the boundary element method. In our talk we concentrate on the implementation of two main approaches to the treatment of singular kernels, namely the regularized numerical scheme [1,2] and the analytic evaluation [3]. In addition, for the latter case we present a newly developed method of simultaneous evaluation of the surface integrals for constant and linear trial functions.

Aiming at modern processing units with wide SIMD registers, we present techniques such as data alignment, array-of-structures to structure-of-arrays transition, or loop collapsing leading to efficient utilization of the available vector instruction sets. The provided scalability experiments validate the proposed methods and show that vectorization has become a necessary instrument in high performance computing.

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Scalable parallel BEM solvers on many-core clusters

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Keywords: Boundary element method, Parallelization, Graphics Processing Units

Our aim is to solve large scale problems discretized by the boundary element method. To this end, we propose to use parallel computers equipped with graphics processing units to assemble and solve the linear systems involved in the discretization. Depending on the application case, we either assemble the full dense system matrix (in parallel) or we compress the matrix by hierarchical matrices with adaptive cross approximation. In either case, Krylov subspace solvers are applied to solve the linear system. Our multi-GPU parallel implementation is achieved by porting a sequential CPU BEM code to GPUs and by applying a multi-GPU library for generic Krylov subspace solvers (`MPLA`, [1, 3]) and a GPU-based hierarchical matrix library (**`hmglib`**, [2]). In our presentation, we will give details on the parallel implementation and we will show our latest parallel performance benchmarks.

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Singular integral equations method for a fracture problem with a surface energy in the Steigmann-Ogden form on the boundary

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Keywords: fracture, integral equations, surface energy

A problem of a straight mixed mode non-interface fracture in an infinite plane is treated analytically with the help of complex analysis techniques. The surfaces of the fracture are subjected to surface elasticity in the form proposed by Steigmann and Ogden [1,2]. The boundary conditions on the banks of the fracture connect the stresses and the derivatives of the displacements. The mechanical problem is reduced to two systems of singular integro-differential equations which are further reduced to the systems of equations with logarithmic singularities. It is shown that modeling of the fracture with the Steigmann-Ogden elasticity produces the stress and strain fields which are bounded at the crack tips. The existence and uniqueness of the solution for almost all the values of the parameters is proved by reducing the systems of singular integro-differential equations to the systems of weakly-singular integral equations. It is shown that introduction of the surface mechanics into the modeling of fracture leads to the size-dependent equations. A numerical scheme of the solution of the systems of singular integro-differential equations is suggested, and the numerical results are presented for different values of the mechanical and the geometric parameters. The results of this research are reported in the paper [1]. The study of a related contact problem for a frictionless contact of a rigid stamp into an elastic semi-plane are reported in [2]. The problem is solved by reduction to a system of singular integro-differential equations as well.

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A binary-tree subdivision method for evaluation of nearly singular integrals and singular integrals in 3D BEM

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Keywords: nearly singular integrals and singular integrals, element subdivision, binary-tree

An adaptive surface element subdivision method for evaluation of nearly singular integrals and singular integrals in three-dimensional(3D) boundary element method(BEM) is presented in this paper. Element subdivision is one of the most widely used methods for evaluating nearly singular and singular integrals. The Spherical Element Subdivision Method [1–3], one of the element subdivision methods, can automatically refine patches as they approach the source point, and evaluate nearly singular integrals and singular integrals accurately and efficiently for cases of arbitrary type fundamental solution, arbitrary shape of element and arbitrary location of the source point. However, this method cannot guarantee the success of element subdivision similar to Advancing Front Method(AFM), which is empirical and popular mesh generation method without strict theory but has been verified by subsequent number of experiments. Therefore, in this paper, we further present a new element subdivision method based on binary-tree. This method splits an element into two patches at each step, and continues the splitting process recursively until meeting a given criteria. This subdivision algorithm is more convenient to implement and can guarantee the convergence of the iterative subdivision based on the given terminating condition. The patches that intersects with the sphere centered at the source point are set to be invalid, and thus the cavity is formed. New patches will be generated along the radial direction of the sphere to fill the cavity motioned above. The distribution of patches close to source point is dense, conversely, the distribution of patches far away form source point is sparse. For singular integral, the patches closed to source point own relatively regular shape. Numerical examples are presented by planar and curved surface elements. Results have demonstrated that not only can our method guarantee successful element subdivision, but also our method can provide much better accuracy and efficiency with fewer Gaussian sample points than the conventional method.

