Journée ANR ISOTACE

Macroscopic models for crowd behavior simulation

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Outline of the talk

Crowd dynamics

Model to reproduce known pedestrian behavior:

- evacuation dynamics: seeking the *fastest* route, avoiding high densities and borders (discomfort)
- desired speed ($\sim 1.34m/s$), depending on situations
- lines and more general patterns formation: self-organization dynamics which minimize interactions with the opposite stream
- oscillations at bottlenecks in opposite streams passing through a narrow passage
- collective auto-organization at intersections: increasing the average efficiency
- etc ...

(Helbing-Farkas-Molnar-Vicsek 2002)

Panic

Crowd behavior changes in **panic** situations and becomes irrational:

- getting nervous → “freezing by heating”
- people try to move faster → clogging → “faster is slower”
- jams building up at exits → fatal pressures
- escape slowed down
- herding behavior (to follow others) → ignorance of available exits
- “phantom panics” due to counterflow and impatience

(Helbing-Farkas-Molnar-Vicsek 2002)

Macroscopic models

$$\Big[\text{number of individuals in } \Omega \subset \mathbb{R}^2 \text{ at time } t\Big] = \int_\Omega \rho(t, \mathbf{x})\, d\mathbf{x}$$

must be conserved!

$$\int_\Omega \rho(t_2, \mathbf{x}) d\mathbf{x} \quad = $$
$$\int_\Omega \rho(t_1, \mathbf{x}) d\mathbf{x} \quad - \quad \int_{t_1}^{t_2} \int_{\partial\Omega} \mathbf{f}(t, \sigma) \cdot \vec{n}\ d\sigma\, dt$$

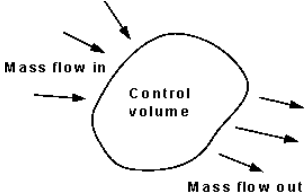
$$\Downarrow$$

divergence theorem for $(\rho, \mathbf{f})$

$$\Downarrow$$

$$\int_{t_1}^{t_2} \int_a^b \partial_t \rho + \mathrm{div}_{\mathbf{x}} \mathbf{f}\ d\mathbf{x}\ dt = 0$$

Ω



Macroscopic models

- Conservation law:

$$\partial_t \rho + \mathrm{div}_{\mathbf{x}} \mathbf{f}(t\, x) = 0$$

- Flux-density relation: $\mathbf{f}(t, \mathbf{x}) = \rho(t, \mathbf{x}) \vec{V}(t, \mathbf{x})$

- Density must be non-negative and bounded: $0 \leq \rho(t, \mathbf{x}) \leq \rho_{\max}$, $\forall \mathbf{x}, t > 0$ (maximum principle?)

- Different from fluid dynamics:
  - preferred direction
  - no conservation of momentum / energy
  - no viscosity
  - $n \ll 6 \cdot 10^{23}$

Continuum hypothesis

$n \ll 6 \cdot 10^{23}$ but ...

Continuum hypothesis

$n \ll 6 \cdot 10^{23}$ but ...

Portland, Oregon, May 2008

Outline of the talk

Scalar conservation laws

We deal with a PDE equation of the form

$$\begin{aligned} &\partial_t \rho + \mathrm{div}_{\mathbf{x}} \mathbf{f}(t, \mathbf{x}, \rho) = 0 \\ &\rho(0, \mathbf{x}) = \rho_0(\mathbf{x}) \\ &+\mathrm{BC} \end{aligned}$$

where $t \in [0, +\infty[$, $\mathbf{x} = (x_1, x_2) \in \mathbb{R}^2$,

$\rho = \rho(t, \mathbf{x}) \in \mathbb{R}$ conserved quantity

$\mathbf{f} : [0, +\infty[\ \times \mathbb{R}^2 \times \mathbb{R} \to \mathbb{R}^2$ flux

Main features:

- ρ NOT smooth
- existence $\leftarrow$ weak solutions
- uniqueness $\leftarrow$ entropy conditions

Kružkov theory (1970)

