Primal-dual interior point methods for state and input constrained optimal control

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Overview

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Original problem
\[ \min_{c(x) \leq 0} f(x) \]

Penalized problem
\[ \min_x f(x) - \epsilon \log(-c(x)) \]
Primal penalized problem

The primal problem consists in solving for $x$ the following first order conditions

$$f'(x) - c'(x) \cdot \frac{\epsilon}{c(x)} = 0$$

Primal dual penalized problem

The primal-dual problem consists in solving for $x$ and $\lambda$ the following first order conditions

$$f'(x) + c'(x) \cdot \lambda = 0$$

$$\lambda c(x) + \epsilon = 0 \iff \lambda - c(x) - \sqrt{\lambda^2 + c(x)^2 + 2\epsilon} = 0$$
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Objective of the paper

Adapt primal-dual interior point methods from numerical optimization to pure state and input constrained optimal control.

State of the art

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Problem Statement

\[ \min_{u \in \mathcal{U}} J(u) = \int_{0}^{T} \ell(x_u(t), u(t)) dt + \varphi(x_u(T)) \]
\[ \dot{x}_u(t) = f(x_u(t), u(t)) \]
\[ x_u(0) = x^0 \]
\[ \mathcal{U} := L^\infty([0, T]; U_{ad} \subset \mathbb{R}^m) \]
\[ g(x_u(t)) \leq 0 \]

Where \( T > 0, \) \( x^0 \) are fixed and \( x_u \) is the solution of the ODE with control \( u. \) And we define the classical pre-Hamiltonian function \( H : \mathbb{R}^n \times \mathbb{R}^m \times \mathbb{R}^n \to \mathbb{R} \) as follows

\[ H(x, u, p) := \ell(x, u) + p.f(x, u) \]
Main assumptions

Assumptions

(A1) The functions $\ell : \mathbb{R}^n \times \mathbb{R}^m \mapsto \mathbb{R}$, $f : \mathbb{R}^n \times \mathbb{R}^m \mapsto \mathbb{R}^n$ and $g_i : \mathbb{R}^n \mapsto \mathbb{R}$ are at least twice continuously differentiable.

(A2) The dynamics $f : \mathbb{R}^n \times \mathbb{R}^m$ from satisfies a sublinear growth property

$$\exists D < +\infty, \forall x \in \mathbb{R}^n, \forall u \in [-1, 1]^m \text{ s.t. } \| f(x, u) \| \leq D(1 + \| x \|)$$

(A3) Interior accessibility

$$\{ u \text{ s.t. } g(x_u) \leq 0 \} \subseteq \text{cl}_{L^1}(\{ u \text{ s.t. } g(x_u) < 0 \})$$

(A4) The OCP has a unique solution $u^*$ and $\exists \beta \geq 0$ and $r > 0$ such that

$$J(u) - J(u^*) \geq \beta \| u - u^* \|_{L^2}^2, \quad \forall u \in B_{L^2}(u^*, r) \cap \mathcal{U}$$
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First order optimality conditions

State constrained Pontryagin Maximum Principle

Any optimal solution \((\bar{x}_u, \bar{u})\) of the presented COCP is a Pontryagin extremal, i.e. \((\bar{x}_u, \bar{u}, \bar{p}, \bar{\mu})\) is solution of

\[
\begin{align*}
\dot{\bar{x}}_u(t) &= f(\bar{x}_u(t), \bar{u}(t)) \\
-d\bar{p}(t) &= H_x'(%(\bar{x}_u(t), \bar{u}(t), \bar{p}(t))dt + g'(\bar{x}_u(t))d\bar{\mu}(t) \\
\bar{x}_u(0) &= x^0 \\
\bar{p}(T) &= \varphi'(\bar{x}_u(T)) \\
H(\bar{x}_u(t), \bar{u}(t), \bar{p}(t)) &= \inf_{v \in \mathcal{U}_{ad}} H(\bar{x}_u(t), v, \bar{p}(t)) \\
d\bar{\mu} &\geq 0 \\
\int_0^T g(\bar{x}_u(t))d\bar{\mu}(t) &= 0
\end{align*}
\]
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Let $\epsilon > 0$, the penalized optimal control problem writes

