



Physics Informed Neural Networks for Maxwell Equations

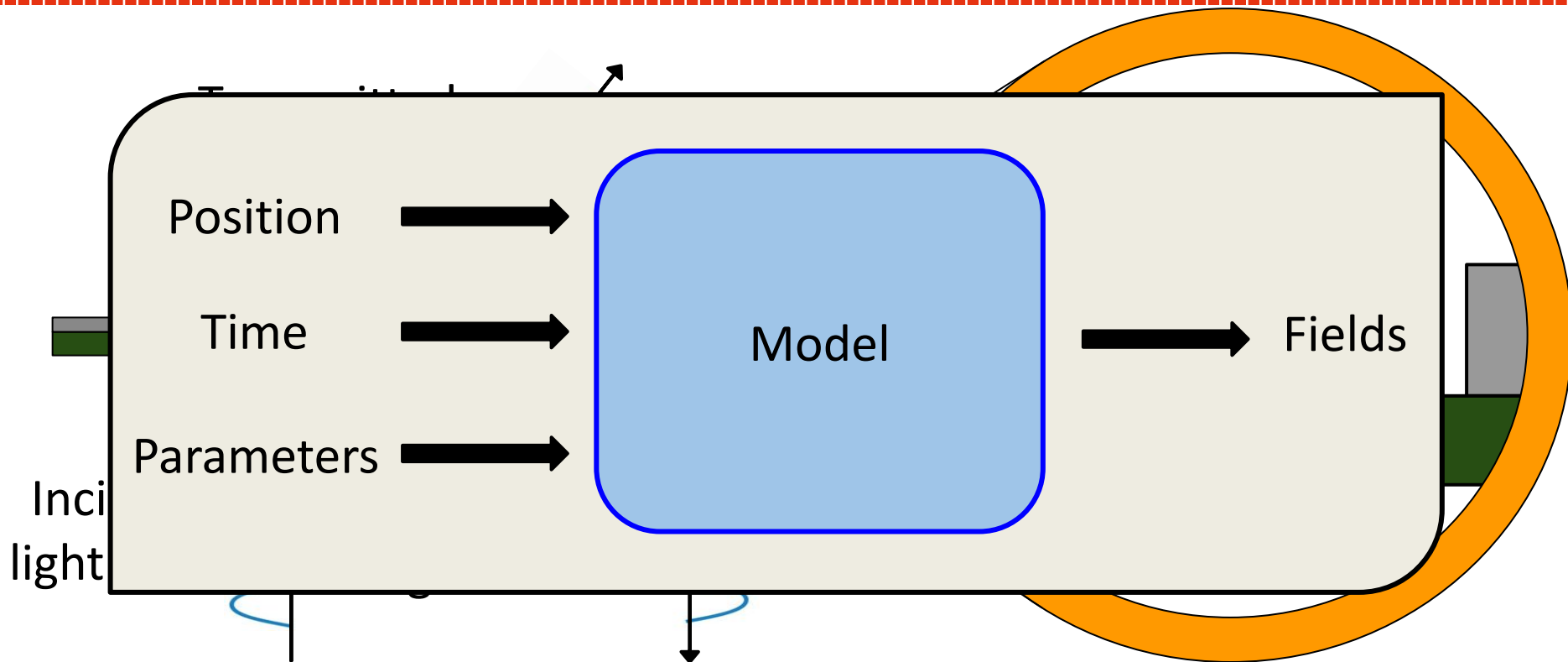
Workshop SciML

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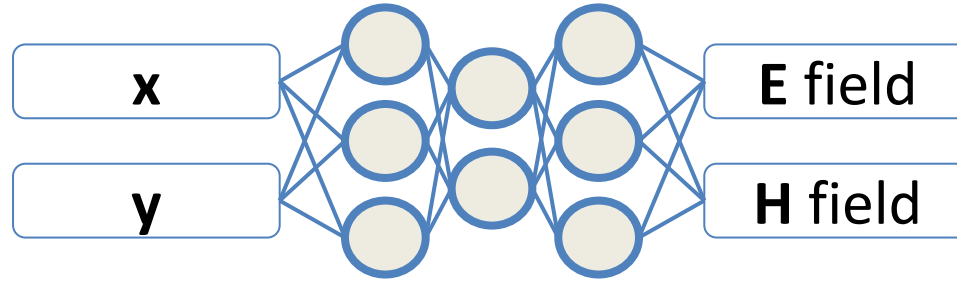
Atlantis project team
Centre Inria d'Université Côte d'Azur

15th October 2026

Context



Physics Informed Neural Networks



Problem: We have no or insufficient **training data**, e.g. results of simulations using classical methods

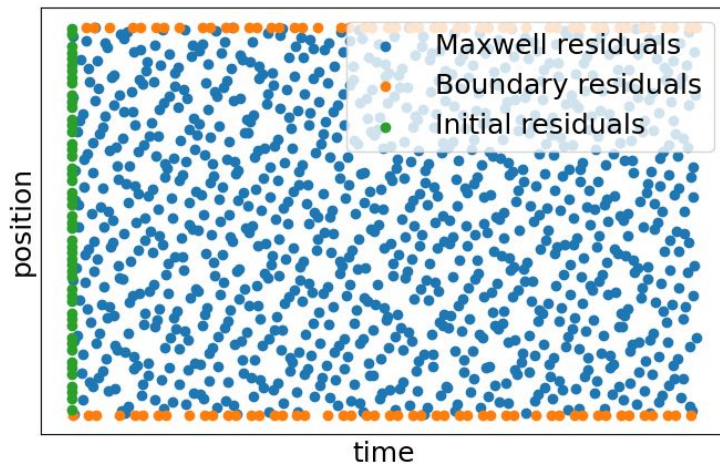
Idea: Using the **squared residuals** of the equations as loss functions

Sampling

Using Monte-Carlo principle:

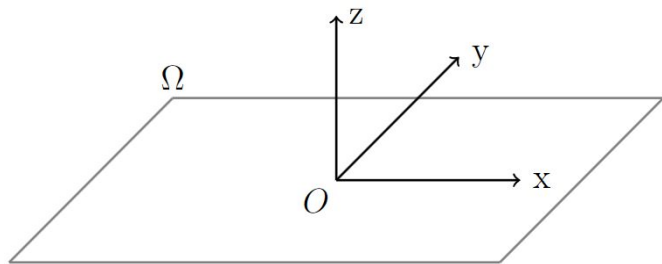
$$\mathcal{D}(u)(x) = f(x) \text{ for } x \in \Omega \iff \frac{1}{N} \sum_{i=1}^N [\mathcal{D}(u)(x_i) - f(x_i)]^2 = 0$$

- Maxwell equations
- Bound conditions
- Initial conditions



Our Framework

- The problem is **invariant** alongside the axis (Oz).