- smooth flux: $\mathbf{f} \in \mathcal{C}_c^1\left([0,+\infty[\,\times\mathbb{R}^2 \times \mathbb{R}\right)$
- entropy weak solution: $\forall k \in \mathbb{R}$ and $\varphi \in \mathcal{C}_c^1\left([0,+\infty[\,\times\mathbb{R}^2\right)$

$$\int_0^{+\infty}\iint_{\mathbb{R}^2} |\rho - k|\partial_t\varphi + \operatorname{sgn}(\rho - k)(\mathbf{f}_i(t,\mathbf{x},\rho) - \mathbf{f}_i(t,\mathbf{x},k))\partial_{x_i}\varphi - \operatorname{sgn}(\rho - k)\partial_{x_i}\mathbf{f}_i\,\varphi\, d\mathbf{x}dt \geq 0$$

- well posedness: existence, uniqueness, stability

Outline of the talk

Eikonal equation

Consider $\Omega \subset \mathbb{R}^2$ walking facility $(\partial\Omega = \partial\Omega_{wall} \cup \partial\Omega_{in} \cup \partial\Omega_{exit})$;
we look for $\phi : \Omega \to \mathbb{R}$ solution of the PDE equation

$$|\nabla_{\mathbf{x}}\phi| = C(\mathbf{x}) \quad \text{in } \Omega$$

$$\phi(t, \mathbf{x}) = 0 \qquad \text{for } \mathbf{x} \in \partial\Omega_{exit}$$

where $C = C(\mathbf{x}) \geq 0$ is the *running cost*:

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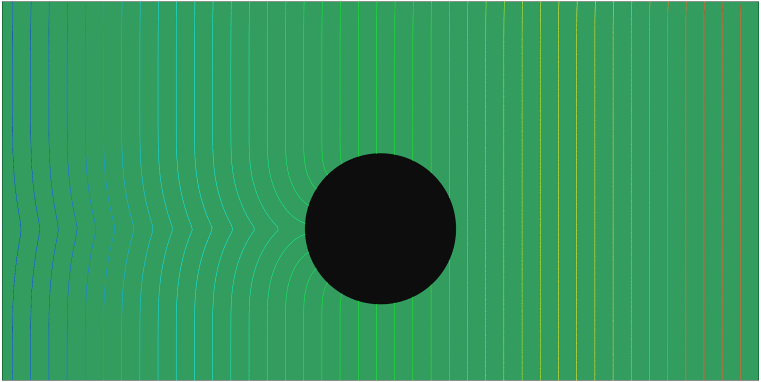
where $C = C(\mathbf{x}) \geq 0$ is the *running cost*:

the solution $\phi(\mathbf{x})$ represents the (weighted) distance of the position $\mathbf{x}$ from the target $\partial\Omega_{exit}$

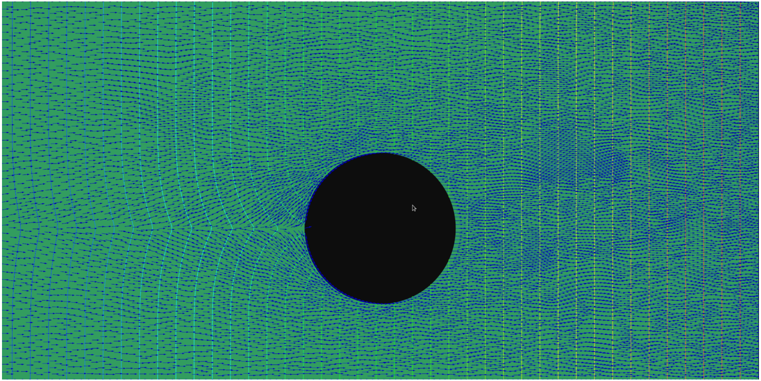
$$\text{if } C(\mathbf{x}) \equiv 1 \text{ and } \Omega \text{ concave then } \phi(\mathbf{x}) = d(\mathbf{x}, \partial\Omega_{exit})$$

(existence and uniqueness under some regularity assumption on C)

Eikonal equation: level set curves for $|\nabla_{\mathbf{x}}\phi| = 1$



Eikonal equation: vector field $\vec{N} = -\frac{\nabla_{\mathbf{x}}\phi}{|\nabla_{\mathbf{x}}\phi|}$



Outline of the talk

Hughes' model (2002)

Mass conservation

$$\partial_t \rho + \mathrm{div}_{\mathbf{x}} \left(\rho \vec{V}(\rho) \right) = 0 \quad \text{in } \mathbb{R}^+ \times \Omega$$

where

$$\vec{V}(\rho) = v(\rho)\vec{N} \quad \text{and} \quad v(\rho) = v_{\max} \left(1 - \frac{\rho}{\rho_{\max}} \right)$$

