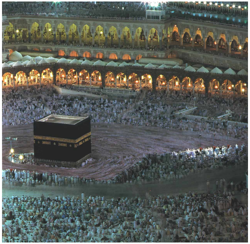
Des foules comme des jeux

Aimé Lachapelle
mfg labs

Orsay, 3 avril 2013

Part I : modeling discussion

Crowds



(lane formation movie)

- Today half of the human population lives in urban areas, in 1950 $\sim$ 30%, prediction for 2050 $\sim$ 70%.
- Fatal accidents in the last decades increased, e.g. Hadj in Mekka, Love Parade in Duisburg, Water Festival in Phnom Penh . . . .
- Empirical studies of human crowd started about 50 years ago, based on observations, photographs and video data.
- Mathematical modeling and simulations have been used successfully to secure dangerous situations (e.g. Jamaraat Bridge after 1000 pilgrim death in 10 years).

Mathematical modeling of human crowds I

- **Microscopic approaches:**
  Individuals are treated as agents whose motion is determined by the interaction with the surrounding agents and the goal to reach a desired destination.

  Behavioral force models - Helbing and Molnar (1995), Helbing et al. (2002), ...

  Cellular automata models - Burstedde et al. (2001), Kirchner and Schadschneider (2002), Adler and Blue (2000), ...

  Optimal control - Hoogendorn and Bovy (2003)

  Microscopic, granular-flow - Granular flowMaury et. al.

- **Mesoscopic approaches:**
  Mainly kinetic models, ideas from gas kinetic theory are used.

  Henderson (1971), Hoogendorn and Bovy (2000)

Mathematical modeling of human crowds II

- **Macroscopic approaches:**
  Here the crowd is treated like a density.

  Fluid dynamics - Henderson (1974), Hughes (2002), Colombo and Rossini (2005), Chalons (2007), Venuti et al. (2007), Bellomo and Dogbé (2008), ...

  Optimal transportation - Maury et al. (2010)

  Nonlinear convection diffusion equations - Burger et al. (submitted, 2010) , . . .

  Mean field games - Lachapelle (2010), Dogbé (2010), . . .

- **Multiscale approaches:**
  Coupling of micro- and macroscopic modeling approaches.

  Time evolving measures - Piccoli and Tosin (2009), Cristiani et al. (2010), . . .

Mathematical modeling of human crowds II

- **Macroscopic approaches:**
  Here the crowd is treated like a density.

  Fluid dynamics - Henderson (1974), Hughes (2002), Colombo and Rossini (2005), Chalons (2007), Venuti et al. (2007), Bellomo and Dogbé (2008), ...

  Optimal transportation - Maury et al. (2010)

  Nonlinear convection diffusion equations - Burger et al. (submitted, 2010) , . . .

  Mean field games - Lachapelle (2010), Dogbé (2010), . . .

- **Multiscale approaches:**
  Coupling of micro- and macroscopic modeling approaches.

  Time evolving measures - Piccoli and Tosin (2009), Cristiani et al. (2010), . . .

This list is by no means complete !!!!!

Crowd modeling and game theory I

- Individuals in a crowd have preferences and goals $\rightarrow$ they **optimize**
- They are rational and strategical $\rightarrow$ they choose **actions**
- They interact locally or globally with the others $\rightarrow$ **decentralized equilibrium**

...

- The crowd dynamic results from this process

$\rightarrow$ It is very natural to use some well-known economic concepts

Game theory (non-cooperative) provides a convenient framework:

- individuals are players
- they optimize a criterion (utility function, pay-off) that can take into account several effects (aversion, positioning, shortest path to an exit, etc.)
- they play "actions": e.g. choose velocity and direction, in regards to other players' actions
- decentralized equilibrium may exit as a result of the aggregation of optimal choices of all the players:

Pros & cons: pedestrian are "smart" individuals rather than sophisticated robots, beyond offering a description of how pedestrian behave in a crowd, it also answers the questions why and how the crowd moves ...but ...doesn't allow to model small cooperative groups

Crowd modeling and game theory II

Surprisingly, game theory approaches have been poorly investigated as compared to other classes of models (e.g. cellular automata).

