# Sensitivity analysis of set-valued models

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## Abstract

Sensitivity analysis (SA) plays a central role in mathematical modeling, providing valuable insight into how input parameters variations affect a model output. This tool allows practitioners to evaluate the robustness and reliability of their models by quantifying the response to changes in key variables. Historically, SA methods have been tailored for scalar-valued models. However, some physical problems involve more complex outputs, such as set-valued models, where each model evaluation represents a subspace within a larger space $\mathcal{X}$. Such models occur in several domains, for example, when the output of interest is the geographic area where a measured pollutant concentration exceeds a specified threshold (Figure 1). It becomes critical to identify which input parameters (e.g., traffic or environmental factors) significantly affect the polluted area. Traditional SA methods are not directly applicable or adaptable to set-valued models, posing a challenge in quantifying uncertainties in the set-valued response.

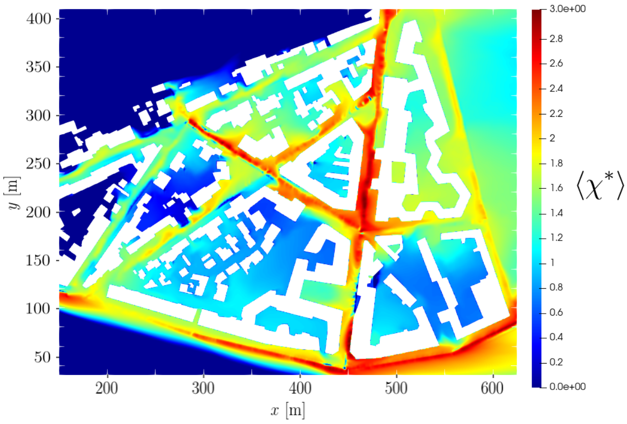


Figure 1: Concentration maps of pollutant dispersion at the urban scale

To fill this gap, we propose three different approaches for defining sensitivity indices when dealing with set-valued models. Our first two are Sobol-like indices $S_i^V$ based on random set tools [3] and universal indices $S_i^{Univ}$ [2] adapted to set outputs. The third approach uses kernel-based sensitivity indices, specifically HSIC and HSIC-ANOVA [1], with sets. This requires the introduction of a kernel, denoted $k_{set}$, defined on sets, which we thoroughly investigate. In

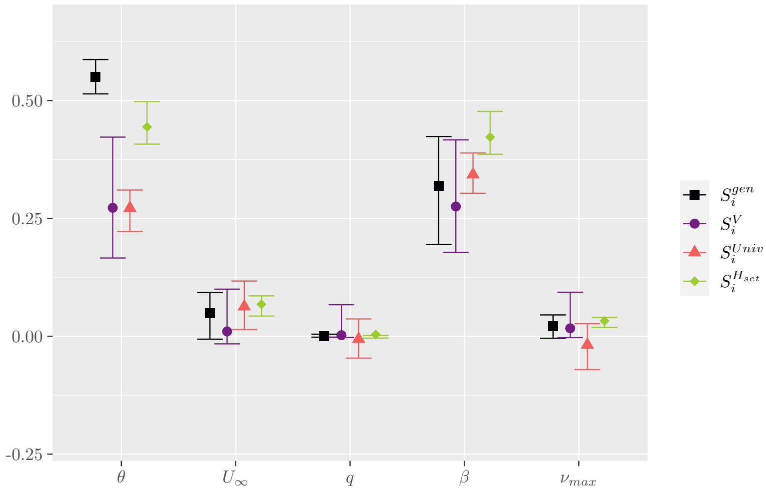


Figure 2: Comparison of the four indices on the set-valued model of pollutant concentration maps. 1000 model evaluations are used to estimate indices and confidence intervals

particular, we demonstrate its *characteristic* property, based on a result of [4], an essential property within the HSIC framework. $k_{set}$ is then used to estimate HSIC with set-valued outputs, but this estimation is complicated by the presence of sets, which forces $k_{set}$ to be estimated as well. Therefore, we propose and study a nested Monte Carlo estimator of the indices.

In Figure 2, our three new indices $S_i^V$, $S_i^{Univ}$, and $S_i^{H_{set}}$ are compared to the generalized point by point Sobol index $S_i^{gen}$ on the pollutant concentration maps. The SA results are similar, but $S_i^{H_{set}}$ stands out as it has less variability than the other three and has the strength to be used for both screening and ranking.

Another ongoing application is the use of these indices in chance-constrained problems and robust optimization by quantifying the impact of uncertain inputs on excursion sets. Knowing which uncertain inputs affect the optimization could allow simplifying steps within Bayesian optimization.

## Short biography

I am in the third year of my PhD, which is taking place at the Ecole Centrale de Lyon and IFP Energies Nouvelles. My work is about using sensitivity analysis methods to simplify robust optimization problems. I'm supervised by Céline Helbert, Christophette Blanchet from ECL and Adrien Spagnol, Delphine Sinoquet from IFPEN. My thesis is part of the CIROQUO consortium.

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