Direction of the motion: $\vec{N} = -\dfrac{\nabla_{\mathbf{x}} \phi}{|\nabla_{\mathbf{x}} \phi|}$ is given by

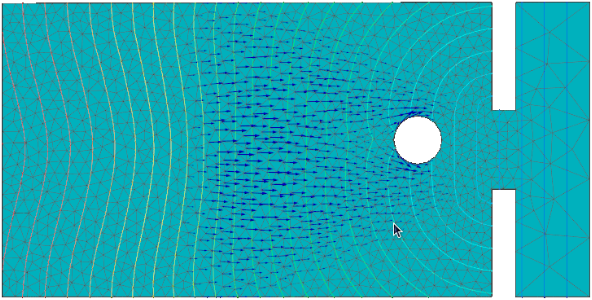
$$|\nabla_{\mathbf{x}} \phi| = \frac{1}{v(\rho)} \quad \text{in } \Omega$$

$$\phi(t, \mathbf{x}) = 0 \qquad \text{for } \mathbf{x} \in \partial\Omega_{exit}, \forall t \geq 0$$

- pedestrians tend to minimize their estimated travel time to the exit
- pedestrians temper their estimated travel time avoiding high densities
- CRITICS: instantaneous global information on entire domain

Hughes' model

Evolution of the velocity field:

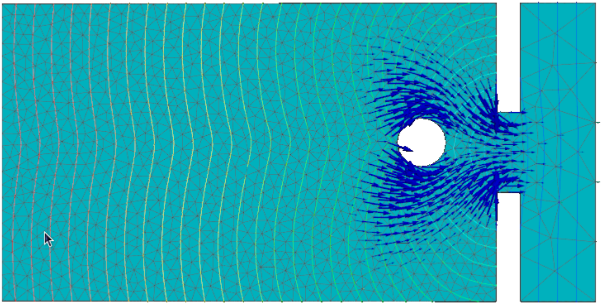


$T = 0.2$

(Twarogowska-Aissiouene-Duvigneau-Goatin, 2012)

Hughes' model

Evolution of the velocity field:

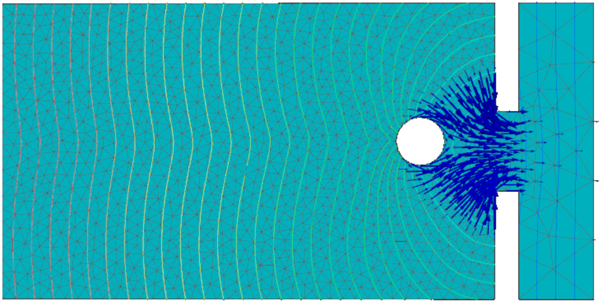


$T = 0.5$

(Twarogowska-Aissiouene-Duvigneau-Goatin, 2012)

Hughes' model

Evolution of the velocity field:

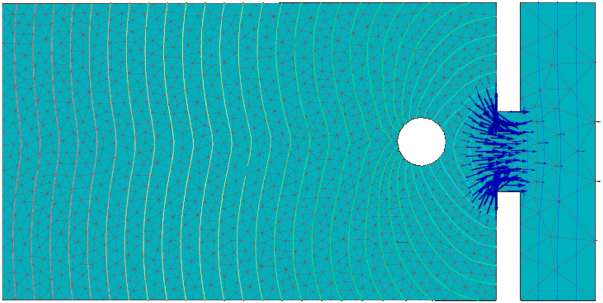


$T = 0.8$

(Twarogowska-Aissiouene-Duvigneau-Goatin, 2012)

Hughes' model

Evolution of the velocity field:



$T = 1.1$

(Twarogowska-Aissiouene-Duvigneau-Goatin, 2012)

Dynamic model with memory effect

Mass conservation

$$\partial_t \rho + \mathrm{div}_{\mathbf{x}} \left(\rho \vec{V}(\rho) \right) = 0 \quad \text{in } \mathbb{R}^+ \times \Omega$$

where

$$\vec{V}(\rho) = v(\rho)\vec{N} \quad \text{and} \quad v(\rho) = v_{\max}\left(1 - \frac{\rho}{\rho_{\max}}\right)$$

