Quantifying uncertainties in seismic waves propagation with a Fourier Neural Operator surrogate model

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Abstract

Physics-based numerical simulations are key tools in earthquake engineering. They complement the existing datasets of recorded earthquakes by computing on-demand the ground motion generated by any realistic earthquake scenario. However, high-fidelity earthquake simulations are computationally demanding since they require solving the hyperbolic elastic wave equation in large three-dimensional (3D) domains and up to high frequencies.

In addition, simulation parameters are highly uncertain due to the difficulty of conducting geophysical experiments. Parameters of particular interest are i) the ground properties that control the waves velocity, ii) the position of the earthquake source, iii) the properties of the earthquake source (i.e. orientation and magnitude). Due to the cost of numerical simulations, repeated calls to the numerical solvers are unaffordable and efficient surrogate models are required to quantify uncertainties. Existing surrogate models based on e.g. Gaussian processes [1] or Polynomical Chaos Expansion [5] do not allow 3D applications with highly heterogeneous domains.

In this work, we propose a surrogate model of seismic waves propagation using a Factorized Fourier Neural Operator (F-FNO [6]), a deep learning method tailored to Partial Differential Equations (PDEs). The F-FNO views integral operators as convolutional kernels of learnable weights and writes the convolution as a product of Fourier coefficients. This leads to efficient neural operators that have strong relationships with physical equations.

Our training database is built from our HEMEW-3D database of 30,000 High-Performance Computing (HPC) simulations [2]. For each simulation, a heterogeneous propagation domain is designed with 3D random fields that represent variations of the rock properties inside the ground (Fig. 1, left). The source position and orientation are also randomly chosen for each simulation. Then, seismic waves are propagated from the source up to the surface where they are recorded by a grid of virtual sensors. Therefore, for each set of input parameters (ground properties, source position, source orientation), the F-FNO learns to predict the time-dependent surface wavefields (Fig. 1).

The F-FNO is compared to baseline models and we show that it is an efficient surrogate model whose accuracy improves when the network complexity increases [3]. Prediction errors are also quantified in the frequency domain, which indicates that the well-known spectral bias hinders

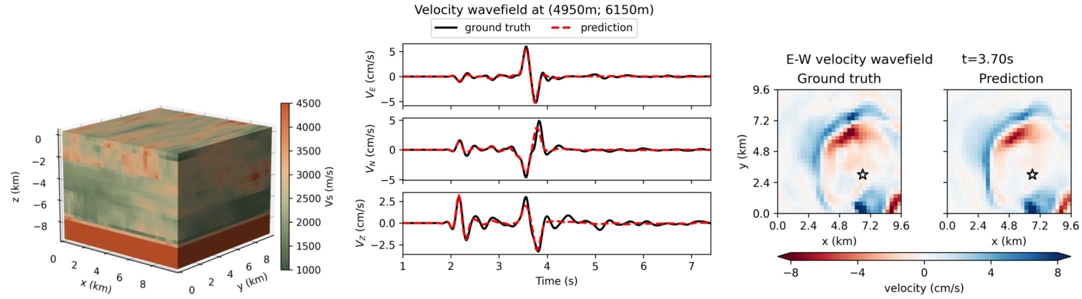

Figure 1: For a heterogeneous geology (left), timeseries of surface ground motion (middle) and the surface wavefields at $t = 3.7$s (right). F-FNO predictions (dashed red lines, middle) are compared with the outputs of the numerical simulations (black lines, middle).

high-frequency accuracy. In addition, prediction errors are well explained by properties of the inputs, thereby giving insights on the expected accuracy before making the prediction.

To quantify the influence of geological uncertainties on surface wavefields, transfer learning was applied to a small dataset of real geologies in Southeastern France [4]. Thanks to the (almost) negligible cost of the F-FNO evaluation, once duly trained, we obtained meaningful confidence intervals that are of great significance for the earthquake engineering community.

Short biography (PhD student)

I graduated from the mathematics department of the ENS Paris-Saclay in 2021. I am doing my PhD at CEA DAM, which partners with academic institutions like Centrale Supélec to perform seismic hazard analyses. This PhD builds on several years of development of the HPC code SEM3D to design a meta-model enabling fast and accurate predictions of seismic ground motion. The meta-model will then be used to investigate site effects for seismic hazard analyses.

References

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