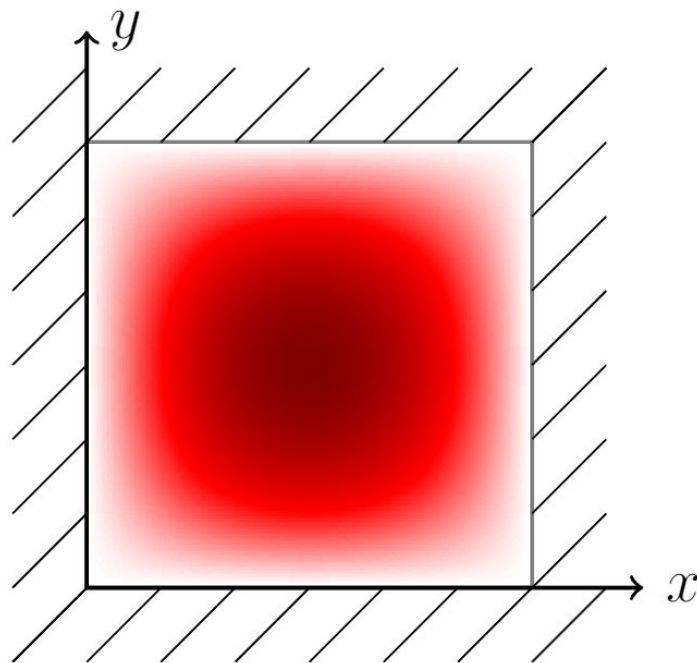


- Two decoupled systems
 - **Transverse magnetic (TM)**
 - **Transverse electric (TE)**

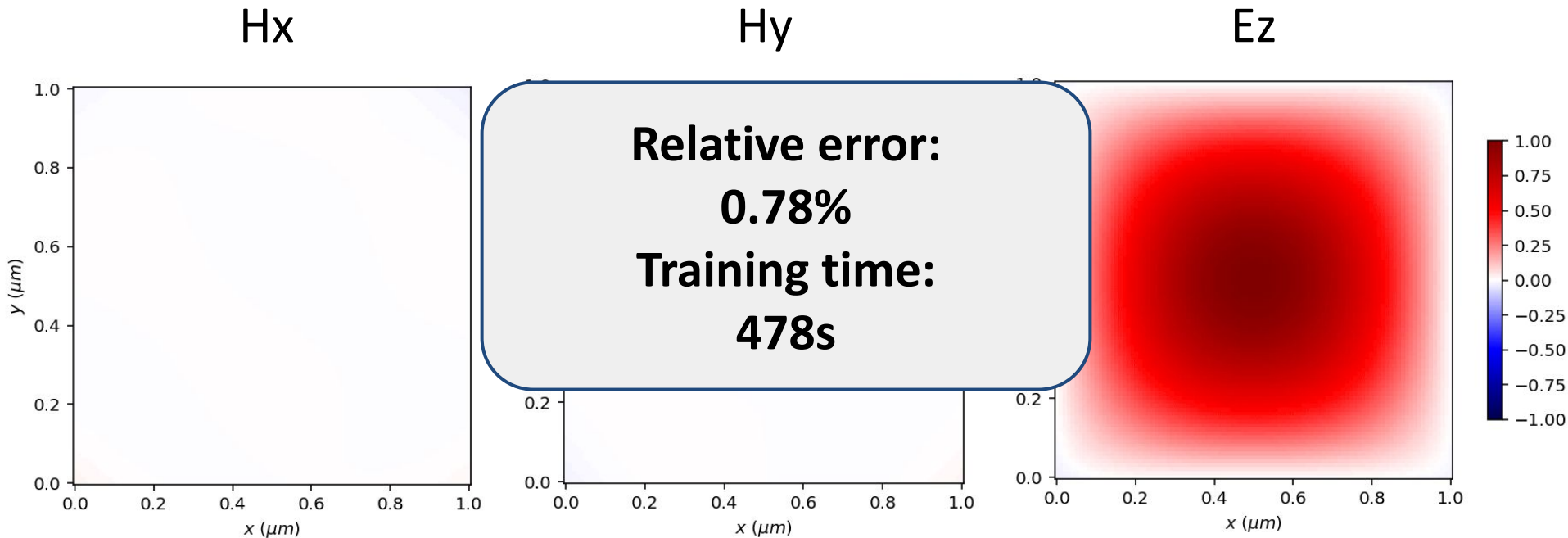
$$\begin{cases} \epsilon \frac{\partial E_z}{\partial t} - \frac{\partial H_y}{\partial x} + \frac{\partial H_x}{\partial y} = 0 \\ \mu \frac{\partial H_x}{\partial t} + \frac{\partial E_z}{\partial y} = 0 \\ \mu \frac{\partial H_y}{\partial t} - \frac{\partial E_z}{\partial x} = 0 \end{cases}$$

