Mitigating Spectral Bias in Physics-Informed Neural Networks for Transport-Dominated Problems

Bikky Kumar*, Manoranjan Mishra

Department of Mathematics, Indian Institute of Technology Ropar, Rupnagar, Punjab, 140001 India
Email: bikky.23maz0009@iitrpr.ac.in, manoranjan@iitrpr.ac.in

Physics-informed neural networks (PINNs) have emerged as an effective technique for solving partial differential equations (PDEs) in a wide range of domains. However, standard PINNs fail on transport-dominated problems with discontinuous initial conditions due to spectral bias[1]. One cause is that neural networks have a tendency to learn low-frequency solution components, making it difficult to capture sharp interfaces. Another possible reason is saturated activation functions caused by large input coordinate values; input coordinates spanning broad numerical ranges lead to ill-conditioning that exacerbates spectral bias and causes progressive loss growth as training proceeds. To mitigate spectral bias and enable stable long-time simulation, we normalize the entire spatiotemporal domain to a unit domain using min-max normalization[2]. This transforms the PDE and its coefficients to a consistent scale, preventing the ill-conditioning that exacerbates spectral bias at sharp interfaces. The PINN is trained on the unit domain, and the predicted solution is rescaled to recover the original physical quantities.

Through this approach, we systematically investigate PINN performance on a hierarchy of transport equations: diffusion, convection–diffusion, Burgers' equation, and convection–diffusion with solution-dependent [3]. Results are validated against analytical solutions where available; for nonlinear cases without closed-form solutions, high-fidelity reference data are generated using Comsol Multiphysics. Numerical results demonstrate that the proposed approach have reduction of Relative L^2 error by one to two orders of magnitude compared to standard PINNs. These findings indicate that scaling and smooth initialization of initial data act as effective preconditioning mechanisms for PINNs, extending their applicability to challenging nonlinear transport problems . Our studies establishes a computational foundation for extending physics-informed neural networks to the study of various kinds of hydrodynamic instabilities, for instance, Kelvin-Helmholtz instability, Rayleigh-Taylor instability and Saffman-Taylor [4].

References

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