$$
\min_{u \in U} J_\epsilon(u) = \int_0^T \ell(x_u(t), u(t)) - \epsilon \log(-g(x_u(t))) dt + \varphi(x_u(T))
$$

$$
\dot{x}_u(t) = f(x_u(t), u(t))
$$

$$
x_u(0) = x^0
$$

$$
U := L^\infty([0, T]; U_{ad})
$$

and the corresponding penalized pre-Hamiltonian is

$$
H[\epsilon](x, u, p) = H(x, u, p) - \epsilon \log(-g(x))
$$
Any Pontryagin extremal \((x_{u,\epsilon}, u_\epsilon, p_\epsilon)\) of the penalized problem, i.e. solution of

\[
\begin{align*}
\dot{x}_{u,\epsilon}(t) &= f(x_{u,\epsilon}(t), u_\epsilon(t)) \\
\dot{p}_\epsilon(t) &= -H[\epsilon]'(x_{u,\epsilon}(t), u_\epsilon(t), p_\epsilon(t)) \\
x_{u,\epsilon}(0) &= x^0 \\
p_\epsilon(T) &= \varphi'(x_{u,\epsilon}(T)) \\
H[\epsilon](x_{u,\epsilon}(t), u_\epsilon(t), p_\epsilon(t)) &= \inf_{v \in U_{ad}} H[\epsilon](x_{u,\epsilon}(t), v, p_\epsilon(t))
\end{align*}
\]
converges to \((\bar{x}_u, \bar{u}, \bar{p}, \bar{\mu})\) to a Pontryagin extremal of the original problem as follows

\[
\begin{align*}
\lim_{\epsilon \downarrow 0} \|u_\epsilon - \bar{u}\|_{L^2} &= 0 \\
\lim_{\epsilon \downarrow 0} \|x_{u,\epsilon} - \bar{x}_u\|_{L^\infty} &= 0 \\
\lim_{\epsilon \downarrow 0} \|p_\epsilon - \bar{p}\|_{L^1} &= 0 \\
\lim_{\epsilon \downarrow 0} \frac{-\epsilon}{g(x_{u,\epsilon})} dt &\overset{*}{\rightharpoonup} d\bar{\mu}
\end{align*}
\]
State Lipschitz continuity

From the sublinear growth property, \( \forall u_1, u_2 \in \mathcal{U}, \exists \text{const}(f) < +\infty \) such that
\[
\| x_{u_1} - x_{u_2} \|_{L^\infty} \leq \text{const}(f) \| u_1 - u_2 \|_{L^1}
\]

State-constraint measure

For all \( u \in \mathcal{U} \) and for all \( E \subseteq g \circ x[u]([0, T]) \) we note \( m[u, g] \) the push-forward \( g \)-measure of \( E \) defined as follows
\[
m[u, g](E) := \text{meas}\left( (g \circ x_u)^{-1}(E) \right)
\]

Proposition

For all \( u \in \mathcal{U} \), let \( E \subseteq g \circ x_u([0, T]) \subseteq \mathbb{R} \) be a Lebesgue-measurable set, the \( g \)-measure is lower bounded as follows
\[
m[u, g](E) \geq \text{const}(f, g) \text{meas}(E)
\]
Set of state saturated control

Let us define the set of saturated-state control $\mathcal{U}_g^0$ as follows

$$\mathcal{U}_g^0 := \{u \in \mathcal{U} \text{ s.t. } \sup_t g(x_u(t)) = 0\}$$

Set of near state-saturated times

For all $u$ s.t. $g(x_u) \leq 0$ and $\forall \delta \geq 0$ we define the set of near state-saturated times, noted $S_u(\delta)$ as follows

$$S_u(\delta) := (g \circ x_u)^{-1} \left([-\delta, +\infty)\right)$$
Preliminary results (3/3)

Proposition

There exists $\Gamma_g > 0$ and $v \in \mathcal{U}$ satisfying

$$\sup_t g(x_v(t)) \leq -2\Gamma_g$$ (1)

