SENSE: Abstraction-Based Synthesis of Networked Control Systems

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NCS: Motivation

Networked control systems (NCS):
- Remote access/control.
- Flexibility in deployment.
- Easy maintenance/upgrading.
- Reduced cost and complexity.

Wide range of applications:
- Factory automation.
- Space exploration.
- Remote diagnosis and troubleshooting.
- Aircrafts and automobiles.


NCS: a Complex Cyber-Physical System

NCS combine:
- Physical environment: a plant, sensors and actuators.
- Communication networks: transfer state information and control actions.

Imperfections of communication networks:
- Time-varying sampling/transmission intervals.
- Time-varying communication delays
- Limited bandwidth.
- Quantization errors.
- Packet dropouts.

Control of NCS:
- Traditionally: classical control, existing methods are limited to stabilizability problem.
- Recently: formal methods, automated synthesis, correct-by-construction, developing research area.
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Control of NCS:
- Traditionally: classical control, existing methods are limited to stabilizability problem.
- Recently: formal methods, automated synthesis, correct-by-construction, developing research area.
- **SENSE**: a framework for symbolic control of NCS
SENSE: Modeling the NCS $\tilde{\Sigma}$

- A control system (plant): $\Sigma = (\mathbb{R}^n, U, \mathcal{U}, f)$.
- A sensor/sampler (time-driven): non-varying sampling-time: $\tau$.
- A zero-order-hold (ZOH).
- Two communication channels:
  - Delays as integer multiples of $\tau$: $\Delta_{sc}^k := N_{sc}^k \tau$ and $\Delta_{ca}^k := N_{ca}^k \tau$.
  - Delays are bounded:
    $N_{sc}^k \in [N_{sc}^{min}, N_{sc}^{max}] \subset \mathbb{N}_+$
    $N_{ca}^k \in [N_{ca}^{min}, N_{ca}^{max}] \subset \mathbb{N}_+$.

Considered Imperfections:
- Bounded delays.
- Packet dropouts (emulation).
- Limited bandwidth.
- Quantization errors.

Next
- What is symbolic control?
- How to apply it to NCS?
Symbolic Control: Abstraction-based Synthesis

Abstraction\(^1\):
- **Plants**: physical systems (e.g. differential equations).
- **Symbolic models**: finite state/input systems representing plants up to some predefined accuracy.

Specifications:
- Linear temporal logic (LTL) formulae; or
- Automata on infinite strings.

\[^1\text{P. Tabuada, Verification and control of hybrid systems. New York, Springer, 2009.}\]
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Controller Synthesis:

- Automated synthesis: Fixed-point or graph-search algorithms.
- Complex specifications can be handled.
- Symbolic controllers are refined with suitable interfaces.

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Symbolic Control of NCS: Plants as Systems

Definition 1: Systems
A system $S$ is a tuple $S = (X, X_0, U, \rightarrow)$ consisting of:
- a (possibly infinite) set of states $X$.
- a (possibly infinite) set of initial states $X_0 \subseteq X$.
- a (possibly infinite) set of inputs $U$.
- a transition relation $\rightarrow \subseteq X \times U \times X$.

Describing the plant as a system:
Given a sampling period $\tau$,

$$\Sigma : \dot{\xi}(t) = f(\xi(t), v(t))$$

is described by the system:

$$S_{\tau}(\Sigma) = (X_{\tau}, X_{\tau,0}, U_{\tau}, \rightarrow_{\tau})$$

which encapsulates all the information contained in the control system $\Sigma$ at sampling times $k\tau$, $k \in \mathbb{N}_0$, where:

- $X_{\tau} = \mathbb{R}^n$, $X_{\tau,0} = \mathbb{R}^n$.
- $U_{\tau} = \{v : \mathbb{R}^+_0 \rightarrow U | v(t) = v((s - 1)\tau), t \in [(s - 1)\tau, s\tau[, s \in \mathbb{N}\}$. 
- $x_{\tau} \xrightarrow{v_{\tau}} x'_{\tau} \iff \exists \xi_{x_{\tau},v_{\tau}} : [0, \tau] \rightarrow \mathbb{R}^n$ in the system $\Sigma$ s.t. $\xi_{x_{\tau},v_{\tau}}(\tau) = x'_{\tau}$. 
Symbolic Control of NCS: Symbolic Models of Plants

- Used relation: feedback refinement relation\(^2\) (denoted by \(Q\)).
- \(S_q(\Sigma) = (X_q, X_{\tau}, U_{\tau}, \rightarrow_q)\)
- \(X_q\) is a finite cover/partition over \(X_{\tau}\)
- Advantage: only a static quantization map to refine the synthesized controller.