Partial differential equations for the TM mode

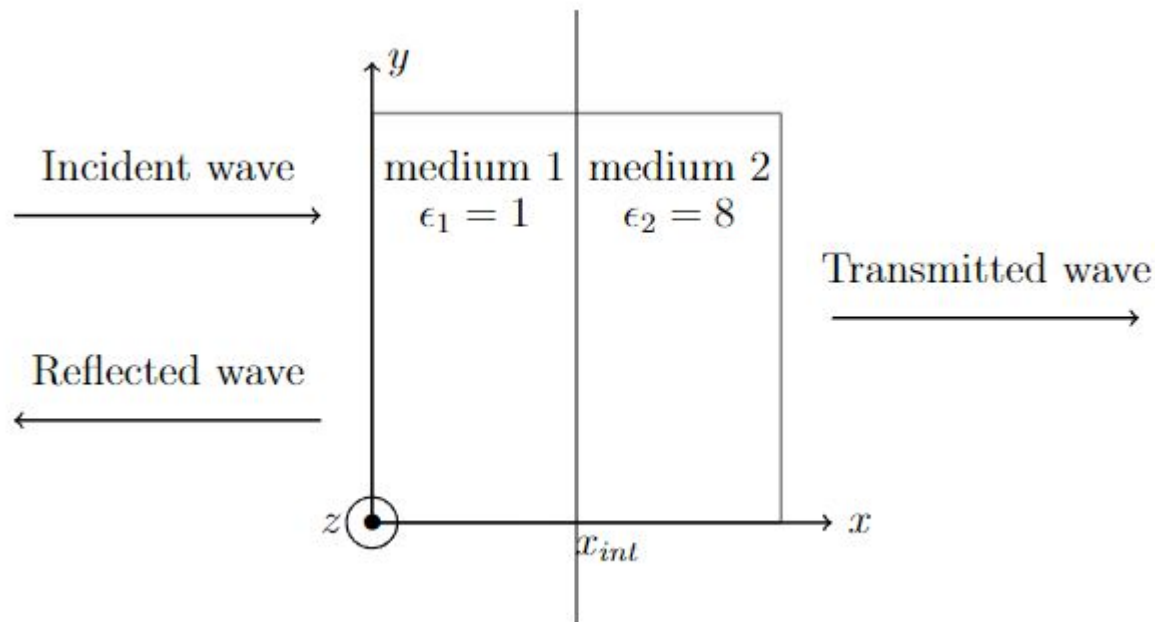
The Resonant Cavity



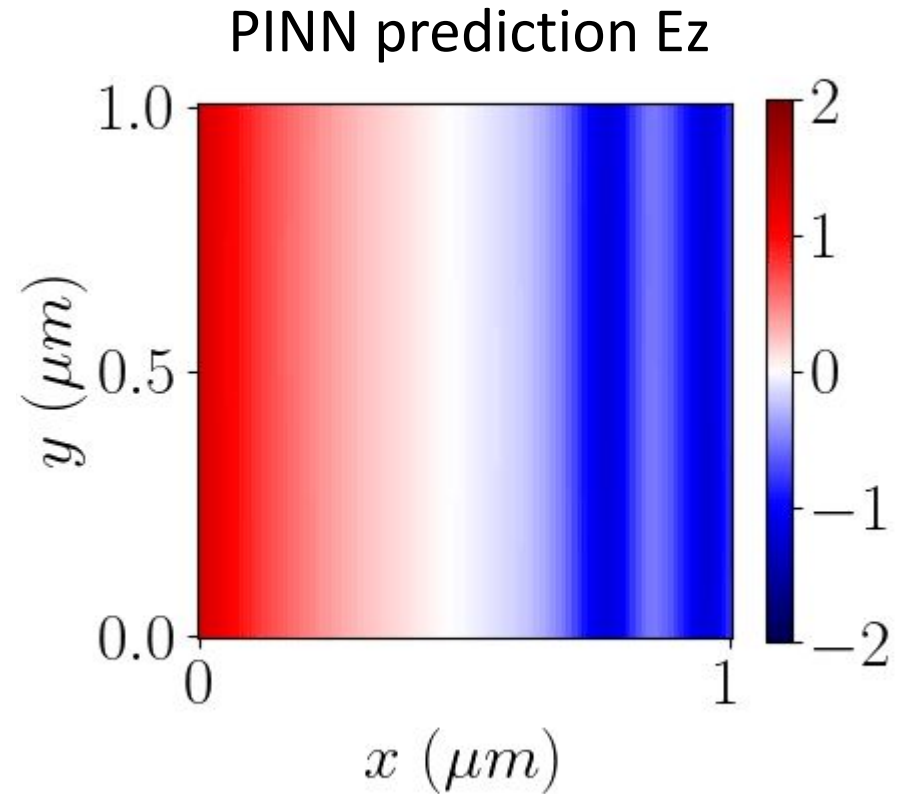
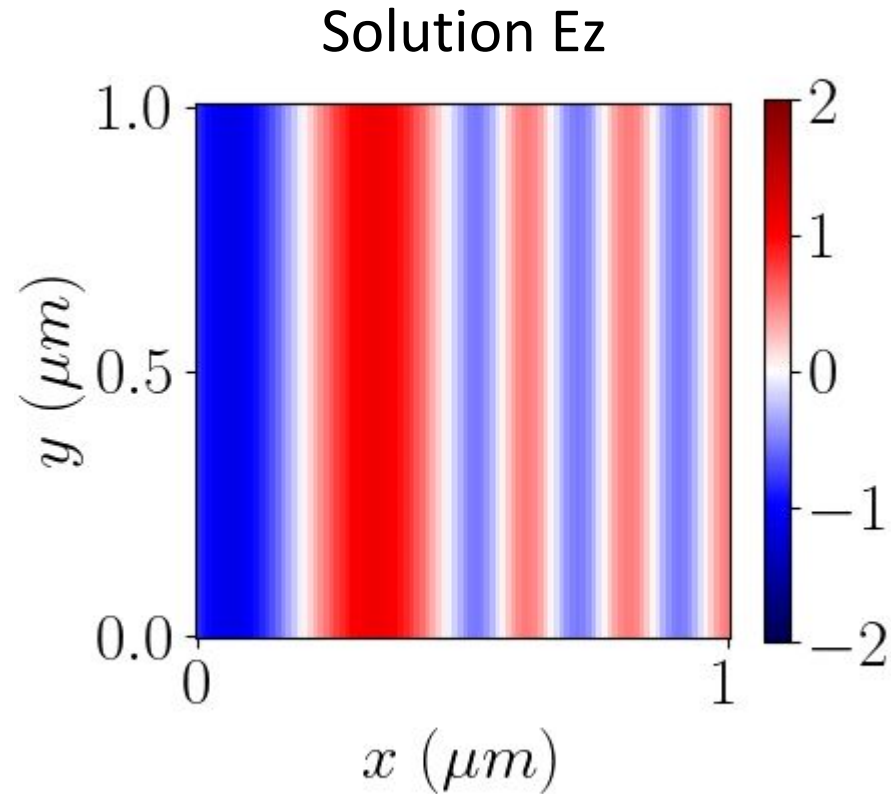
Results On The Resonant Cavity



The Flat Interface Between Two Media



Results On The Flat Interface



Can we help our model in any way?

Techniques: Fourier embeddings

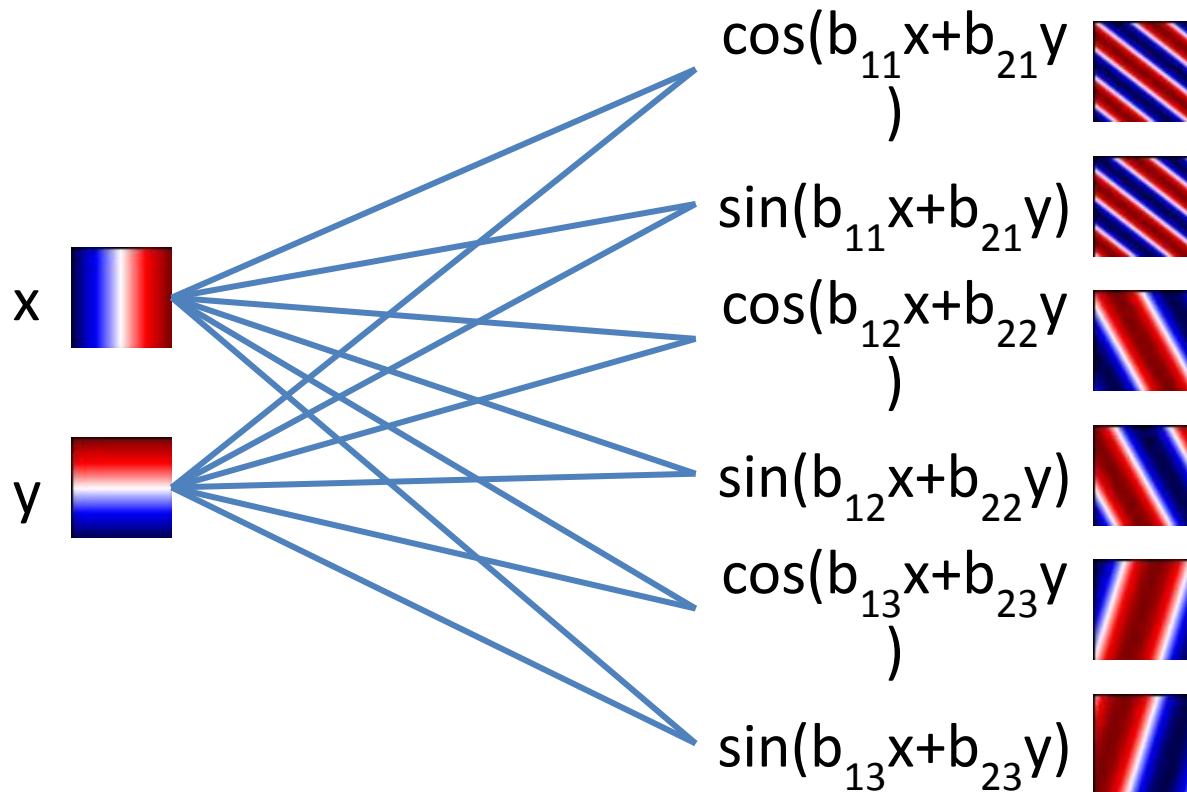
- Problem: **High frequencies** are difficult to learn.
- Solution: Encourage learning random **Fourier modes**.

We replace the inputs by:

$$\gamma(\mathbf{X}) = \begin{bmatrix} \cos(B\mathbf{X}) \\ \sin(B\mathbf{X}) \end{bmatrix}$$

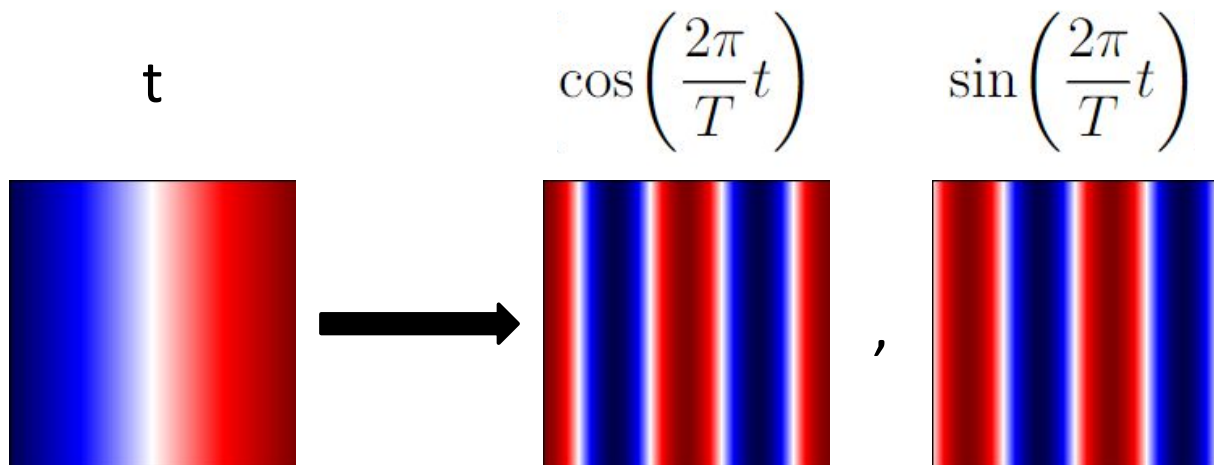
1. $\mathbf{X}$: input vector
2. B : random matrix

Techniques: Fourier embeddings



Techniques: Imposing Periodicity

- Problem: We want to make the solution **periodic**.
- Solution: Only use the **expected frequency** as input.