Direction of the motion: $\vec{N} = -\dfrac{\nabla_{\mathbf{x}}(\phi + \omega D)}{|\nabla_{\mathbf{x}}(\phi + \omega D)|}$ where

$$|\nabla_{\mathbf{x}} \phi| = \frac{1}{v_{\max}} \quad \text{in } \Omega, \quad \phi(\mathbf{x}) = 0 \text{ for } \mathbf{x} \in \partial\Omega_{exit},$$

$$D = D(\rho) = \frac{1}{v(\rho)} + \beta\rho^2 \quad \text{discomfort}$$

- pedestrians seek to minimize their estimated travel time based on their knowledge of the walking domain
- pedestrians temper their behavior locally to avoid high densities

(Xia-Wong-Shu, 2009)

Second order model

Euler equations with relaxation

$$\partial_t \rho + \nabla \cdot (\rho \vec{V}) = 0$$

$$\partial_t (\rho \vec{V}) + \nabla \cdot (\rho \vec{V} \otimes \vec{V}) = \frac{1}{\tau}(\rho v_e(\rho) \vec{N} - \rho \vec{V}) + \nabla P(\rho)$$

(Jiang-Zhang-Wong-Liu, 2010)

Second order model

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where

$$v_e(\rho) = v_{\max} \exp\left(-\alpha \left(\frac{\rho}{\rho_{\max}}\right)^2\right), \qquad P(\rho) = p_0 \rho^\gamma$$

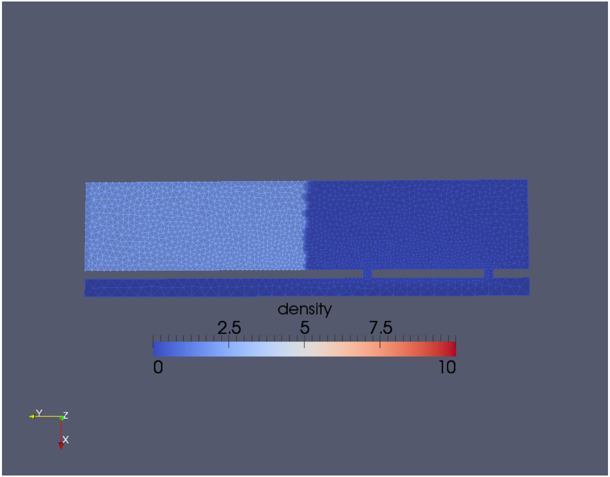
and boundary conditions: $\nabla_{\mathbf{x}} \rho \cdot \vec{n} = 0$ and $\vec{V} \cdot \vec{n} = 0$

(Jiang-Zhang-Wong-Liu, 2010)

Outline of the talk

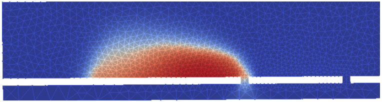
The fastest route ...

... needs not to be the shortest!

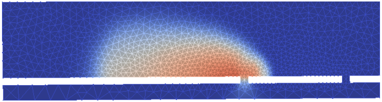


The fastest route ...

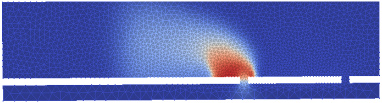
... depends on the model!



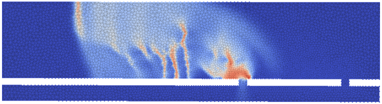
$|\nabla_{\mathbf{x}}\phi| = 1$



$\nabla_{\mathbf{x}}(\phi + \omega D)$



$|\nabla_{\mathbf{x}}\phi| = 1/v(\rho)$



second order

(Twarogowska-Duvigneau-Goatin, 2012)

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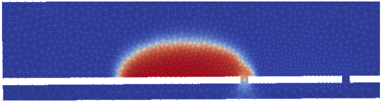

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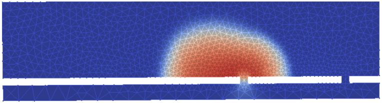

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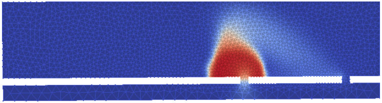

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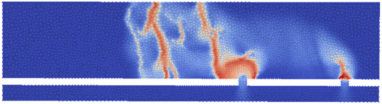

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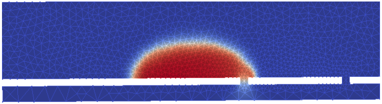

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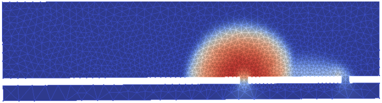
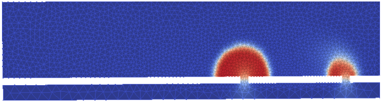