SENSE: Symbolic Models of NCS

- The $\mathcal{L}$-map is introduced$^3$:

$$\mathcal{L} : \mathcal{T}(U, X) \times \mathbb{N}^4 \to \mathcal{T}(U, \tilde{X}).$$

$$S_*(\tilde{\Sigma}) = (\tilde{X}, \tilde{X}_0, U_r, \longrightarrow^*)$$

$$= \mathcal{L}(S_q(\Sigma), N^{sc}_{\min}, N^{sc}_{\max}, N^{ca}_{\min}, N^{ca}_{\max}),$$

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  \[
  \mathcal{L}(S_q(\Sigma), N_{sc min}^{sc}, N_{max}^{sc}, N_{min}^{ca}, N_{max}^{ca}),
  \]
  where
  - $\tilde{X} = \{X_q \cup q\} N_{max}^{sc} \times U_r N_{max}^{ca} \times [N_{min}^{sc}; N_{max}^{sc}] N_{max}^{sc} \times [N_{min}^{ca}; N_{max}^{ca}] N_{max}^{ca}$, where $q$ is a dummy symbol.
  - $\tilde{X}_0$ and $\rightarrow$ are depicted visually.

---

Remark 1
The map $\mathcal{L}$ provides a systematic technique of constructing the NCS from its plant and the delays.

Theorem 1
Consider an NCS $\tilde{\Sigma}$ and suppose there exists a simple finite system $S_q(\Sigma)$ such that $S_{\tau}(\Sigma) \preceq_Q S_q(\Sigma)$. Then we have $S(\tilde{\Sigma}) \preceq_{\tilde{Q}} S_{\tau}(\tilde{\Sigma})$, for some feedback refinement relation $\tilde{Q}$, where $S_{\tau}(\tilde{\Sigma}) := \mathcal{L}(S_q(\Sigma), N_{\text{sc}}^{\text{min}}, N_{\text{sc}}^{\text{max}}, N_{\text{ca}}^{\text{min}}, N_{\text{ca}}^{\text{max}})$ is finite.
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SENSE: Efficient Symbolic Abstractions of NCS

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Remark 2
a- **Normal methodology**: one should first derive an infinite system $S(\tilde{\Sigma})$ that captures all NCS information and then, construct the symbolic model $S_*(\tilde{\Sigma})$ from it.

b- **Efficient methodology**: systematically construct the symbolic abstraction $S_*(\tilde{\Sigma})$ directly from the symbolic model $S_q(\Sigma)$.

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SENSE: Abstraction Construction Engine

- BDD-based abstraction construction:
  - $S_q(\Sigma)$ is given as a BDD-object.
  - The library SCOTS can be used to generate $S_q(\Sigma)$.
  - Apply $\mathcal{L}$ to $S_q(\Sigma)$ via BDD operations.
  - NCS settings guides the engine about the class of NCS.
  - Fully customizable: different classes of NCS.

- Efficient implementation of the map $\mathcal{L}$
  - BDD bit-list of $S_q(\Sigma)$ gets more bits for: $\{X_q \cup q\}^{N_{sc\max}}$, $\{U^{N_{ca\max}}, [N_{sc\min}^{N_{sc\max}}]^{N_{sc\max}}$ and $[N_{ca\min}^{N_{ca\max}}]^{N_{ca\max}}$.
  - Transition relation of $S_*(\tilde{\Sigma})$: BDD operations on expanded $S_q(\Sigma)$. 
SENSE: Controller Synthesis and Refinement

- Synthesis of symbolic controllers $C_*$:
  - BDD-based fixed-point operations on $S_*(\tilde{\Sigma})$.  
  - Supports safety, reachability, persistence and recurrence specifications.
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- **Solution:** delay prolongation:
  - All packets suffer same delay.
  - Realizable via buffers.
  - Results in simpler map $\mathcal{L}$.
  - Less conservative for existence of controllers.
  - Controllers refined with state reconstructors.
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**Remark 3**

SENSE constructs abstractions of any class of NCS. However, only to resolve the refinement issue, prolonged-delay NCS need to be considered.
SENSE: Simulation, Analysis and Code Generation

Interfaces to simulate the closed-loop:

- MATLAB: closed-loop simulation.
- OMNet++: realistic NCS simulation and visualization.

