A Combined Medial Object and Frame Approach to Compute Mesh Singularity Lines

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Abstract. In this work, a novel method for calculating mesh singularity lines (chains of irregular edges in an otherwise structured hex mesh) in a 3D computational domain is presented. It explores how combining the medial object of a domain with frames can provide insights to the placement of singularity lines. The medial object's vertices, edges and surfaces provide a framework upon which frames representing advancing mesh fronts can be determined. By analyzing adjacent frames singularity lines are located. Examples are given for validation purposes.

Keywords: Hex Meshing, Medial object, Frames, Singularity lines

1 Introduction - Contribution

Generating a structured mesh has been a topic of research since the 1970s. A limited class of objects can be meshed by using sweeping algorithms [1], [2] or plastering methods [3] , which is the 3D equivalent of the paving method in 2D [4]. Attempts have also been made to use the 3D medial object to decompose an object into sub regions which can be hex meshed using mapping techniques or midpoint subdivision [5]–[7]. Finally, more techniques have recently emerged which try to either parameterize or subdivide 3D domains based on frame fields [8]–[12].

In this work, the medial object of a 3D domain and the frame representation of hexahedra are combined to identify critical lines, called singularity lines, where, instead of four elements meeting at an edge, either three (a negative singularity) or five (a positive singularity) or more meet.

Price et al. [5] define the medial object as the locus of the center of an inscribed sphere of maximal diameter as it rolls around the interior of an object. A sphere is maximal if no other inscribed sphere that contains it exists. The medial object consists of medial surfaces, edges and vertices. In non-degenerate cases medial surfaces are constructed by centers of spheres that touch two boundary faces, while medial edges and vertices by centers of spheres that touch three and four respectively.

A frame consists of 3 mutual perpendicular unit vectors $\{\boldsymbol{u}, \boldsymbol{v}, \boldsymbol{w}\}$. These vectors represent the orientation in 3D space of a cube whose faces are normal to them.

The method proposed here relies on the generation of frames on top of the medial object. By analyzing frames, singularities that lie on the medial object, or that are normal to it, will be identified. Frames are constructed based on touching vectors that start from points on the medial object, end on the boundary of the domain and have length equal to the radius of the maximal sphere at each point. They hold the connection between the medial object and the boundary and can be thought of as showing directions where advancing fronts from boundary entities collide. The similarity between the singularities generated by progressive boundary offsetting, medial object approaches and 2D cross fields has recently been described [13]. The method is divided into the following steps (Fig. 1):

- Generate 3D frames on medial edges and vertices based on touching vectors (Fig. 1a).
- Identify singularities that lie on medial edges or enter medial surfaces through them (Fig. 1a inset).
- Align frames with the normal vector for each medial surface.
- Derive 2D cross fields on medial surfaces based on 3D frames on medial edges and vertices (Fig. 1b).
- Use 2D cross fields to trace singularities running along medial surfaces (Fig. 1c) and identify singularities normal to medial surfaces (Fig. 5).

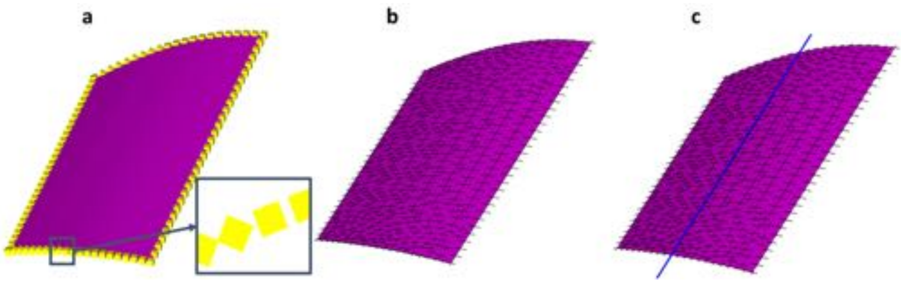

Fig. 1. Steps of the method to identify singularities.