$\nabla_{\mathbf{x}}(\phi + \omega D)$



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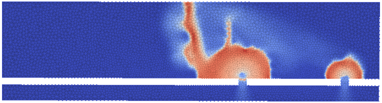

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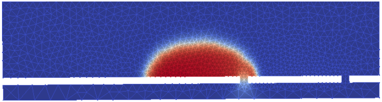

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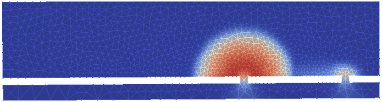

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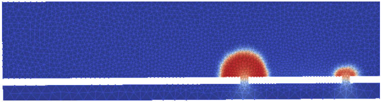

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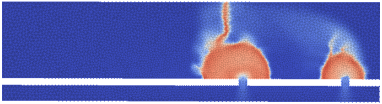
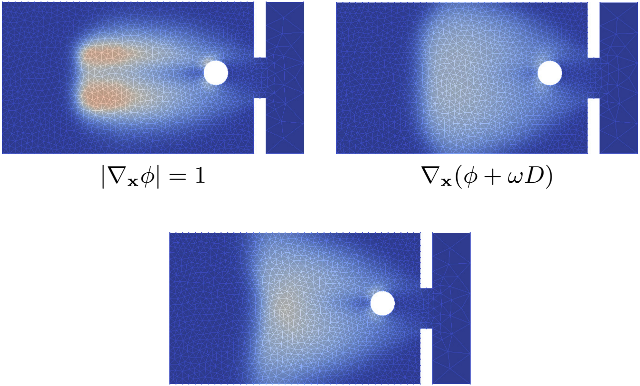


second order

(Twarogowska-Duvigneau-Goatin, 2012)

Braess' paradox?

A column in front of the exit can reduce inter-pedestrians pressure and evacuation time?



$|\nabla_{\mathbf{x}}\phi| = 1$ $\nabla_{\mathbf{x}}(\phi + \omega D)$

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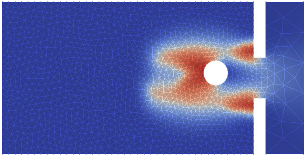

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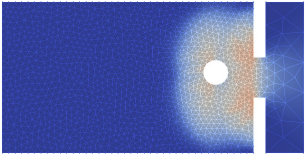

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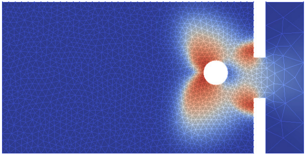

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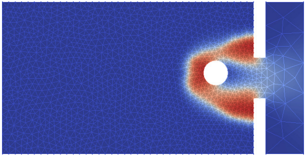

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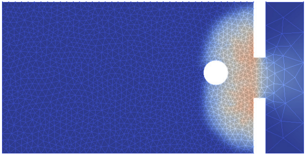

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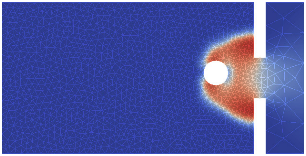

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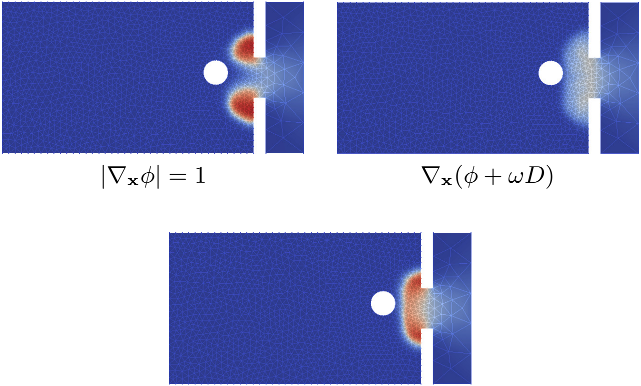

$|\nabla_{\mathbf{x}}\phi| = 1$

$\nabla_{\mathbf{x}}(\phi + \omega D)$

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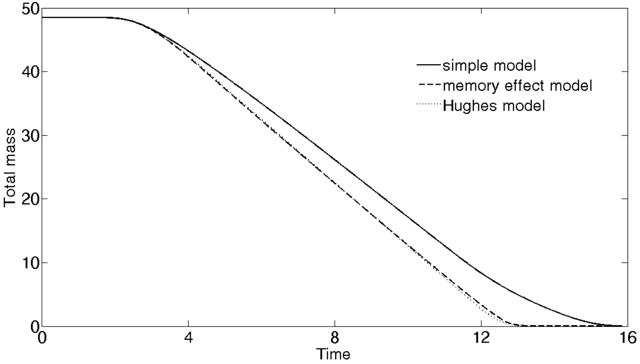
(Twarogowska-Duvigneau-Goatin, 2012)

Braess' paradox?