The seminal paper is Hoogendorn and Bovy (2003): a game theory formalism at the microscopic scale.
Until 2010, this paper was the only one involving game theoretic modeling of crowds.

Main characteristics of their model:

- Deterministic
- Finite number of players. Perfect knowledge of the positions of all other players
- $\dot{x}_i(t) = \alpha_i(t)$, where $x_i(t)$ is player's i location at time t, and $\alpha_i(t)$ is the control parameter (action) chosen by the player.
- Anticipation is exogenous (a predefined function of states)

...as we will see in a few instants, it is a limited approach (nowadays, new mathematics allow for instance to consider a continuum of players)

Game theory in a single slide

- Modern theory "old as the hills": Von Neumann & Morgenstern (40s), Nash (50s), with many applications: economics, engineering, social sciences ...
- A usual non-cooperative game is made of: N-players, that choose actions $\alpha = (\alpha_i)$ and optimize their pay-offs: $\max_{\alpha_i} J_i(\alpha_i, \alpha_{-i})$ (that depend on player's i action α_i and other players' actions α_{-i})
- A Nash equilibrium α^* is such that every player chooses the best strategy given what others do:

$$\alpha_i^* = \text{argmax}_{\alpha_i} J_i(\alpha_i, \alpha_{-i}^*), \forall i$$

- Many situations impose to consider dynamic games: repeated games (discrete time) and differential games (continuous time)

(Stochastic) Differential Games

- Optimal control framework
- State of each player : $dx_i(t) = \alpha_i(t)dt\ (+\sigma_i(t)dW_t^i)$, $x_i \in \mathcal{X}$
  e.g. $x = \textit{position}$, $\alpha = \textit{velocity}$, various structure of noise correlation can be considered (most classical: W_i are independent Brownian Motions)
- Pay-off maximization: $\mathbb{E} J_i(T, x, \alpha)$
  e.g. $J_i(T, x_i, \alpha_i) = -\int_0^T f_i(\alpha_i(t), x_i(t), x_{-i}(t))dt + g_i(x_i(T), x_{-i}(T))$
  $\rightarrow f$ models the moving cost and the interactions between pedestrians, g is the goal function

What class of games to model crowd motions?

What happens in crowds?

- Players are anonymous (eq. symmetrical): $J_i = J$, $f_i = f$, $g_i = g$
- They are numerous (cf definition of a crowd): $N \to \infty$
- They dynamically control there trajectory
- They have no strategic power

$\rightarrow$ suitable class of games:

Mean Field Games (MFG)

=

Stochastic differential games with a continuum of anonymous players

MFG: a continuum of players

The key idea of modeling a crowd of agents (players) as a continuum ...

... i.e. a non atomic measure m on agents' state space $\mathcal{X}$ ...

...has been introduced by Robert Aumann (in the early 70s) as a breakthrough in macroeconomic theory (general equilibrium theory).

" continuum " of agents has many important consequences, e.g.:

- each individual agent has no strategic power
- the "number " of agents is a real number, and not an integer

Aumann's **non atomic game** + **dynamic and stochastic** setting = **MFG**

MFG: a limit of N-player stochastic differential games

- For N-player games: each player has a value function $U_i(t, x_i, x_{-i})$ that satisfies a PDE (extension of HJB for a single player optimization)

- Key hypothesis : the game is invariant under any player permutation
  N-player game:
  $\max J_i(T, x_i, \alpha_i) = \int_0^T f_i(\alpha_i(t), x_i(t), x_{-i}(t))dt + g_i(x_i(T), x_{-i}(T))$
  $dx_i(t) = \alpha_i(t)dt + \sigma_i(t)dW_t^i$
  $f_i = f, g_i = g$ and have the dependency form $(\alpha_i, x_i, \sum_{j \neq i} x_j)$

- Thus the N value functions can be replaced by one function $U^N(x, \hat{m})$ where $\hat{m}$ is the discrete distribution of other players: $\hat{m} = \sum \delta_y/(N-1)$