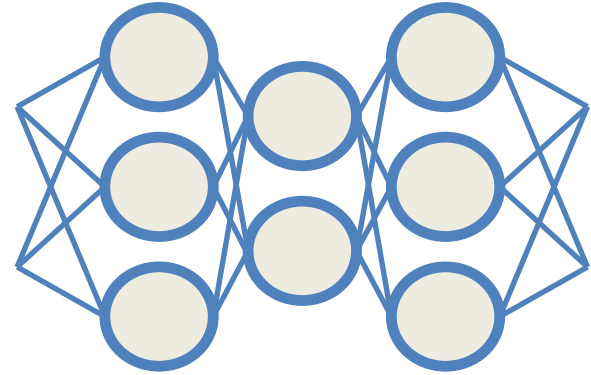


Example: New inputs

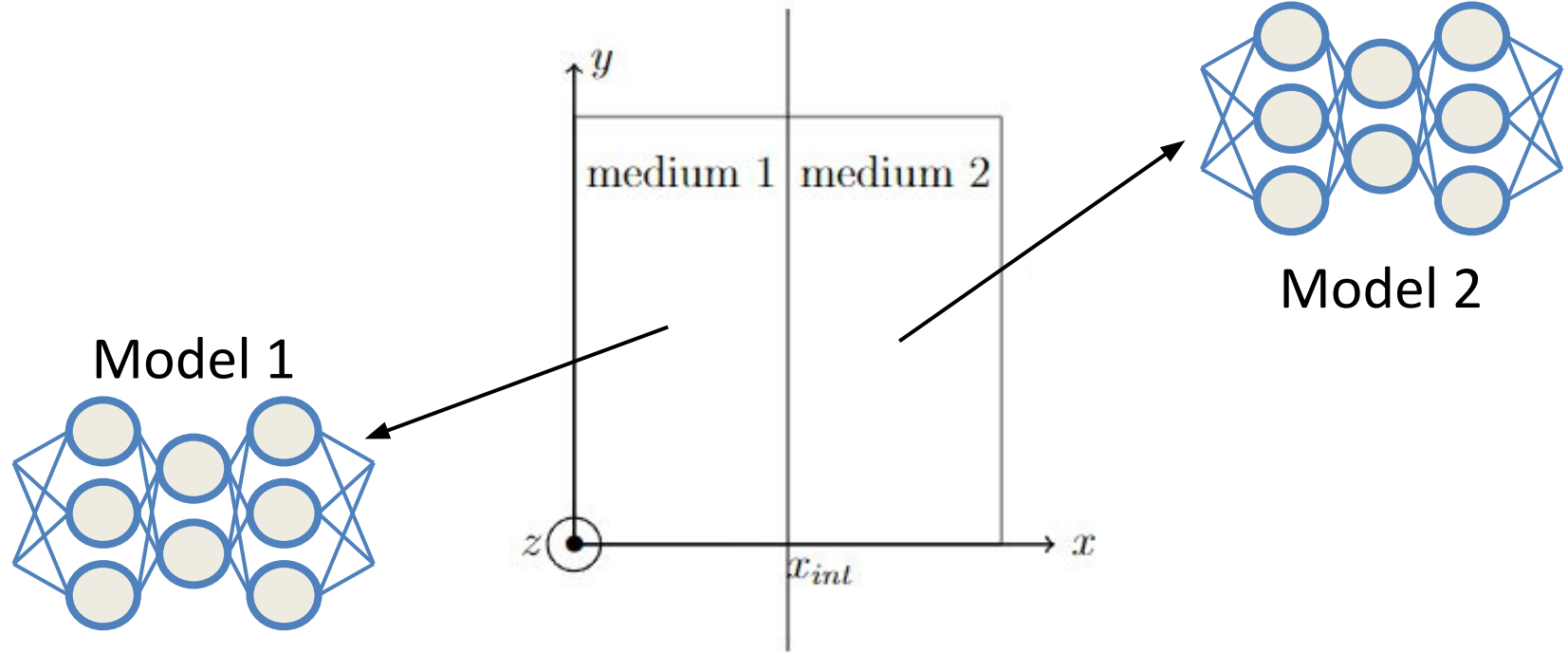
x $\longrightarrow$ Fourier modes of x $\longrightarrow$

y $\longrightarrow$ $\cos\left(\frac{2\pi}{L}y\right)$, $\sin\left(\frac{2\pi}{L}y\right)$ $\longrightarrow$

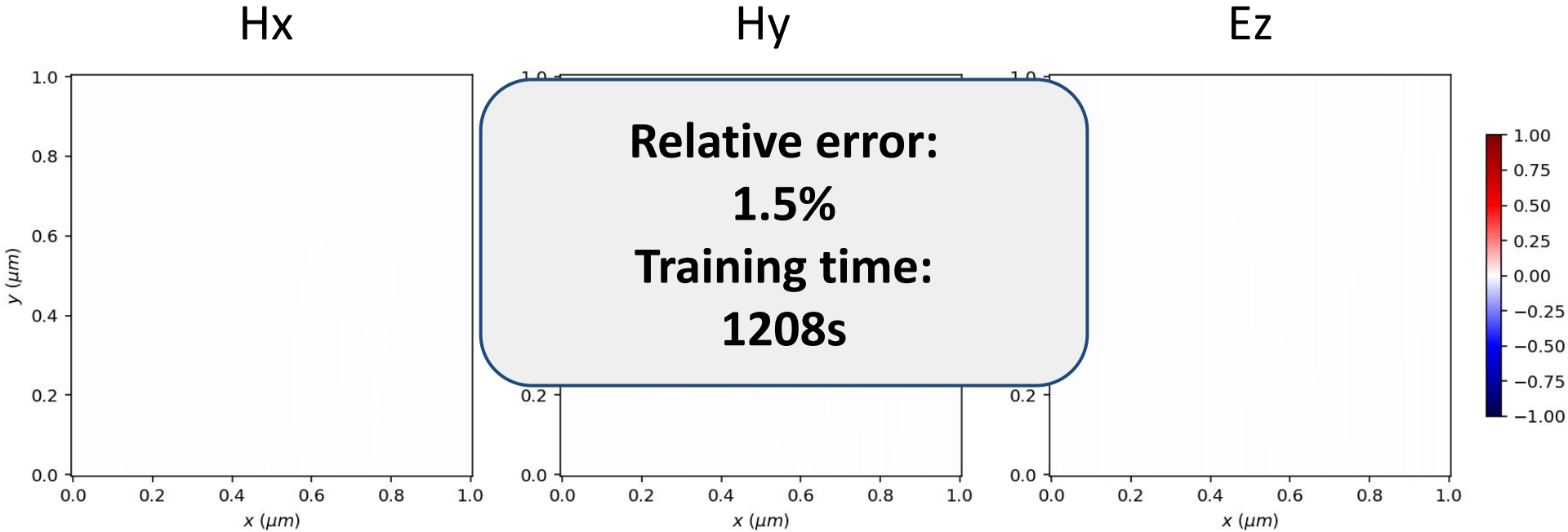
t $\longrightarrow$ $\cos\left(\frac{2\pi}{T}t\right)$, $\sin\left(\frac{2\pi}{T}t\right)$ $\longrightarrow$