Thus, for all $u \in \mathcal{U}_g^0$ this yields

$$g(x_v(t)) \leq g(x_u(t)) - \Gamma_g, \ \forall t \in S_u(\Gamma_g)$$ (2)

Proof

Let $\delta > 0$

$$\Gamma_g = -\frac{1}{2} \sup_{u \in \mathcal{U}_g^0} \left\{ \inf_{v \in B_{L^1}^1(u, \delta) \cap \mathcal{U}} \left\{ \sup_t g(x[v](t)) \right\} \right\}$$
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State penalized optimal control

**State-penalized problem**

\[
\min_u G_\epsilon(u) = \int_0^T \ell(x_u(t), u(t)) - \epsilon \log(-g(x_u(t))) \, dt
\]

\[
\dot{x}_u(t) = f(x_u(t), u(t))
\]

\[
x_u(0) = x^0
\]

\[
u \in \mathcal{U}
\]
Lemma

Any optimal solution of state penalized problem $u_\epsilon$ satisfies

$$g(x_{u_\epsilon}(t)) < 0, \forall t \in [0, T]$$

and $\exists K_\psi < +\infty$ such that $\forall \epsilon \in (0, \epsilon_0)$ one has

$$\left\| \frac{\epsilon}{g(x_{u_\epsilon})} \right\|_{L^1} \leq K_\psi$$
State Interiority proof (1/6)

\[ G_\varepsilon(v) - G_\varepsilon(u_\varepsilon) = \Delta_\ell(u_\varepsilon, v) + \varepsilon \Delta_{\log}(u_\varepsilon, v) \]

with

\[ \Delta_\ell(u_\varepsilon, v) = \int \ell(x_v, v) - \ell(x_{u_\varepsilon}, u_\varepsilon) dt \]

\[ \Delta_{\log}(u_\varepsilon, v) = \int -\log(-g(x_v)) + \log(-g(x_{u_\varepsilon})) dt \]
\[ \Delta_\ell(u_\epsilon, v) = \int \ell(x_v, v) - \ell(x_{u_\epsilon}, u_\epsilon)dt \]
\[ \leq \int \text{const}(\ell)(\|x_v(t) - x_{u_\epsilon}(t)\| + \|v(t) - u_\epsilon(t)\|)dt \]
\[ \leq \text{const}(\ell, f, T, \Gamma_g) \]

\[ \epsilon \Delta_{\log}(u_\epsilon, v) = \epsilon \int_{S_{u_\epsilon}(\Gamma_g)} - \log(-g(x_v)) + \log(-g(x_{u_\epsilon}))dt \]
\[ + \epsilon \int_{[0,T] \setminus S_{u_\epsilon}(\Gamma_g)} - \log(-g(x_v)) + \log(-g(x_{u_\epsilon}))dt \]
\[ := \epsilon \Delta_S + \epsilon \Delta_{S^c} \]
\( \epsilon \Delta_{S_c} \leq \text{const}(g, f, T, \Gamma_g, \epsilon_0, \psi) \)

\[
\epsilon \Delta_S = \epsilon \int_{S_{u\epsilon}(\Gamma_g)} - \log(-g(x_v)) + \log(-g(x_{u\epsilon})) \, dt
\]

\[
= \epsilon \int_{S_{u\epsilon}(\Gamma_g)} - \frac{1}{g(x[u\epsilon])} (g(x[v]) - g(x[u\epsilon])) \, dt
\]

\[
\leq -\epsilon \Gamma_g \int_{S_{u\epsilon}(\Gamma_g)} - \frac{1}{g(x[u\epsilon])} \, dt
\]
Now, let us prove interiority of $u_\epsilon$ by contradiction. Assume $u_\epsilon$ an optimal solution such that $\forall \rho \in (0, \Gamma_g)$, $(g \circ x_{u_\epsilon})^{-1}((-\Gamma_g, -\rho)) \subset [0, T]$

$$\epsilon \int_{S_{u_\epsilon}(\Gamma_g)} -\frac{1}{g(x[u_\epsilon])} dt \geq \epsilon \int_{(g \circ x[u_\epsilon])^{-1}((-\Gamma_g, -\rho))} -\frac{1}{g(x[u_\epsilon])} dt$$