- The N-PDE system is replaced by a single equation on U^N

- when $N \to \infty$, U^N tends to a function $\mathcal{U}(x, m)$ (up to subsequences and under some general continuity assumptions), where m is a measure on agents state space X. $\mathcal{U}$ satisfies the **"master equation"** which is a very specific PDE integrating the concept of MFG equilibrium

The general case is extremely tricky and mathematically challenging

But many (sub) classes of the general case deserve focus and their own specific mathematical tools, results and numerical approximation methods:

- **Class A : only shared risk**, players' state space $\mathcal{X}$ is finite, $\mathcal{X} = \{1, \ldots, n\}$
  $\rightarrow$ monotone systems. Existence + Uniqueness results. Iterative methods exist.
- **Class B : individual independent risks**
  • The dynamic of the population is **deterministic**
  •The value function depends explicitly only on time and agent's state: $\mathcal{U}(t, x, m)$ is replaced by $v(t, x)$
  • The Master Equation becomes mostly unnecessary :
  the equilibrium can be computed through the HJB/FP forward/backward coupled PDE system

...mix models could be considered (in progress)

$\rightarrow$ our crowd motion models belong to **Class B : individual independent risks**

Mean field games: finite & infinite horizon

Microscopic model

- N-player stochastic differential game

$$\inf J_i(\alpha) = \mathbb{E}\left[\int_0^T f(t, X_t^{i,\alpha}, \hat{X}_t^{-i,\alpha}, \alpha_t^i)dt + g(X_T^{i,\alpha}, \hat{X}_t^{-i,\alpha})\right],$$

$$dX_t = \alpha_t dt + \sigma dW_t,\ \hat{x}^{-i} = \frac{1}{N-1}\sum_{j\neq i}\delta_{x^j}$$

Macroscopic model

- Limiting equations as $N \to \infty$ gives time dependent mean field game:

$$\partial_t v + \frac{\sigma^2}{2}\Delta v + H(t,x,\nabla v, m) = 0,\ v|_{t=T} = g(m_T)$$
$$\partial_t m - \frac{\sigma^2}{2}\Delta m + \mathrm{div}(m\, \partial_p H(t,x,\nabla v, m)) = 0,\ m|_{t=0} = m_0,$$

where H is the Legendre transform of the running cost f, v is the value function and m the density of the crowd.

Important remark: notice the red term in FP. Backward driven strategy in the continuation equation: $\alpha = \partial_p H(t,x,\nabla v, m)$

Mean field games: finite & infinite horizon

Microscopic model

- N-player stochastic differential game

$$J_i^s(\alpha) = \mathbb{E}\left[\int_0^\infty f_i(t, X_t^\alpha, \alpha_t^i) e^{-rt} dt\right],$$

$$dX_t = \alpha_t dt + \sigma dW_t,\ \hat{x}^{-i} = \frac{1}{N-1}\sum_{j\neq i}\delta_{x^j}$$

Macroscopic model

- Stationary problem: Find (u, m, λ) such that

$$\begin{aligned} & \frac{\sigma^2}{2}\Delta v + H(x, \nabla v, m) - rv = 0 \\ - \quad & \frac{\sigma^2}{2}\Delta m + \operatorname{div}(m\,\partial_p H(x, \nabla v, m)) = 0 \\ & \int m dx = 1,\ m > 0,\ \int v dx = 0. \end{aligned}$$

where H is the Legendre transform of the running cost f.