Supporting tools:

- bdd2implement: automatically generate C/C++ or VHDL/Verilog for controllers.
- bdd2fsm: generate files following the FSM data format to be used for visualization.
- bddDump: extract the meta-data information stored inside BDD files.
- contCoverage: fast terminal-based ASCII-art visualization.
- sysExplorer: testing the expanded transition relation.
SENSE: Work-Flow Summary

- Compute $S_q(\Sigma)$ as a BDD using SCOTS.
- Pass the NCS delays and BDD file of $S_q(\Sigma)$ to SENSE.
- SENSE takes the specifications as input.
- SENSE generates BDD files for $S_*(\tilde{\Sigma})$ and $C_*$. 
- Use the interfaces for MATLAB and/or OMNet++ to simulate/visualize the closed-loop NCS.
- Use bdd2implement to generate C/C++ or VHDL/Verilog source codes for implementations of final controllers.
- Use the interfaces for MATLAB and/or OMNet++ to simulate/visualize the closed-loop NCS.
- Use bdd2implement to generate the final controller for implementation.
SENSE: Test Drive

- Available download/manual in: https://www.hcs.ei.tum.de

- Requires:
  - C/C++ Compiler.
  - CUDD Library.
  - MATLAB or OMNet++ (optional).

- Install and usage: Video!
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**Example:**
Robot in 2D arena:

\[
\begin{bmatrix}
\dot{\xi}_1 \\
\dot{\xi}_2
\end{bmatrix} =
\begin{bmatrix}
v_1 \\
v_2
\end{bmatrix}
\]

\((\xi_1, \xi_2) \in X = [0, 64] \times [0, 64]\) quantized by \((1, 1)\).

\((v_1, v_2) \in U = [-1, 1] \times [-1, 1]\) quantized by \((1, 1)\).

**Specifications:**

\[
\psi = \left( \bigwedge_{i=1}^9 \square (\neg \text{Obstacle}_i) \right) \land \ Diamond \text{ (Target1)} \land \ Diamond \text{ (Target2)},
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Results:

\(S_*(\bar{\Sigma})\): in 0.49 seconds ( 15 KB).
\(C_*\): in 8 seconds ( 11 KB).
Visualized by OMNet++.
Video: Installation and Usage of SENSE.
Thanks! Any Questions?