Fig. 2 shows the motivation behind this method. On Fig. 2a, the medial axis of a simple 2D profile is given. On Fig. 2b, frames approximating touching vectors on a medial edge are shown. The sudden change in frame orientation (Fig. 2b inset) indicates the position of a positive singularity. The implied block topology is given. After extruding this profile to get a 3D object (Fig. 2c) a medial surface and its cross field provide the base to trace this singularity and create a complete singularity line. Singularities normal to the medial object or lying on medial edges can also exist.

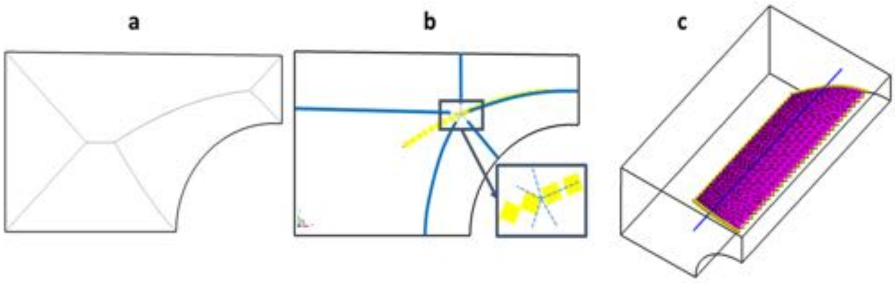

Fig. 2. Block topology for a five-sided prism. A positive singularity lies on a medial surface.

2 Method

2.1 Functional Representation of a Frame

A functional representation of frames restricted to the unit sphere S^2 is used which exhibits their 24 symmetries. As described by [11], this function can be decomposed into the basis of nine spherical harmonics namely $B = (Y_{4,-4}, Y_{4,-3}, \dots, Y_{4,4})$. The function of each frame is described as $F = Ba$ where the representation vector a is 9-dimensional and describes the influence of each harmonic. The difference between two frames i and j is given by $\int_{S^2}(F_i(x) - F_j(x))^2 dx$. Since the function basis B is orthonormal, this integral can be simplified as $\left\|(a_i - a_j)\right\|^2$. The proximity of two frames is now simply described by the squared distance of two vectors.

2.2 Frames on Medial Edges and Medial Vertices

Frames are generated on medial edges and vertices using touching vectors. Here, it is explained how frames on medial edges that touch 3 boundary faces can be calculated. The extension to medial vertices is straight-forward if more vectors are taken into account.

Let $\overrightarrow{n_1}$, $\overrightarrow{n_2}$ and $\overrightarrow{n_3}$ be three touching vectors on a medial edge. Firstly, a frame F_{12} that is aligned with vector $\overrightarrow{n_1}$ is created, by using three vectors $\{\overrightarrow{n_1},\overrightarrow{n_{12}},\overrightarrow{n_{112}}\}$, where $\overrightarrow{n_{12}} = \overrightarrow{n_1} \times \overrightarrow{n_2}$ and $\overrightarrow{n_{112}} = \overrightarrow{n_1} \times \overrightarrow{n_{12}}$. Similarly, frame F_{13} which is also aligned with the vector $\overrightarrow{n_1}$ is found. F_{12} and F_{13} are candidate frames for the touching vector $\overrightarrow{n_1}$. Each one has its representation vector, namely a_{12} and a_{13}. To have one frame corresponding to touching vector $\overrightarrow{n_1}$ a representation vector a_1 that is close to both a_{12} and a_{13} is found by minimizing the function

$$E = \|a_1 - a_{12}\|^2 + \|a_1 - a_{13}\|^2 \tag{1}$$

a_1 is a function of the Euler angles $a_1 = a_1(\vartheta_1, \theta_2, \theta_3)$ and consequently the minimization problem seeks for the optimum Euler angles.