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A binary-tree subdivision method for volume integrals in BEM

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Keywords: BEM, singular integral, nearly singular integral, binary element subdivision, Gaussian quadrature

An adaptive volume element subdivision method for the numerical evaluation of singular domain integrals and nearly singular domain integrals using binary tree in three-dimensional (3D) boundary element method (BEM) is presented. The application of BEM for solving boundary value problems with body forces, time dependent effects or certain class of non-linearities generally leads to an integral equation which contains domain integrals. The singular integrals arise when the source point is inside, at vertices or on the boundary of the volume element. While, the nearly singular integrals arise when the source point is close to but not inside or on the boundary of element. For the solution of transient heat conduction problem when using time-dependent boundary integral equation method, as the time step value is very small, the integrand in the domain integral is close to singular. And the domain integrals cannot be evaluated accurately and efficiently by standard Gaussian quadrature, thus rendering accurate evaluation of the integral difficult. The Spherical Element Subdivision Method [1], proposed by the first author, by which the singular integrals and nearly singular integrals can be evaluated accurately and efficiently for cases of arbitrary type fundamental solution, arbitrary shape of element and arbitrary location of the source point, is used for subdividing an element into a number of patches through a sequence of spheres with decreasing radius. Although this method overcomes all the difficulties associated with integration in the BEM, it cannot guarantee successful element subdivision for some situations. Hence, a general, adaptive and efficient volume element subdivision method using binary tree is presented in this paper, which can generate patches successfully under any circumstances. In this method, the volume element is subdivided into a number of patches by binary tree. A binary tree is a tree data structure in which each node has at most two children. A significant advantage of this structure is that a single data structure can handle volume element subdivision very efficiently. For singular domain integrals, the subdivision rule is based on coupling minimum subdivision size and the ratio of circumradius of element to the distance between the source point and center of element. The subdivision will be executed continuously when the ratio is greater than the reference value. The minimum subdivision size is used to avoid the infinite loop and control the number of patches generated. For nearly singular domain integrals, only the ratio(as mentioned above) should be regarded as the terminating condition without considering the minimum subdivision size. With the proposed method, the patches obtained are automatically refined and each patch of the integration element is acceptable in shape and size for standard Gaussian quadrature. Numerical examples demonstrate that our method is more robust and reliable to provide much better accuracy and efficiency than conventional subdivision methods [2].

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How to achieve the goal of 5aCAE based on BIE

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Keywords: CAD-CAE integration, BIE, BFM, FMM, ACA

Within a successful Computer Aided Engineering (CAE) driven product development, an ideal CAE tool should possess the following five features:

1) Automatic meshing and analysis for any complicated structures with complex geometries;
2) Accuracy much better than existing FEM tools is achievable;
3) Arbitrary geometries and material compositions of structures can be easily handled;
4) Accelerated by the fast methods, such as the Fast Multipole Method, the Hierarchical Matrix;
5) Adaptive solution procedures to guarantee the reliability of the computational results.

We name this kind of CAE as 5aCAE. In this talk, I will first explain why the BIE has advantages over FEM to achieve the gaol of 5aCAE, also considering the currently popular meshless methods and isogeometric analysis, and then briefly introduce some new algorithms we have proposed for software implementation. These algorithms are as follows:

1) A Boundary Face Method (BFM) [1], which combines the BIE with Computer Graphics and is a truly isogeometric analysis method, can perform CAE analysis on a CAD model directly.

2) A Dual Interpolation Method [2], which combines the traditional element interpolation and meshless approximation, unifies the conforming and nonconforming elements that are separately used in traditional BEM implementations.

3) A simplified binary tree meshing method, which realizes entirely automatic meshing for arbitrarily complex structures, even with geometrically 'dirty' part of their CAD models.

4) A Spherical Element Subdivision Method [3], by which the singular integrals and nearly singular integrals can be evaluated accurately and efficiently for cases of arbitrary type fundamental solution, arbitrary shape of element and arbitrary location of the source point.

5) An adaptive Fast Multipole Method [4] and a Geometric Mapping Cross Approximation (GMCA) Method.

6) A domain sequence optimization method for multi-domain problems, which can deal with arbitrary inter-domain connections with optimized band of the assembling system matrix.

Finally, I will introduce the software developed using above algorithms and Integrating the BFM into the commercial CAD software UG-NX, making the CAE entirely within the CAD environment. I will also show a number of examples of real complicated industrial products and engineering structures.

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An efficient adaptive solution technique for periodic Stokes flow

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Keywords: Stokes flow, periodic geometry, boundary integral equations, adaptive discretization, fast direct solvers

The numerical modeling of objects flowing through confined geometries arise in many applications. Examples includes the modeling of blood coagulation, the development of microfluidic devices, and the understanding of bacteria and other microswimmers. As an alternative to traditional methods, the boundary integral formulation reduces the dimensionality of the problem by one and usually leads to much smaller discretized systems to solve. The trade-off is that it requires dense matrix computations and special techniques for handling the evaluation of integral operators when points are "near" each other in physical space. There has been much work on these two items separately: fast algorithms for dense matrix computations and special quadrature rules for near field evaluations. The work presented in this talk brings these two techniques together hopefully increasing the range of problems for which boundary integral methods can be applied. Specificaly, this talk presents an adaptive discretization technique which locally refines the discretization on the confining walls when the objects or bodies get close to them. At the same time, a fast direct solver for the locally refined discretization is built following the algorithm proposed in [2]. The solver utilizes the fact that the local refinement corresponds to a low rank update to the original system, and thus the inverse of the new system can be applied to a given vector via a Woodbury formula with little cost. Numerical results illustrating the performance of the method for objects flowing through a periodic pipe using the boundary integral formulation from [1] will be presented.

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