**Table**: Results for constructing symbolic models of prolonged-delay NCS using those of their plants.

| Case Study     | $|S_q(\Sigma)|$ | (2,2) | (2,3) | (2,4) | (2,5) | (3,2) | (3,3) | (3,4) | (3,5) | (4,2) | (4,3) |
|----------------|-----------------|------|------|------|------|------|------|------|------|------|------|
| Double         | 2039            | 14096| 56336| 225296| 901136| 22832| 91184| 364592| 1.45×10^6| 38340| 152964|
| Time (sec)     | $< 1$           | $< 1$| $< 1$| $< 1$| $< 1$| $< 1$| $< 1$| $< 1$| $< 1$| $< 1$| $< 1$|
| Memory (KB)    | 2.0             | 2.4  | 3.1  | 2.9  | 3.0  | 2.9  | 3.1  | 3.2  | 5.2  | 5.2  | 4.3  |
| Robot          | 29280           | 4.1×10^6| 6.5×10^7| 1.04×10^10| 1.67×10^10| 3.4×10^7| 5.4×10^8| 8.7×10^9| 1.4×10^11| 2.8×10^8| 4.6×10^9|
| Time (sec)     | $< 1$           | $< 1$| $< 1$| $< 1$| $< 1$| $< 1$| $< 1$| $< 1$| $< 1$| $< 1$| 1.4 |
| Memory (KB)    | 15              | 14   | 17   | 16   | 16   | 21   | 22   | 19   | 35   | 33   |      |
| Jet Engine     | 9.0×10^5        | 1.5×10^11| 1.0×10^13| 6.5×10^14| 4.2×10^16| 7.2×10^12| 4.6×10^14| 2.9×10^16| 1.9×10^19| 3.3×10^14| 2.1×10^16|
| Time (sec)     | 1970            | 1637 | 1674 | 2172 | 3408 | 1772 | 2111 | 7107 | 4011 | 2854 |      |
| Memory (KB)    | 4323.3          | 2374.2| 2389.1| 2392.6| 3683.2| 4098.2| 3317.2| 3582.6| 5894.3| 4784.5|      |
| DC-DC Converter| 3.8×10^6        | 8.9×10^7| 1.8×10^8| 3.6×10^8| 7.1×10^8| 5.2×10^8| 1.0×10^9| 2.0×10^9| 4.1×10^9| 3.0×10^9| 6.0×10^9|
| Time (sec)     | 672             | 681  | 530  | 1131 | 13690| 10114| 9791 | 10084| 139693| 137648|      |
| Memory (KB)    | 3347.2          | 3145.0| 3176.7| 2784.8| 10875.2| 11543.8| 11572.0| 34169.2| 5894.3| 4784.5|      |
| Vehicle        | 1.9×10^7        | 4.2×10^12| 2.7×10^14| 1.7×10^16| 1.1×10^18| 2.3×10^14| 1.4×10^16| 9.3×10^17| 5.9×10^19| 1.3×10^16| 7.94×10^17|
| Time (sec)     | 273.3           | 285  | 238.4 | 173  | 22344| 54919| 27667| 36467| 39065| 145390|      |
| Memory (KB)    | 1638.4          | 1945.6| 1843.2| 1945.6| 23040| 40652.8| 30208| 22425.6| 21094.4| 36556.3|      |
| Inverted Pendulum| 4.3×10^8       | 2.4×10^14| 1.5×10^16| 1.0×10^18| 6.5×10^19| 4.6×10^16| 2.9×10^18| 1.9×10^20| 1.2×10^22| 9.2×10^16| 5.8×10^20|
| Time (sec)     | 361.4           | 349.1| 340.9 | 347.8 | 942  | 793  | 1218 | 910  | 58411| 57110 |      |
| Memory (KB)    | 723.1           | 808.4| 349.6 | 1012.7| 2958 | 2840 | 3104 | 2898 | 35907| 35628 |      |
Appendix 1: More detailed model

- A control system (plant): $\Sigma = (\mathbb{R}^n, U, \mathcal{U}, f)$.
- A sensor/sampler (time-driven):
  - Non-varying sampling-time: $\tau$.
  - $x_k := \xi(s_k)$.
  - Triggered: $s_k := k\tau$, $k \in \mathbb{N}_0$.
- A zero-order-hold (ZOH).
- Two communication channels:
  - Delays are integer multiples of the sampling time: $\Delta_k^{sc} := N_k^{sc}\tau$ and $\Delta_k^{ca} := N_k^{ca}\tau$.
  - Delays are bounded: $N_k^{sc} \in [N_{\text{min}}^{sc}; N_{\text{max}}^{sc}]$, $N_k^{ca} \in [N_{\text{min}}^{ca}; N_{\text{max}}^{ca}]$.
  - Packet dropouts: emulation (i.e. increasing the delays), assuming subsequent dropouts are bounded.
- Packet rejection:
  - $v(t) = u_{k+j^*} - N_{\text{max}}^{ca}$.
  - $j^*$: time-index shifting used to determine the correct control input by taking care of message rejection due to out of order packet arrival: two packets arrive at the same time, reject the older one.
  - Applied to both channels.
Appendix 2: Controller Synthesis and Refinement

- Fixed-point operations on $X \times U$.
- Possible LTL: $\Box \varphi_S$, $\Diamond \varphi_T$, $\Diamond \Box \varphi_S$, $\Box \Diamond \varphi_T$.
- A Pre-map is a base for all four algorithms:

$$\text{pre}(Z) = \{(x, u) \in X_q \times U_q | \emptyset \neq F_q(x, u) \subseteq \pi_{X_q}(Z)\}$$

$$\pi_{X_q}(Z) = \{x \in X_q | \exists u \in U_q (x, u) \in Z\}$$

SCOTS$^5$:

- Symbolic Models: via FRR and encoded in BDD.
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