A similar optimization problem is solved for all touching vectors. In the end, three representation vectors (frames), a_1 , a_2 and a_3 are identified, all referring to the same position on the medial edge. Finally, the frame that best fits all three frames is found by minimizing the function

$$E = \|a - a_1\|^2 + \|a - a_2\|^2 + \|a - a_3\|^2. \tag{2}$$

2.3 Singularity Identification

Singularities that enter medial surfaces can be identified by calculating the net rotation of three adjacent frames on medial edges calculated by (1). Fig. **3** (left) shows how frames along two neighboring points a and b on a medial edge can be compared. A singularity can be seen (right) to enter at the position where frames suddenly “flip”. This singularity will be traced along the medial surface as was also done in the case of Fig. 1.

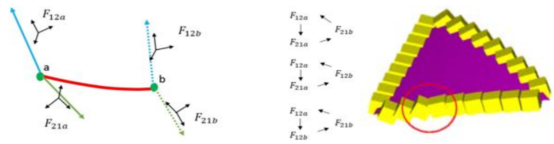

Fig. 3. Sudden changes in frame orientation indicate the position of a singularity.

Singularities that run along medial edges (Fig. 4b) can be identified by calculating the net rotation of the three different frames at each position on the medial edge given by (2) (Fig. 4a). In Fig. 4c frames around a medial edge can have different orientation and imply a singularity (top) or have all the same orientation (bottom) and imply a block topology that does not require singularities.

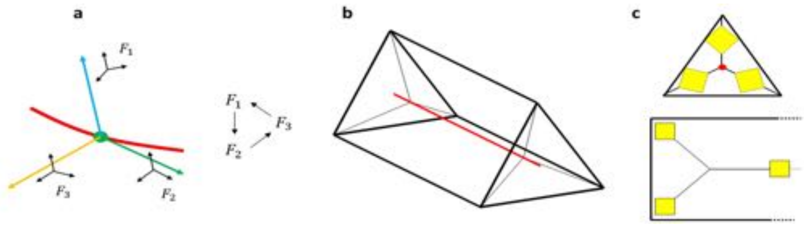

Fig. 4. Rotation around a medial edge. A negative singularity (red) runs across the medial edge.

Finally, singularities normal to medial surfaces can be identified by analyzing 2D cross fields on medial surfaces that depend on crosses on medial edges and vertices [14]. Since such crosses should lie on planes that are tangent to the medial surface, frames approximating touching vectors are rotated to align with the normal vector of the medial surface. Examples of such cross fields were given in Fig. 1 and Fig. 2. The simple examples of thin triangular and pentagonal prisms are shown in Fig. 5. Such singularities are mapped to the boundaries to generate complete singularity lines.

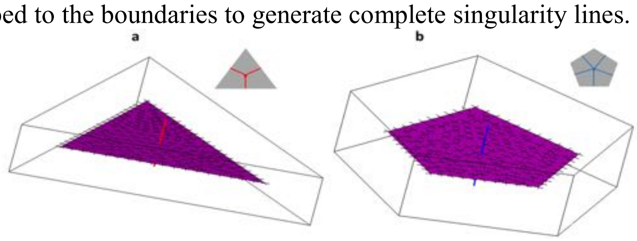

Fig. 5. Cross field on medial surfaces of thin triangular and pentagonal prisms.

3 Results

The aforementioned method was tested in various models. A plate with two rounded notches running across its upper and lower surfaces diagonally is shown on Fig. 6. As it can be seen in Fig. 7, the angle of the notches affects the singularity network and the method can capture the changes. The implied surface block topology is also given (Fig. 7c).

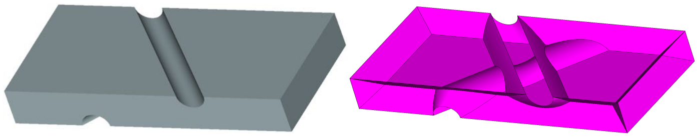

Fig. 6. Plate with rounded notches. On the right, the medial object is shown in purple.