Link to deterministic optimal control problems

If the running cost f has the form

$$f(x, t, \alpha, m) = L(t, x, \alpha) + V[x, m]$$

and V and g are the Gateaux derivative of the potentials Φ and Ψ, then the MFG system can be written as the following optimal control problem:

$$\inf_{\alpha} \left[\int_0^T \int_\Omega L(t, x, \alpha) m(t, x) dxdt + \Phi(m) + \Psi(m(T)) \right]$$

under the constraint that

$$\frac{\partial m}{\partial t} - \nu \Delta m + \mathrm{div}(\alpha m) = 0.$$
$$m(x, 0) = m_0(x).$$

Optimality conditions:

$$\alpha = \frac{\partial H}{\partial p}(x, \nabla v), \quad \text{and } \partial_t v + \frac{\sigma^2}{2} \Delta v + H(t, x, \nabla v, m) = 0, \; v|_{t=T} = g(m_T).$$

MFG: main characteristics and differences with H&B modeling

Equilibrium with rational expectations:

- Each player considers the probability law of the crowd stochastic dynamic as given (this information is common knowledge)
- The law will not be impacted by her own decision
- Nevertheless, this law is the result of the individual choices (by integration over the density m)

Hoogendorn and Bovy	**MFG**
Perfect knowledge (*over knowledge*)	Common knowledge of an approximation of the states of all the pedestrians
Rational players, predefined anticipation method	Rational expectations (endogenous anticipation)
Two at a time interactions	Mean field type interactions
Individual strategical power	No individual strategical power

Part II : applications

Joint work with M.-T. Wolfram

Features of applications

Situations modeled:

- Two group interactions
- Aversion and xenophobic behavior
- Congestion

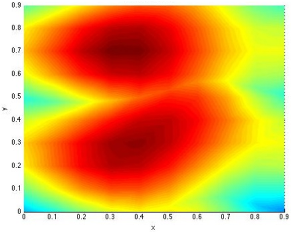
Numerical solvers:

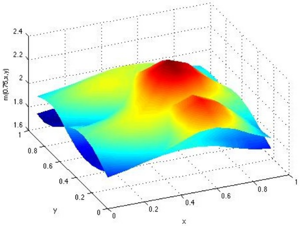
- Both finite difference and finite elements
- Gradient descent method
- Hybrid Discontinuous Galerkin method

Outputs:

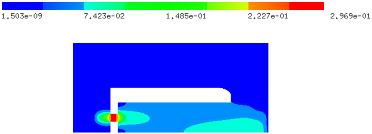
- Symmetry breaking
- Endogenous self-organizing
- Lane formation
- Anticipation behaviors

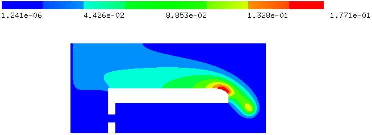
Symmetry breaking





Anticipation behavior





Case 1: Aversion and xenophobic pedestrians

- 2-group dynamical interactions, **convexity = crowd aversion**

- $\Omega = \mathbb{T}^2$, $Q = [0, T] \times \Omega$, 2 groups ($i = 1, 2$) made of a continuum of players

- **Population i criterion, given $(\alpha_t^{i-}, m_t^{i-})$ is**

$$J_\lambda^i(\alpha) := \int_Q \frac{|\alpha^i(t,x)|^2}{2} m^i(t,x)dx + \Phi_\lambda^i(m_t^1, m_t^2)dt + \int_\Omega \Psi^i m^i|_T$$

m^i weak solution to FP: $\partial_t m^i - \frac{\sigma^2}{2}\Delta m^i = -\mathrm{div}(\alpha^i m^i), m^i(0,.) = m_0^i(.)$

- $\Phi_\lambda^i(m_t) := \int_\Omega (m_t^i)^2 + \lambda m_t^1 m_t^2$, $\lambda \geq 0$ ($\lambda \leftrightarrow$ **xenophobia**)

Nash problem, $i = 1, 2$

$$(\mathcal{N}) \text{ Find } \bar{\alpha} = (\bar{\alpha}^1, \bar{\alpha}^2) \text{ s.t.: } J^i_\lambda(\bar{\alpha}) = \inf_{\alpha^i \in \mathcal{M}_b(Q,\mathbb{R}^d)} J^i_\lambda(\alpha^i, \bar{\alpha}^{i-})$$