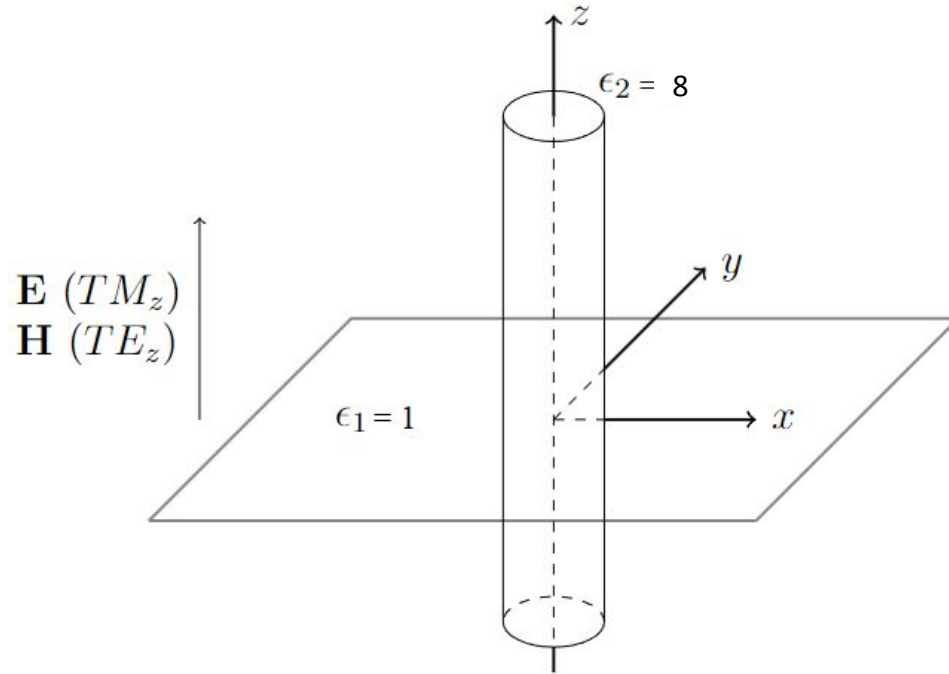
Domain Decomposition



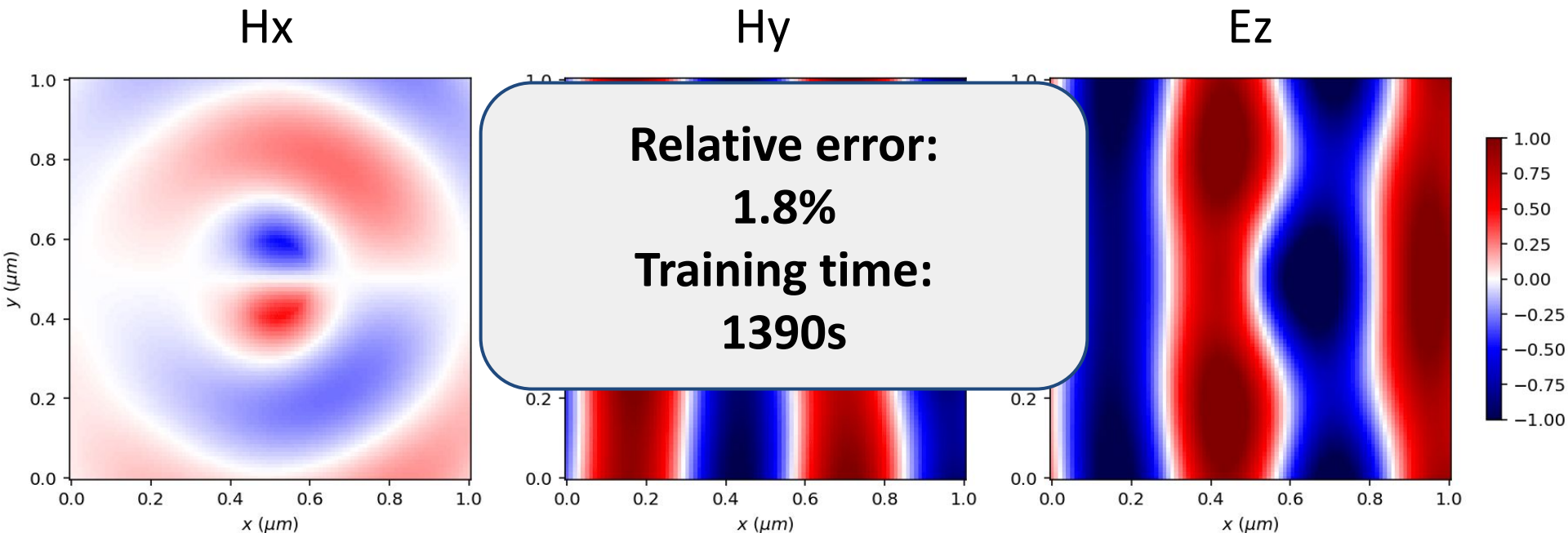
Results With All Techniques



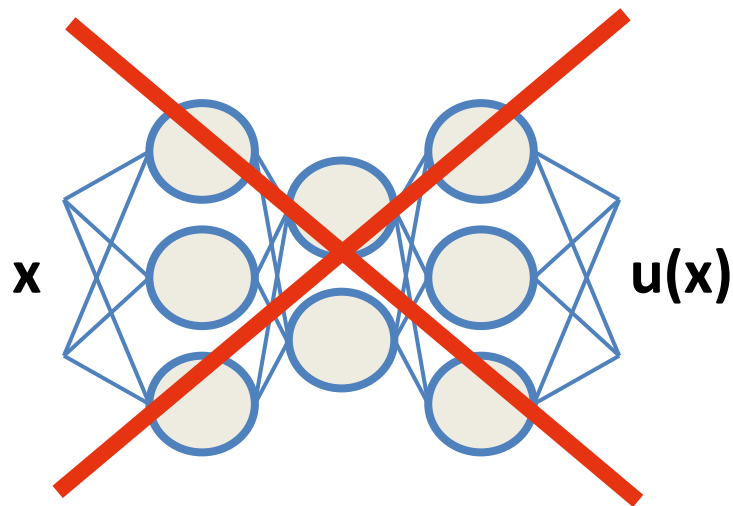
Dielectric Cylinder



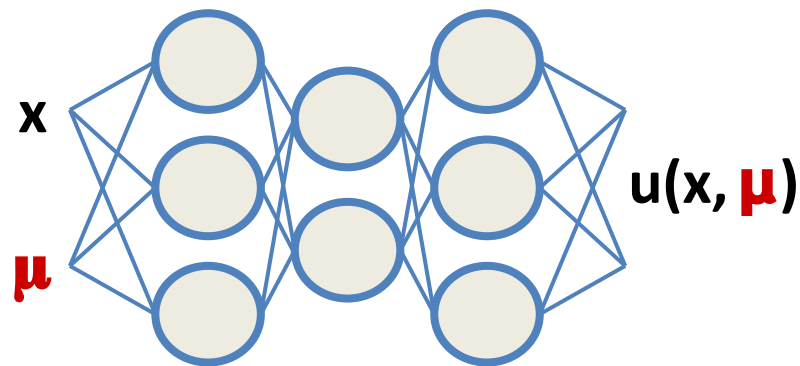
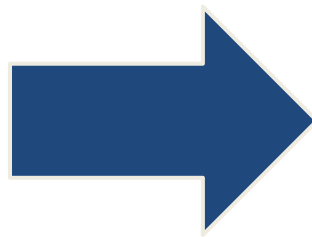
Results For The Dielectric Cylinder



Solving The Parametric Problem



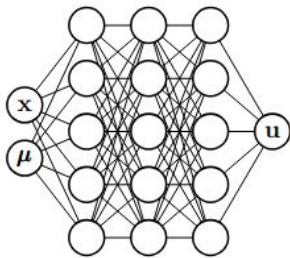
$$\begin{aligned}\forall x \in \Omega, L(u)(x) &= f(x) \\ \forall x \in \partial\Omega, B(u)(x) &= g(x)\end{aligned}$$



