Analysis and improvement of the convergence of physics-informed neural networks

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Abstract

Partial Differential Equations (PDEs) are traditionally solved with numerical methods requiring grids and ad-hoc discretization schemes. Physics-Informed Neural Networks (PINNs) have recently emerged as a flexible alternative, relying on feed-forward networks trained by minimizing a loss that combines PDE residuals, boundary conditions and data. PINNs can address forward, inverse or parametric problems and integrate measurements, but often face convergence issues. This work aims at analysing and improving their convergence, from a multi-criterion optimization viewpoint.

A benchmark involving a stationary flow in a differentially heated cavity, governed by incompressible Navier–Stokes equations with thermal transport, shows that PINNs converge slowly yet preserve physical consistency, unlike pure data-fitting. The slow convergence is linked to an ill-conditioned Hessian, suggesting the need for second-order optimizers. Moreover, antagonism between PDE residual losses and boundary/data terms contributes to this ill-conditioning.

To address these difficulties, we study directional conflicts between the derivatives with respect to network parameters for each loss term, enabling clustering and the design of multi-criterion learning strategies.