MFG, $i = 1, 2$

$$(\mathcal{MFG}) \begin{cases} \partial_t m^i - \frac{\sigma^2}{2}\Delta m^i + \mathrm{div}(m^i \nabla v^i) = 0 \ , \ m^i(0,.) = m^i_0, \\ \partial_t v^i + \frac{\sigma^2}{2}\Delta v^i + \frac{|\nabla v^i|^2}{2} = \Phi^i_\lambda(m)' \ , \ v^i(T,.) = \Psi^i. \end{cases}$$

Joint Problem

$$(\mathcal{Q}) \inf_{\alpha=(\alpha^1,\alpha^2)} J_\lambda(\alpha) := J^1_{\lambda/2}(\alpha) + J^2_{\lambda/2}(\alpha)$$

Proposition 1

If $\lambda \leq 2$, then the following are equivalent:

1. $\overline{\alpha} \in \mathcal{M}_b(Q, \mathbb{R}^d)$ solves $(\mathcal{N})$ and $\bar{m}$ verifies FP for $\alpha = \overline{\alpha}$,
2. $\overline{\alpha} \in \mathcal{M}_b(Q, \mathbb{R}^d)$ solves $(\mathcal{Q})$ and $\bar{m}$ verifies FP for $\alpha = \overline{\alpha}$,
3. $(\bar{m}, \bar{v})$ solve $(\mathcal{MFG})$ eand $\overline{\alpha} = \nabla \bar{v}$.

If $\lambda > 2$ then we only have 2. $\Rightarrow$ 1.,3.

Proposition 2

$(\mathcal{Q})$ has a unique solution if $\lambda \leq 2$ and $m_0 \in L^2$.

Algorithm & simulations (1) : gradient descend method

- $\lambda \leq 2 \rightarrow$ joint gradient descend to solve $(\mathcal{Q})$
- $\lambda > 2 \rightarrow$ alternate gradient descends (partial convexity)
- Classical change of variable: $q^i = \alpha^i m^i$
- Initialization: $(q^{i(0)}, m^{i(0)})$

- Step $k+1$:
  1. $-\partial_t \theta^i - \frac{\sigma^2}{2}\Delta\theta^i = \frac{-|q^{i(k)}|^2}{(m^{i(k)})^2} + (2m^{i(k)} + \lambda m^{i^-(k)})$, $\theta^i|_{t=T} = \Psi^i$
     Rq: finite difference discretization
  2. $\nabla J(q^{i(k)}, q^{i^-(k)}) = \frac{2q^{i(k)}}{m^{i(k)}} + \nabla\theta^i$
  3. $q^{i(k+1)} = q^{i(k)} - \rho_k \nabla J(q^{i(k)}, q^{i^-(k)})$

- $\lambda = 20$ (high level of xenophobia)
- $T = 1$ et $\frac{\sigma^2}{2} = 0.1$
- Difference between groups: initial positions and objectives

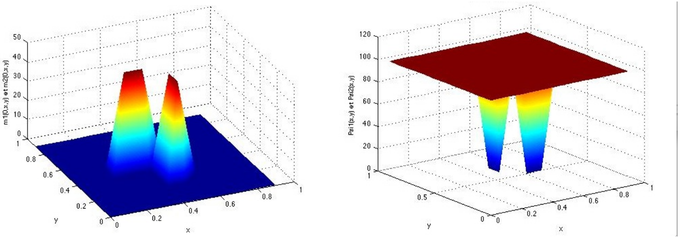
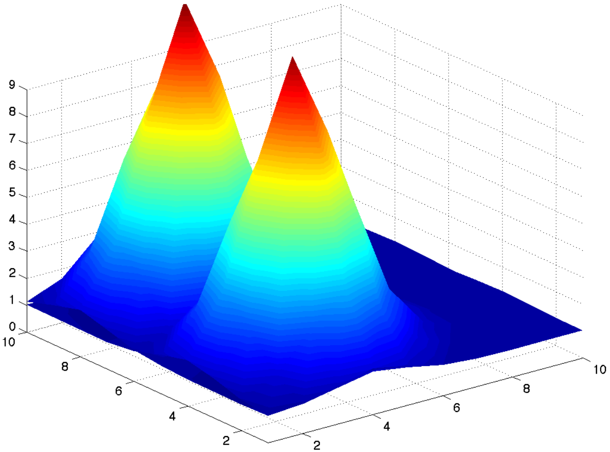