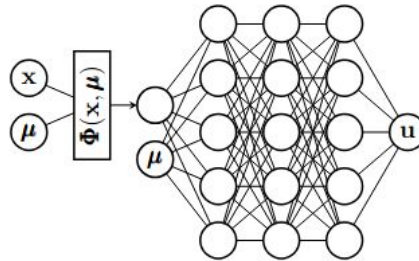
$$\begin{aligned}\forall x \in \Omega, L_{\mu}(u)(x) &= f_{\mu}(x) \\ \forall x \in \partial\Omega, B_{\mu}(u)(x) &= g_{\mu}(x)\end{aligned}$$

Parametric Embedding

Basic



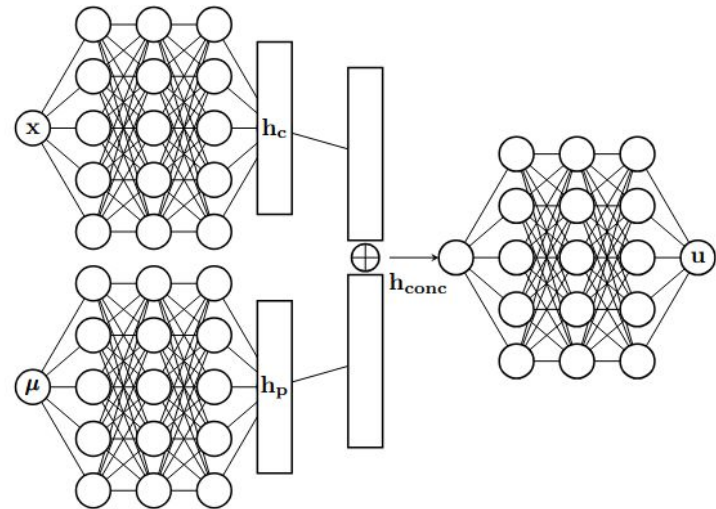
Feature



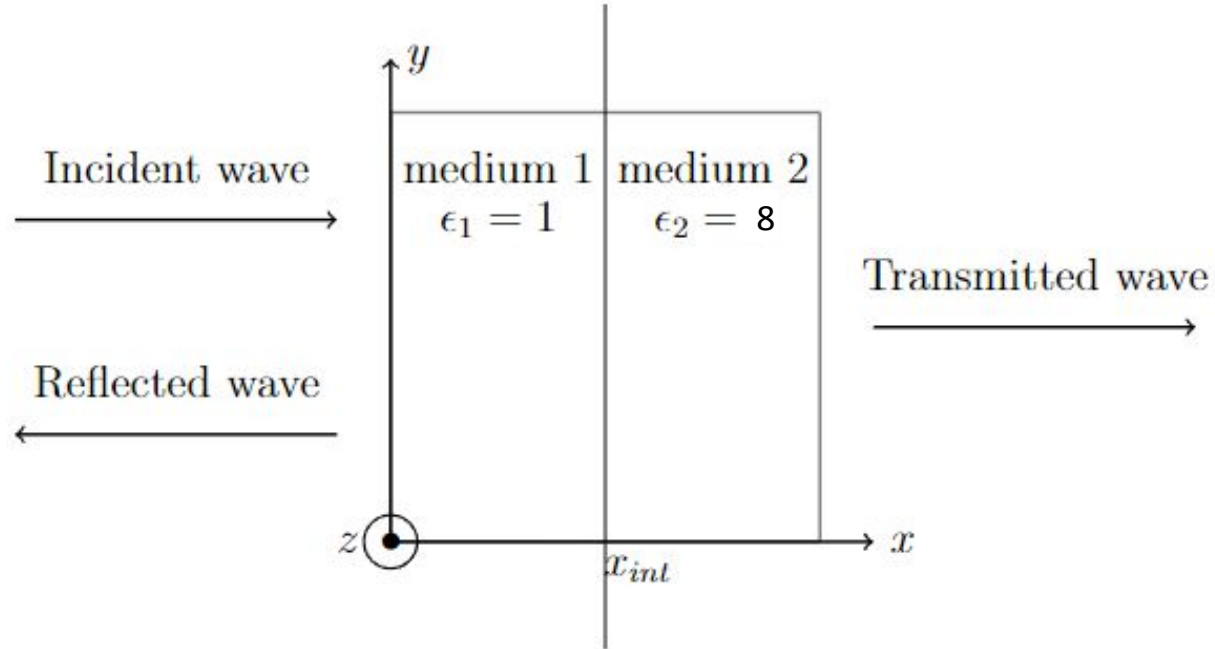
For example: $\Phi(x, \omega) = \omega x$

$$\text{or: } \Phi(\mathbf{x}, \omega) = \begin{bmatrix} \cos(\omega \mathbf{B} \mathbf{x}) \\ \sin(\omega \mathbf{B} \mathbf{x}) \end{bmatrix}$$