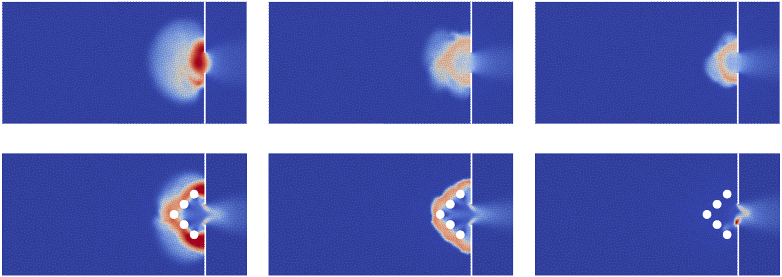
Evacuation time:



(Twarogowska-Duvigneau-Goatin, 2012)

Braess' paradox?

The second order model displays a better behavior:



(Twarogowska-Duvigneau-Goatin, 2013)

Braess' paradox?

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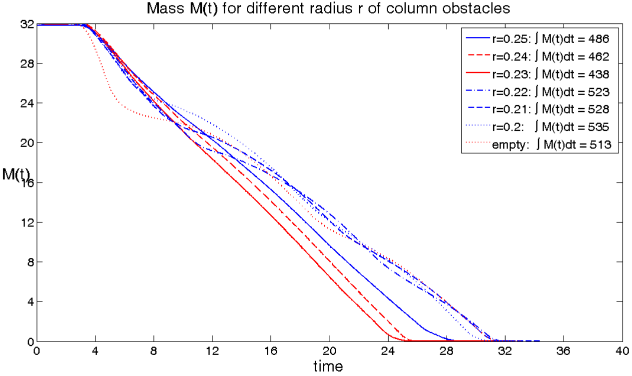
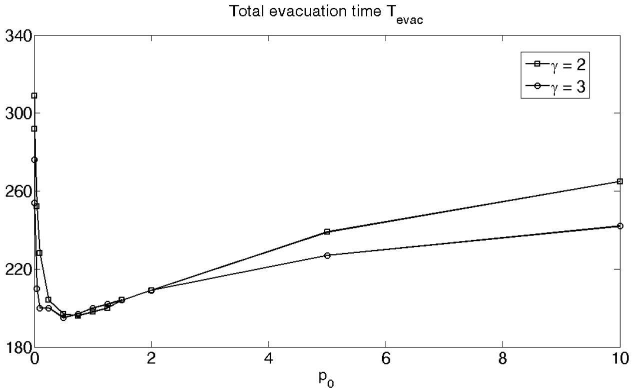


(Twarogowska-Duvigneau-Goatin, 2013)

Dependence on p_0

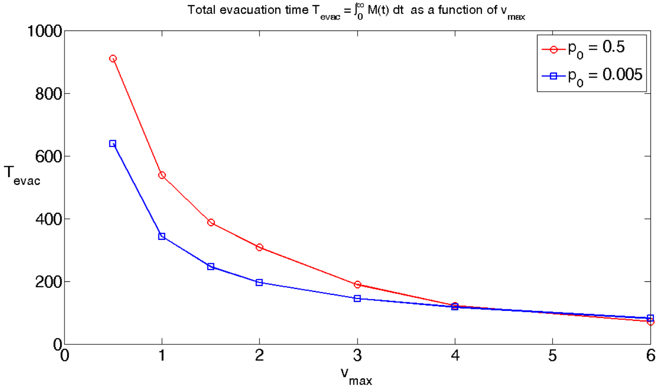
Total evacuation time optimal for $p_0 = 0.5$



(Twarogowska-Duvigneau-Goatin, 2013)