Figure: Input data

Algorithm & simulations (3) : movie



We consider the following stochastic control problem

$$\inf_{\alpha_t} \mathbb{E}\left(\int_0^\infty (\frac{|\alpha_t|^q}{q}(m(X_t,s))^a + k(X_t,s))e^{-rs}ds\right)$$
$$dX_t = \sigma dW_t + \alpha_t dt$$

The corresponding mean field game is given by

$$\nu\Delta u - \frac{1}{p}\frac{|\nabla u|^p}{m^b} - ru = k$$
$$-\nu\Delta m - \operatorname{div}(m\frac{(\nabla u)^{p-1}}{m^b}) = 0, \quad m(x,0) = m_0(x),$$
$$\int udx = 0, \quad \int m\ dx = 1.$$

where $b = \frac{a}{q-1}$ and $p = \frac{q}{q-1}$.

Congestion model for two species I

Two species model - each density would like to avoid congestion within its own group as well as with the other.

$$
\begin{aligned}
&\inf_{\alpha_t^i} \mathbb{E}\left(\int_0^\infty \left(\frac{|\alpha_t^i|^q}{q}(m^i(X_t,s))^a(m^j(X_t,s))^{\tilde{a}} + k(X_t,s)\right)e^{-rs}ds\right)\\
&dX_t = \sigma dW_t + \alpha_t^i dt
\end{aligned}
$$

for i=1,2. The corresponding mean field game for both species reads as

$$
\begin{aligned}
-\nu\Delta u_i + \frac{1}{p}\frac{|\nabla u_i|^p}{m_i^b m_j^{\tilde{b}}} - ru_i &= k\\
-\nu\Delta m_i - \operatorname{div}\left(m_i\frac{(\nabla u_i)^{p-1}}{m_i^b m_j^{\tilde{b}}}\right) &= 0\\
\int u_i dx = \ 0, \quad \int m_i \ dx &= 1.
\end{aligned}
$$

where $b = \frac{a}{q-1}$, $\tilde{b} = \frac{\tilde{a}}{q-1}$ and $p = \frac{q}{q-1}$.

Congestion model for two species II

Two species model - each density would like to avoid congestion within its own group as well as with the other.

$$\inf_{\alpha_t^i} \mathbb{E}\left(\int_0^\infty \left(\frac{|\alpha_t^i|^q}{q}(c + m^i(X_t, s))^a (c + m^j(X_t, s))^{\tilde{a}} + k(X_t, s)\right)e^{-rs} ds\right)$$

$$dX_t = \sigma dW_t + \alpha_t^i dt$$

for i=1,2. The corresponding mean field game for both species reads as

$$-\nu \Delta u_i + \frac{1}{p}\frac{|\nabla u_i|^p}{(c + m_i)^b (c + m_j)^{\tilde{b}}} - r u_i = k$$

$$-\nu \Delta m_i - \mathrm{div}\left(m_i \frac{(\nabla u_i)^{p-1}}{(c + m_i)^b (c + m_j)^{\tilde{b}}}\right) = 0$$

$$\int u_i dx = 0, \quad \int m_i \, dx = 1.$$

for a small positive constant c.

Boundary conditions I

- *Neumann boundary conditions:*

  In- and outflow or people m_i, i.e.

$$\frac{\partial m_i}{\partial n} = j_i^{in} \quad \text{for all } x \in \Gamma_i^{in} \quad \text{and} \quad \frac{\partial m_i}{\partial n} = j_i^{out} \quad \text{for all } x \in \Gamma_i^{out}$$

$$\text{with} \int_{\Gamma_i^{out}} j_i^{out} \cdot n \; ds = \int_{\Gamma_i^{in}} j_i^{in} \cdot n \; ds.$$

$\Rightarrow$ homogeneous Neumann boundary conditions for u_i, i.e. $\frac{\partial u_i}{\partial n} = 0$, for all $x \in \Gamma$.