Encoders

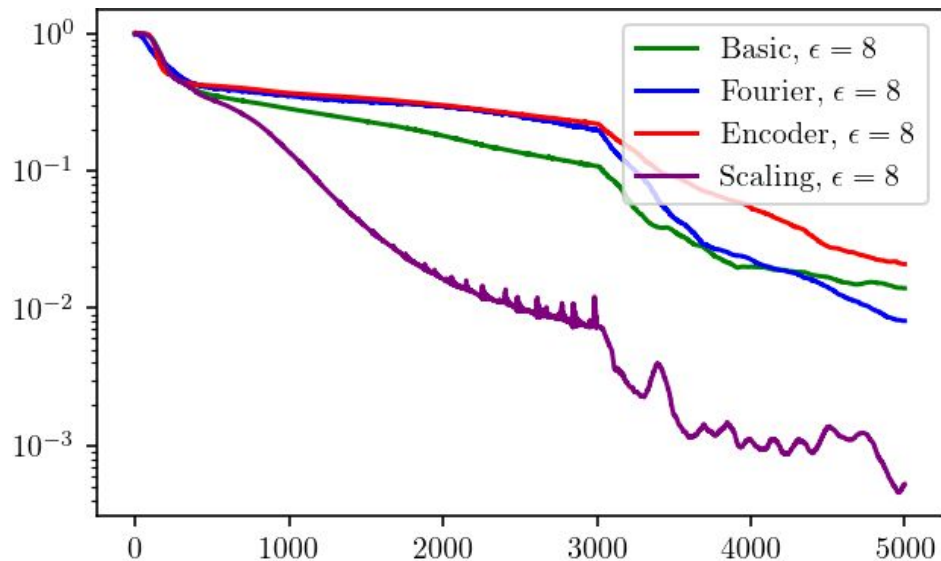


Flat Interface Case

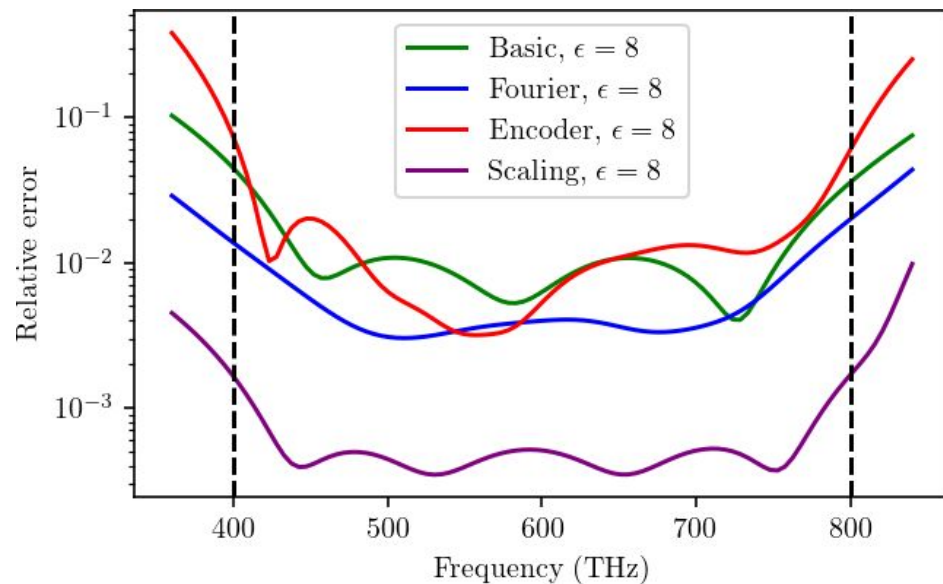


Parameters: frequency

Results

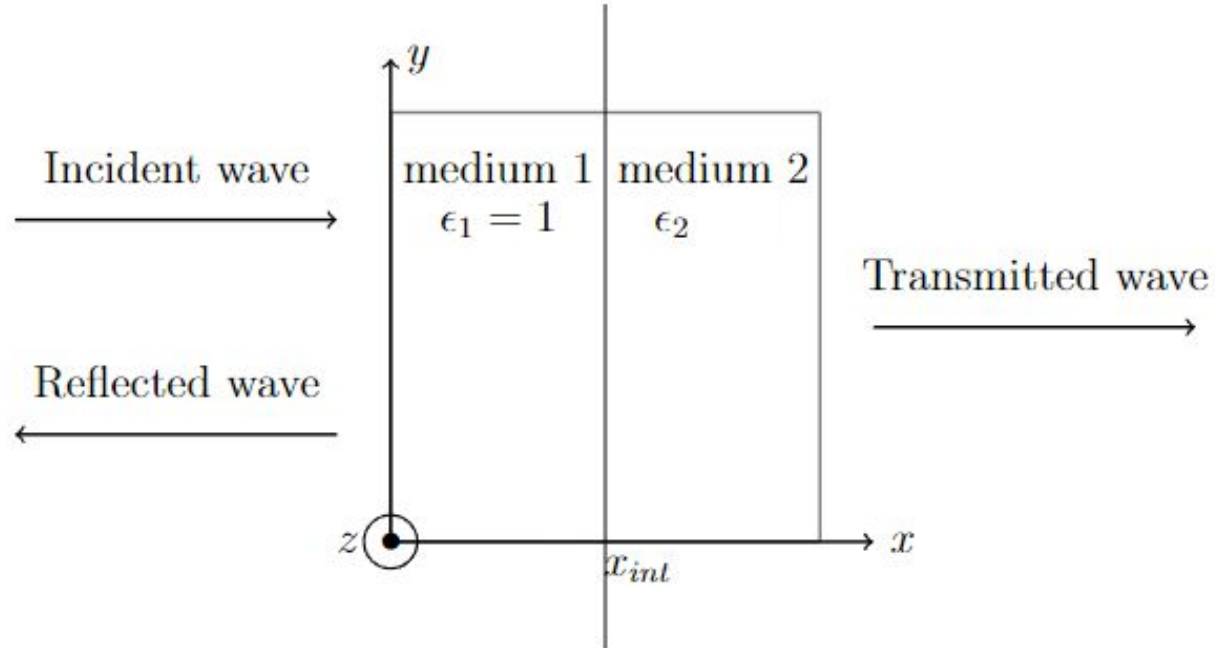


Relative error depending on the number of epochs



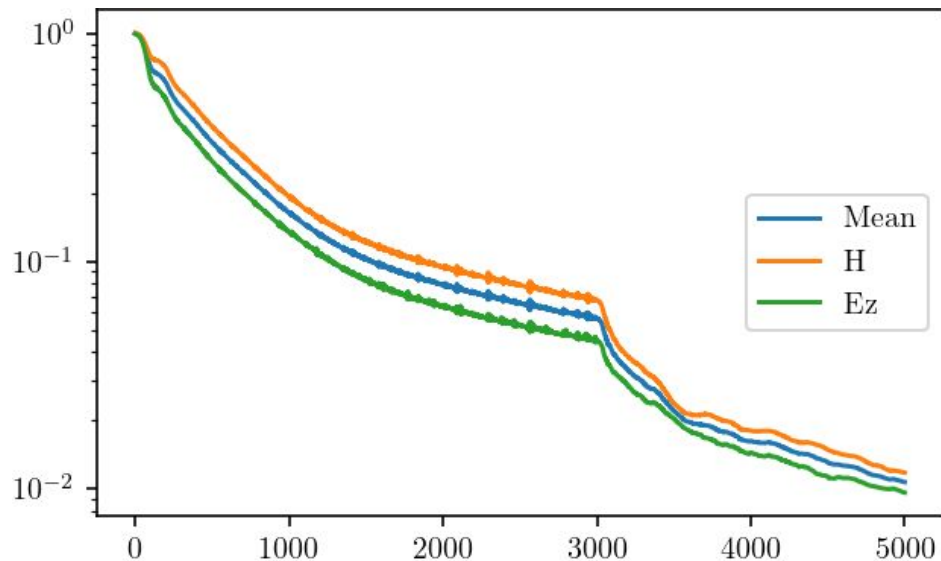
Relative error in function of the frequency