$$= \epsilon \int_{-\Gamma_g}^{-\rho} -\frac{1}{s} m[u_\epsilon, g](ds)$$

Using the lower bound on the state-constraint measure

$$\epsilon \int_{S_{u_\epsilon}(\Gamma_g)} -\frac{1}{g(x[u_\epsilon])} dt \geq \epsilon \text{const}(f, g) \int_{-\Gamma_g}^{-\rho} -\frac{1}{s} ds$$

$$\geq \epsilon \text{const}(f, g) (-\log(\rho) + \log(\Gamma_g))$$
For $\rho$ small enough

$$G_\epsilon(v) - G_\epsilon(u_\epsilon) \leq \text{const}(\ell, f, g, T, \Gamma_g, \epsilon_0, \psi) + \epsilon \text{const}(f, g, \Gamma_g) \log(\rho) < 0$$

which contradicts the optimality of $u_\epsilon$ and proves interiority.
Let us prove the $L^1$-boundedness by contradiction. Assume $u_\epsilon$ optimal such that $\forall K_\psi > 0, \exists \epsilon > 0, \|\epsilon \psi' \circ g(x[u_\epsilon])\|_{L^1} \geq K_\psi$. First, we have

$$(g \circ x[u_\epsilon])^{-1}(\{0\}) = \emptyset$$

thus

$$\left\| \frac{\epsilon}{g(x[u_\epsilon])} \right\|_{L^1} := \lim_{\rho \to 0} \int_{-\infty}^{-\rho} \frac{\epsilon}{g(x[u_\epsilon])} m[u_\epsilon, g](ds) > K_\psi + \frac{\epsilon_0 T}{\Gamma_g}$$

Finally, we have:

$$G_\epsilon(v) - G_\epsilon(u_\epsilon) \leq \text{const}(\ell, f, g, T, \Gamma_g, \psi, \epsilon_0) - \Gamma_g K_\psi$$

Then $\exists K_\psi > 0$, such that $G_\epsilon(v) - G_\epsilon(u_\epsilon) < 0$, which contradicts the optimality of $u_\epsilon$. 
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Primal variables convergence

Primal variables convergence is a well established result

\[
\lim_{\epsilon \downarrow 0} \| u^* - u_\epsilon \|_{L^2} = 0
\]

\[
\lim_{\epsilon \downarrow 0} \| x_{u^*} - x_{u_\epsilon} \|_{L^\infty} = 0
\]
The sequence \( \left( \frac{\epsilon_n}{g(x_{u\epsilon_n})} \right)_n \) is uniformly \( L^1 \)-bounded by \( K_\psi \). Thus

\[
\forall \phi \in L^\infty, |T_{u_{\epsilon_n}}(\phi)| \leq K_\psi \|\phi\|_{L^\infty}
\]

hence \( \|T_{u_{\epsilon_n}}\|_{\mathcal{M}} \leq K_\psi \). By weak \( \star \) compactness of the unit ball of \( \mathcal{M}([0, T]) \), \( \exists \mu \in \mathcal{M}([0, T]) \) such that

\[
\lim_{k \to +\infty} -\frac{\epsilon_{n_k}}{g(x_{u_{\epsilon_n_k}})} dt \star d\mu
\]
Convergence of state penalties (2/2)

Complementarity conditions

Positivity of $\mu$

$$- \frac{\epsilon_n}{g(x_{u\epsilon_n})} \geq 0 \Rightarrow d\mu \geq 0$$

Complementarity conditions

$$\langle \mu, g(x[u^*]) \rangle = \lim_{n \to +\infty} \int - \frac{\epsilon_n}{g(x_{u\epsilon_n})} g(x_{u^*}) dt$$

$$= \lim_{n \to +\infty} \int - \frac{\epsilon_n}{g(x_{u\epsilon_n})} g(x_{u\epsilon_n}) dt$$