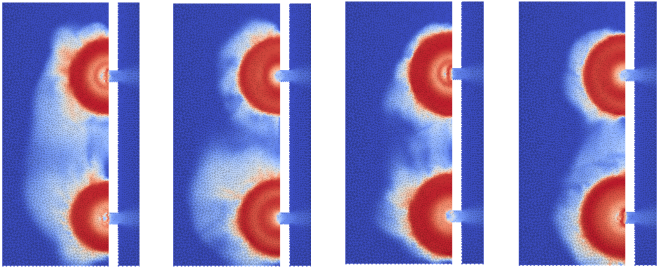
Dependence on $v_{\max}$

Total evacuation time



(Twarogowska-Duvigneau-Goatin, 2013)

Stop-and-go waves



t=90 t=100 t=110 t=120

$$P(\rho) = 0.005\rho^2,\ v_{\max} = 2,\ \rho_{\max} = 7$$

(Twarogowska-Duvigneau-Goatin, 2013)

Outline of the talk

The 1D case: statement of the problem

We consider the initial-boundary value problem

$$\begin{aligned} &\rho_t - \left(\rho(1-\rho)\frac{\phi_x}{|\phi_x|}\right)_x = 0 \\ &|\phi_x| = c(\rho) \end{aligned} \qquad x \in \Omega = \,]-1,1[,\ t>0$$

with initial density $\rho(0,\cdot) = \rho_0 \in \mathrm{BV}(]0,1[)$
and *absorbing* boundary conditions

$$\begin{aligned} &\rho(t,-1) = \rho(t,1) = 0 \qquad \text{(weak sense)} \\ &\phi(t,-1) = \phi(t,1) = 0 \end{aligned}$$

The 1D case: statement of the problem

We consider the initial-boundary value problem

$$\begin{aligned} &\rho_t - \left(\rho(1-\rho)\frac{\phi_x}{|\phi_x|}\right)_x = 0 \\ &|\phi_x| = c(\rho) \end{aligned} \qquad x \in \Omega = \,]-1,1[,\ t > 0$$

with initial density $\rho(0,\cdot) = \rho_0 \in \mathrm{BV}(]0,1[)$
and *absorbing* boundary conditions

$$\begin{aligned} &\rho(t,-1) = \rho(t,1) = 0 \qquad \text{(weak sense)} \\ &\phi(t,-1) = \phi(t,1) = 0 \end{aligned}$$

General cost function c: $[0,1[\ \to [1,+\infty[$ smooth s.t. $c(0) = 1$ and $c'(\rho) \geq 0$
(e.g. $c(\rho) = 1/v(\rho)$)

The 1D case: statement of the problem

The problem can be rewritten as

$$\rho_t - \Big(\operatorname{sgn}(x - \xi(t))\ f(\rho)\Big)_x = 0$$

where the *turning point* is given by

$$\int_{-1}^{\xi(t)} c\,(\rho(t,y))\ dy = \int_{\xi(t)}^{1} c\,(\rho(t,y))\ dy$$

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$\longrightarrow$ the discontinuity point $\xi = \xi(t)$ is not fixed *a priori*, but depends *non-locally* on ρ

The 1D case: preliminary results

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- **wave-front tracking algorithm** and convergence of finite volume schemes
(Goatin-Mimault, 2013)

The 1D case: entropy condition

Definition: entropy weak solution (ElKhatib-Goatin-Rosini, 2012)

$\rho \in \mathbf{C^0}\left(\mathbb{R}^+; \mathbf{L^1}(\Omega)\right) \cap \mathrm{BV}\left(\mathbb{R}^+ \times \Omega; [0,1]\right)$ s.t. for all $k \in [0,1]$ and $\psi \in \mathbf{C_c^\infty}(\mathbb{R} \times \Omega; \mathbb{R}^+)$:

$$
\begin{aligned}
0 \leq & \int_0^{+\infty} \int_{-1}^{1} \left(|\rho - k| \psi_t + \Phi(t,x,\rho,k) \psi_x\right)\, dx\, dt + \int_{-1}^{1} |\rho_0(x) - k| \psi(0,x)\, dx \\
& + \operatorname{sgn}(k) \int_0^{+\infty} \left(f\left(\rho(t,1-)\right) - f(k)\right) \psi(t,1)\, dt \\
& + \operatorname{sgn}(k) \int_0^{+\infty} \left(f\left(\rho(t,-1+)\right) - f(k)\right) \psi(t,-1)\, dt \\
& + 2 \int_0^{+\infty} f(k) \psi\left(t, \xi(t)\right)\, dt.
\end{aligned}
$$

where $\Phi(t,x,\rho,k) = \operatorname{sgn}(\rho - k)\left(F(t,x,\rho) - F(t,x,k)\right)$