Boundary conditions II

- *Dirichlet boundary conditions:*

  Homogeneous Dirichlet conditions for m_i at the exit (people leave the room, hence the density has to be zero) and a homogenous Neumann boundary conditions on the rest of the boundary, i.e.

$$m_i = 0 \quad \text{for all } x \in \Gamma_i^{out} \text{ and } \frac{\partial m_i}{\partial n} = 0 \quad \text{on the rest of the boundary.}$$

  $\Rightarrow$ same boundary conditions for u_i, the integral condition for u_i necessary.

  Integral condition for m_i is replaced by a **source term** in the Kolmogorov equation, i.e.

$$-\nu\Delta m_i - \operatorname{div}\left(m_i \frac{(\nabla u_i)^{p-1}}{(c+m_i)^b (c+m_j)^{\tilde{b}}}\right) = \mathbf{f(x)}.$$

  This source term can be interpreted as an exit of an underground or supermarket.

Hybrid discontinuous Galerkin method for elliptic problems[1]

We consider the Laplace problem on the domain Ω

$$-\Delta u = 0.$$

Notation: $\mathcal{T}_h$ denote the triangulation of Ω into triangles T, $\mathcal{F}_h$ the set of facets F.

Basic idea: Choose discontinuous basis functions on the triangle and enforce continuity via Lagrange functions that live on the element interface (representing the trace of the continuous function u). We choose the following spaces

$$V_h := \{(u, u_F) : u \in P^k(T)\ \forall T \in \mathcal{T}_h, u_h \in L^2(F)\ \forall F \in \mathcal{F}_h\}$$

where P^k denotes the space of polynomials of degree less or equal to k.

[1] Cockburn, B., Gopalakrishnan, J. and Lazarov, R. *Unified Hybridization of Discontinuous, Mixed and Continuous Galerkin Methods for Second Order Elliptic Problems*, SIAM J. Numer. Anal. (47), 1319-1365

Hybrid discontinuous Galerkin method for elliptic problems

We consider the Laplace problem on the domain Ω

$$-\Delta u = 0.$$

Then the hybrid discontinuous Galerkin (HDG) method reads as:

$$\sum_{T \in \mathcal{T}_h} [\int_T \nabla u \nabla v dx - \int_{\partial T} \frac{\partial u}{\partial n}(v - v_F) ds - \overbrace{\int_{\partial T} (u - u_F) \frac{\partial v}{\partial n} ds}^{\text{symmetry}}$$
$$+ \underbrace{\frac{\alpha}{h} \int_{\partial T} (u - u_F)(v - v_F) ds}_{stability}] = 0$$

where α denotes the stability parameter and h the maximum mesh size.

HDG methods for hyperbolic problems

We consider

$$\mathrm{div}(bu) = 0$$

where the normal component of the vector field b is continuous across element interfaces.
The HDG formulation of the problem reads as

$$\sum_{T\in\mathcal{T}_h} \int_T \mathrm{div}(bu) v dx = \sum_{T\in\mathcal{T}_h} [-\int_T ub \cdot \nabla v dx + \int_{\partial T} u^{up} b_n v ds]$$

where b_n denotes the normal component of the vector field b and u^{up} is the upwind value defined by

$$u^{up} = \begin{cases} u & \text{if } b_n > 0 \\ u_F & \text{if } b_n < 0. \end{cases}$$

Problem: element only couple on the downwind element, to obtain a coupling with the upwind element we add the term

$$\int_{T^{out}} b_n (u_F - u) v_F ds \quad \text{where } T^{out} = \{x \in \partial T : b_n > 0\}.$$

HDG for the stationary congestion model

- Stationary problem is a coupled system of four nonlinear partial differential equations $\Rightarrow$ Newton's method.
- Two nonlinear convection-diffusion equations for m_i

$$-\nu \Delta m_i - \mathrm{div}\left(\frac{m_i}{(c+m_i)^b (c+m_j)^{\tilde{b}}} \nabla u_i\right) = f_i(x)$$

$\Rightarrow$ HDG for diffusion and convection part (with upwind).