$$= \lim_{n \to +\infty} -\epsilon_n T$$

$$= 0$$
The optimal adjoint state satisfies the following ODE

\[-dp^* = (\ell'_x(x[u^*], u^*) - f'_x(x[u^*], u^*) \cdot p^*) \, dt + g'(x[u^*]) \, d\mu\]

with \(p^*(T) = 0\) and the penalized adjoint state is solution of

\[\dot{p}[u_{\epsilon_n}] = -\ell'_x(x[u_{\epsilon_n}], u_{\epsilon_n}) - f'_x(x[u_{\epsilon_n}], u_{\epsilon_n}) \cdot p[u_{\epsilon_n}] + \frac{1}{\epsilon g'(x[u_{\epsilon_n}](t))} \frac{1}{g(x[u_{\epsilon_n}])(t))} \]

\[p[u_{\epsilon_n}](T) = 0\]

Then \(p[u_{\epsilon_n}]\) converges pointwise to \(p^*\), and the boundedness of both \(p[u_{\epsilon_n}]\) and \(p^*\) proves \(\lim_{n \to +\infty} \|p[u_{\epsilon_n}] - p^*\|_{L^1}\)
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Any solution \((x_{u^*}, u^*, p^*, \lambda^*)\) of the following Primal Dual TPBVP

\[
\begin{align*}
\dot{x}_{u^*} &= f(x_{u^*}, u^*) \\
\dot{p}^* &= -H'_x(x_{u^*}, u^*, p^*) - \lambda^* g'(x_{u^*}) \\
H(x_u, x_{u^*}, p^*) &= \inf_{v \in U_{ad}} H(x_u, v, p^*) \\
0 &= \lambda^* - g(x_{u^*}) - \sqrt{\lambda^*_2 + g(x_{u^*})^2 + 2\epsilon} \\
x[u^*](0) &= x^0; \quad p^*(T) = \varphi'(x_{u^*}(T))
\end{align*}
\]

converges to \((x^*, u^*, p^*)\) as follows

\[
\begin{align*}
\lim_{\epsilon \downarrow 0} \|u_{\epsilon}^* - u^*\|_{L^2} &= 0; \quad \lim_{\epsilon \downarrow 0} \|x_{u^*} - x[u^*]\|_{L^\infty} = 0; \quad \lim_{\epsilon \downarrow 0} \|p_{\epsilon}^* - p^*\|_{L^1} = 0 \\
\lim_{\epsilon \downarrow 0} \lambda^*_\epsilon dt &\to d\mu
\end{align*}
\]
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We note

\[ S(\epsilon) = (x_{\epsilon^*}, u_{\epsilon^*}, p_{\epsilon^*}, \lambda_{\epsilon^*}) \]

a solution of the primal-dual TPBVP.

**Primal dual Algorithm**

1. Define \( \epsilon_0 > 0, \alpha \in (0, 1), \text{tol} = o(1), k = 0 \)
2. **while** \( \epsilon_k > \text{tol} \) **do**
3. \( S(\epsilon_{k+1}) \leftarrow \) solution of primal-dual TPBVP initialized with \( S(\epsilon_k) \)
4. \( \epsilon_{k+1} \leftarrow \alpha \epsilon_k \)
5. \( k \leftarrow k + 1 \)
6. **end while**
7. **return** \( S(\epsilon_k) \)
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Battery management problem

Optimal control problem

\[
\min_u \int_0^T \text{spot}(t) \left( \max\{p_{\text{compteur}}(t); 0\} - \min\{p_{\text{compteur}}(t); 0\} \right) dt
\]

under the following constraints

\[
p_{\text{compteur}} := \text{cons} - PV + \frac{1}{\rho_c} \max\{u; 0\} + \rho_d \min\{u; 0\}
\]

\[
\dot{x}(t) = u(t)
\]

\[
x(0) = \frac{x^+}{2}
\]

\[
x(t) \in [0; x^+]
\]

\[
u(t) \in [u^-; \rho_c u^+]
\]

\[
T := 8760h
\]
## Comparison with discretize then optimize approach

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Conclusion and perspectives

- Extend theoretical results to mixed state and input constraints and to more general input constraints.
- Provide a predictor-corrector in function space type algorithm to automatically adapt the weighting parameter of IPMs.