The 1D case: maximum principle

Proposition (ElKhatib-Goatin-Rosini, 2012)

Let $\rho \in \mathbf{C^0}\left(\mathbb{R}^+; \mathrm{BV}(\Omega) \cap \mathbf{L^1}(\Omega)\right)$ be an entropy weak solution. Then

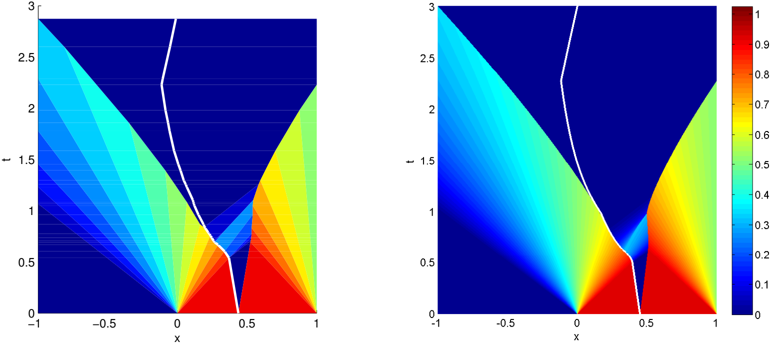
$$0 \leq \rho(t,x) \leq \|\rho_0\|_{\mathbf{L}^\infty(\Omega)}.$$

Characteristic speeds satisfy

$$\begin{aligned} f'\left(\rho^+(t)\right) &\leq \dot{\xi}(t), \text{ if } \rho^-(t) < \rho^+(t), \\ -f'\left(\rho^-(t)\right) &\geq \dot{\xi}(t), \text{ if } \rho^-(t) > \rho^+(t). \end{aligned}$$

The 1D case: wave-front tracking

Riemann-type initial data:

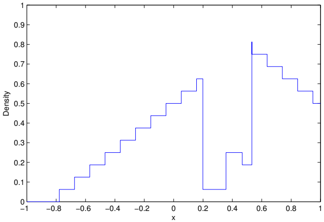


$\Delta\rho = 2^{-4}$ $\Delta\rho = 2^{-10}$

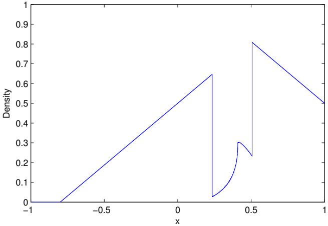
(Goatin-Mimault, 2013)

The 1D case: wave-front tracking

Density profile at $t = 0.8$:



$\Delta\rho = 2^{-4}$

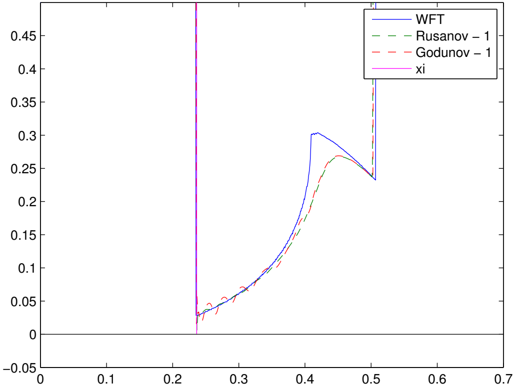


$\Delta\rho = 2^{-10}$

(Goatin-Mimault, 2013)

The 1D case: comparison WFT vs FV

Wave-front tracking with $\Delta\rho = 2^{-10}$ and finite volumes with $\Delta x = 1/1500$



(Goatin-Mimault, 2013)

Outline of the talk

Macroscopic models of pedestrians flows

PDEs describing the evolution of macroscopic quantities (e.g. density):

$$\partial_t u(t,\mathbf{x}) + \mathrm{div}_{\mathbf{x}} f(u(t,\mathbf{x})) = 0 \qquad t>0,\ \mathbf{x}\in I\!R^D,\ u\in I\!R^n$$

- based on the *continuum hypothesis*
- give global description of spatio-temporal evolution
- suitable for posing control and optimization problems

BUT :

- no general analytical theory for multi-D hyperbolic systems ($n>1$): existence and uniqueness in 1D?
- able to recover complexity features of crowd dynamics?
- good agreement with empirical data?

Thank you for your attention!