- Two nonlinear Hamilton Jacobi equations for u_i:

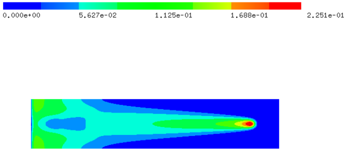
$$-\nu \Delta u_i + \frac{1}{2} \frac{|\nabla u_i|^2}{(c+m_i)^b (c+m_j)^{\tilde{b}}} - r u_i = 0$$

$\Rightarrow$ HDG for diffusion and Hamiltonian (no stabilization).

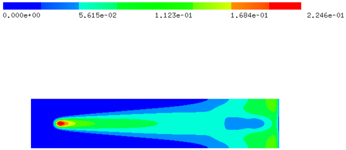
- Computational domain $\Omega = [-1, 1] \times [-0.2, 0.2]$
- Single source of people for every species, i.e. $f(x) = 50 \times \exp(-\frac{(x \pm 0.8)^2 + y^2}{10^{-3}})$
- The parameters are

$$a = 0.5,\ \ \tilde{a} = 0.5,\ \ q = 2,\ \ ,\nu = 0.05,\ \ k = 1,\ \ r = 1.$$

- The maximum mesh size is $h = 0.03$ and we choose $c = 0.01$.



(a) Population m_1

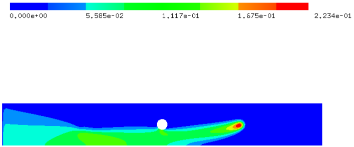


(b) Population m_2

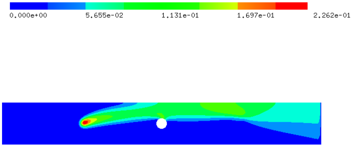
Lane formation

- Computational domain $\Omega = [-1.5, 1.5] \times [-0.2, 0.2]$
- Single source of people for every species, i.e. $f(x) = 50 \times \exp(-\frac{(x \pm 0.75)^2 + y^2}{10^{-3}})$
- The parameters are

$$a = 0.25, \; \tilde{a} = 0.75, \; q = 2, \; , \nu = 0.05, \; k = 1, \; r = 1.$$



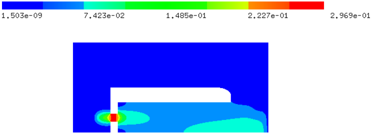
(c) Population m_1



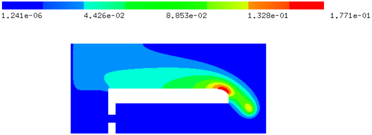
(d) Population m_2

- Rectangular domain with two corridors and a small door (bottleneck).
- Two sources placed in the lower left and lower right corner
- The parameters are

$$a = 0.25,\ \tilde{a} = 2,\ q = 2,\ ,\nu = 0.1,\ k = 1,\ r = 1.$$



(e) Population m_1



(f) Population m_2

Pros & Cons of the approach, further developments

- MFG is a natural macroscopic approach for crowd motion modeling, bringin together three core aspects: dynamical game theory, a continuum of anonymous players with nil individual influence on the crowd density, anticipation behavior

- The computational cost is clearly lower for MFG than for agent-based models

- It allows entries and exits of players (contrary to finite games)

- Limitations: no small groups in the crowd, does not consider nearest neighbor type interactions

- Possible developments: introduce partial blindness (convolution term), global shared risk (emergency evacuation), congestion density constraint (cf. Filippo's talk)

Pros & Cons of the approach, further developments

- MFG is a natural macroscopic approach for crowd motion modeling, bringin together three core aspects: dynamical game theory, a continuum of anonymous players with nil individual influence on the crowd density, anticipation behavior

- The computational cost is clearly lower for MFG than for agent-based models

- It allows entries and exits of players (contrary to finite games)

- Limitations: no small groups in the crowd, does not consider nearest neighbor type interactions

- Possible developments: introduce partial blindness (convolution term), global shared risk (emergency evacuation), congestion density constraint (cf. Filippo's talk)

Thank you very much for your attention !