Flat Interface Case

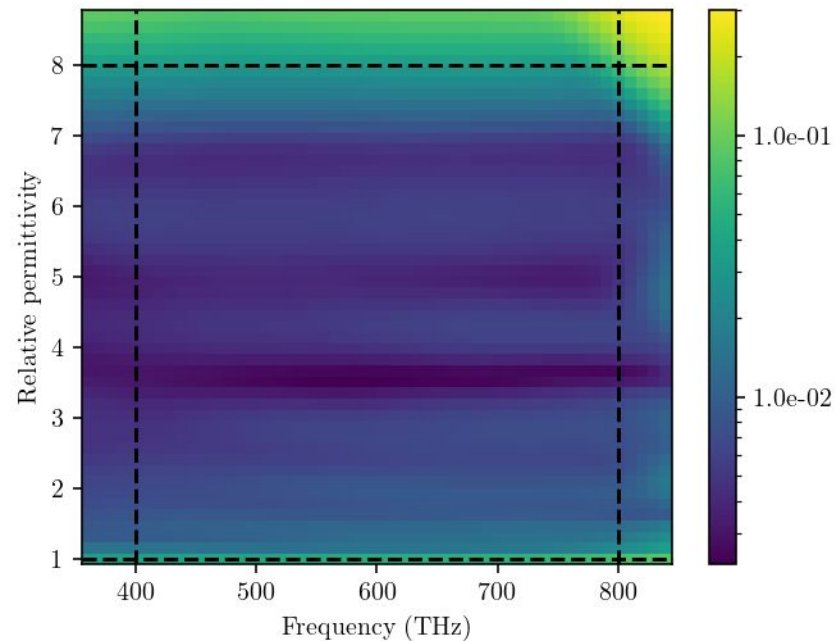


Parameters: frequency, ϵ_2

Results

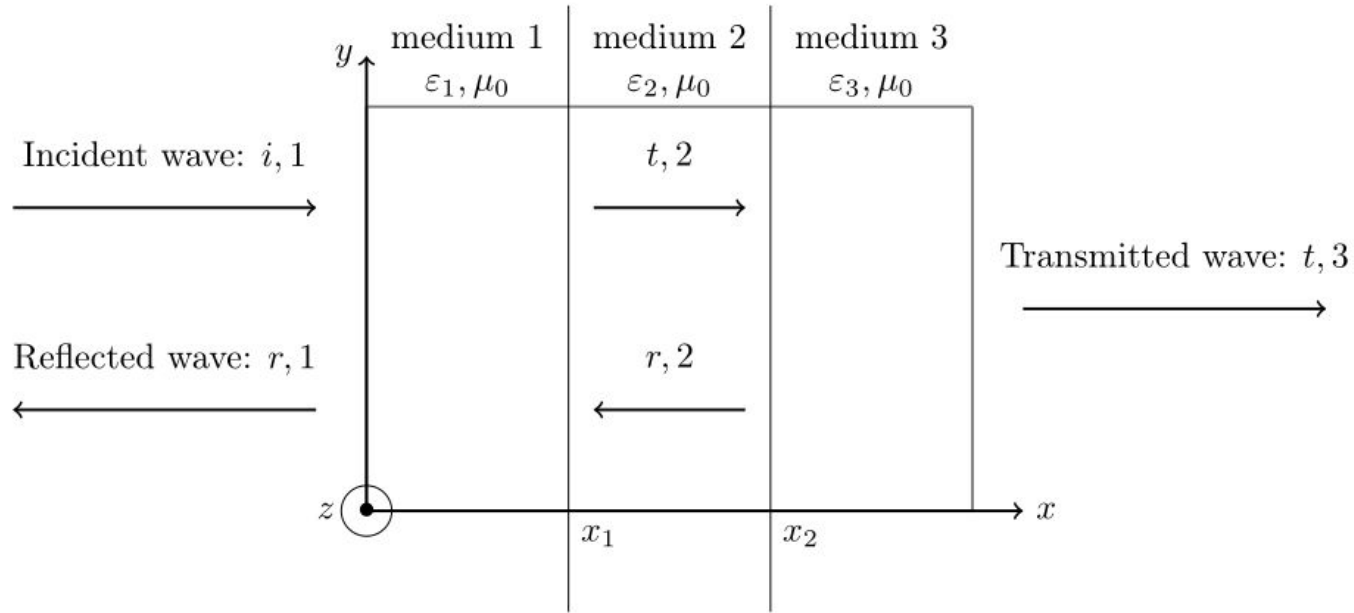


Relative error depending on the number of epochs



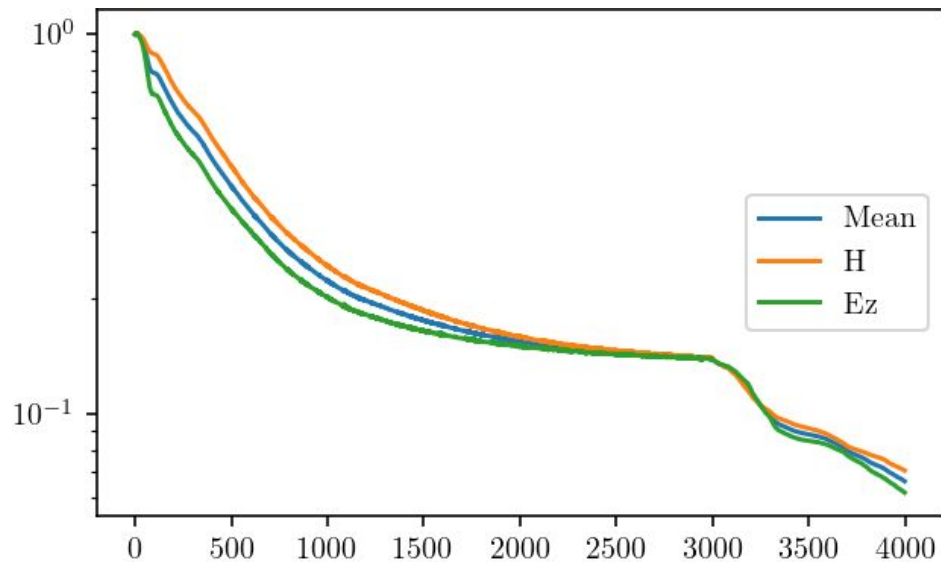
Relative error in function of the parameters

Flat Layers

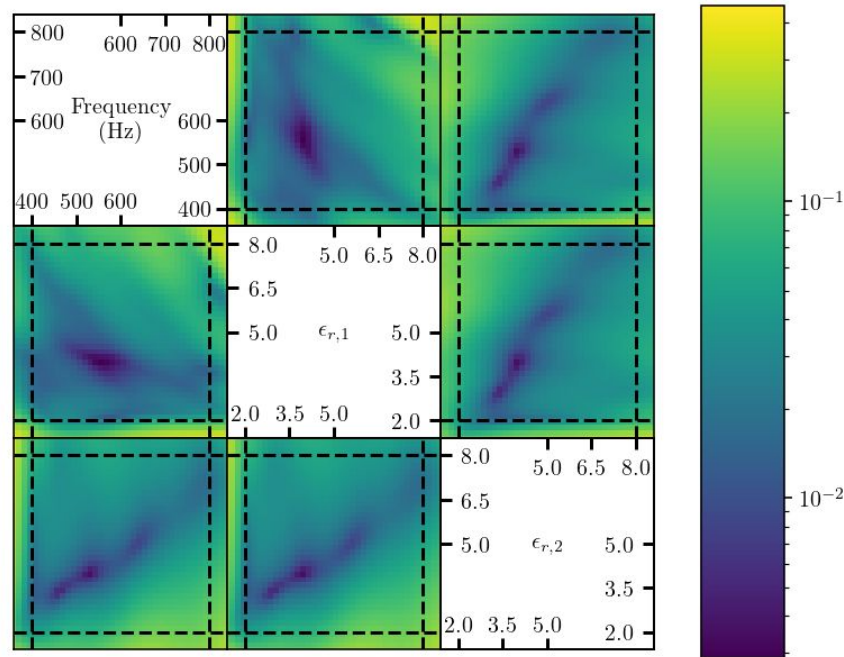


Parameters: frequency, ε_2 , ε_3

Results



Relative error depending on the number of epochs



Relative error in function of the parameters

Conclusion

- PINNs suffer from many pathologies
 - Spectral bias, bad regularity of the solution on interfaces, difficult loss landscapes
- Many of those issues can be alleviated
 - Fourier embeddings, domain decomposition, higher order optimizers
- Solving satisfyingly the forward problem allows to solve the parametric problem
 - Still issues for geometrical parameters
- Many more techniques!
 - SIREN, domain decomposition as preconditioner, Energy Natural Gradient Descent, cusp level set functions

**Thank you for your attention.
Any questions ?**

Appendix

Équations de Maxwell (2D)

Mode transverse magnétique

$$\begin{cases} \varepsilon \frac{\partial E_z}{\partial t} - \frac{\partial H_y}{\partial x} + \frac{\partial H_x}{\partial y} = -j_z \\ \mu \frac{\partial H_x}{\partial t} + \frac{\partial E_z}{\partial y} = 0 \\ \mu \frac{\partial H_y}{\partial t} - \frac{\partial E_z}{\partial x} = 0 \end{cases}$$

Mode transverse électrique

$$\begin{cases} \mu \frac{\partial H_z}{\partial t} + \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = 0 \\ \varepsilon \frac{\partial E_x}{\partial t} - \frac{\partial H_z}{\partial y} = -j_x \\ \varepsilon \frac{\partial E_y}{\partial t} + \frac{\partial H_z}{\partial x} = -j_y \end{cases}$$

Conditions de bord

Conducteur électrique parfait:

$$\text{TM: } E_z = 0$$

$$\text{TE: } n_y E_x - n_x E_y = 0$$

Condition absorbante
de Silver-Müller:

$$\text{TM: } E_z = c\mu(n_y H_x - n_x H_y)$$

$$\text{TE: } c\mu H_z = -n_y E_x + n_x E_y$$

Adimensionalization

$$\left\{ \begin{array}{l} \tilde{\mathbf{H}} = \frac{Z_0}{E_0} \mathbf{H}, \quad \tilde{\mathbf{E}} = \frac{1}{E_0} \mathbf{E} \\ \tilde{t} = \frac{c_0}{L_0} t \\ \tilde{x} = \frac{1}{L_0} x, \quad \tilde{y} = \frac{1}{L_0} y \end{array} \right.$$

L2 Relative Error

$$L_2(\mathbf{y}_{true}, \mathbf{y}_{pred}) = \frac{\|\mathbf{y}_{true} - \mathbf{y}_{pred}\|_2}{\|\mathbf{y}_{true}\|_2}$$