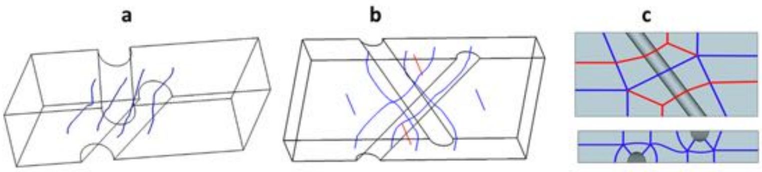

Fig. 7. Singularity network for different angles of the notches.

Fig. **8** shows a turbine blade with tip clearance and blended tip together with the necessary singularities for a multi-block decomposition. The block topology implied by the positive singularities is also given. In Fig. 9 the turning of the blade is greater and more singularities are identified. A possible blocking that these singularities induce is given. Finally, in Fig. 10 two blades are placed parallel to each other. More singularities are needed for a good block topology.

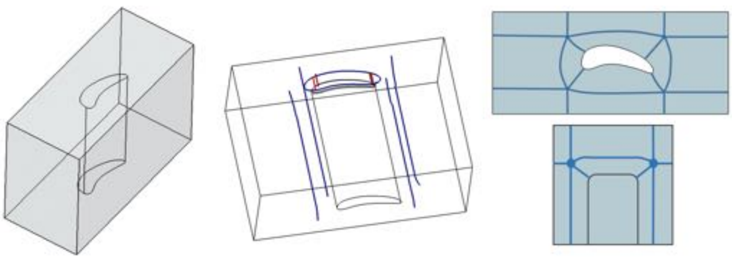

Fig. 8. Singularities and block topology in a fluid domain around a turbine blade with tip clearance and blended tip.

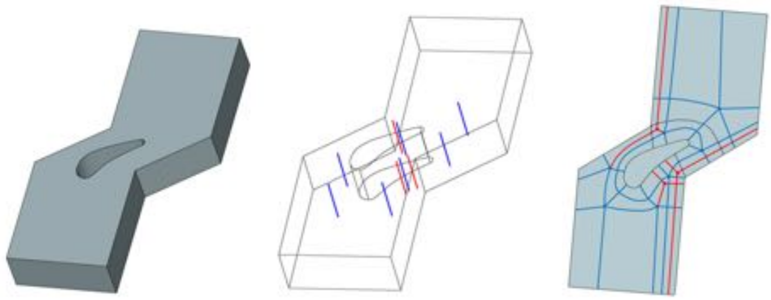

Fig. 9. Singularities around a turbine blade with high turning.

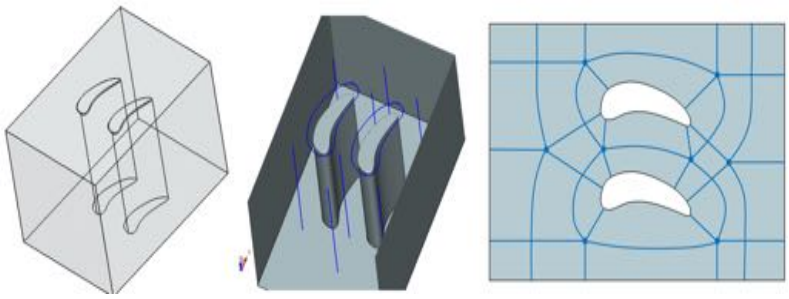

Fig. 10. Singularity lines and block topology for two parallel blades.

4 Conclusion

A novel approach was presented to identify singularity lines on a 3D domain. Unlike previous methods here it is attempted to identify their position based on an analysis of the medial object and then use them to generate decompositions. Future work will focus on making the method more robust and on creating decompositions based on the singularity lines and the structure the medial object provides.

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