Integral B2B Debt Netting

Model, Evaluation Metrics, and Experiments

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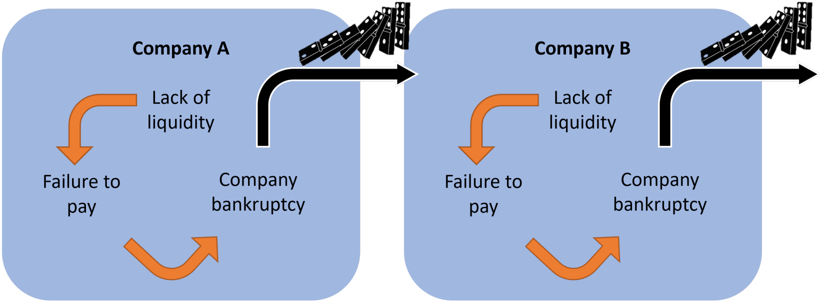
Introduction : Economic fabric

- Companies settle invoices with delays $\rightarrow$ intertwined debts
- Excessive accumulation of debts can cause financial instability



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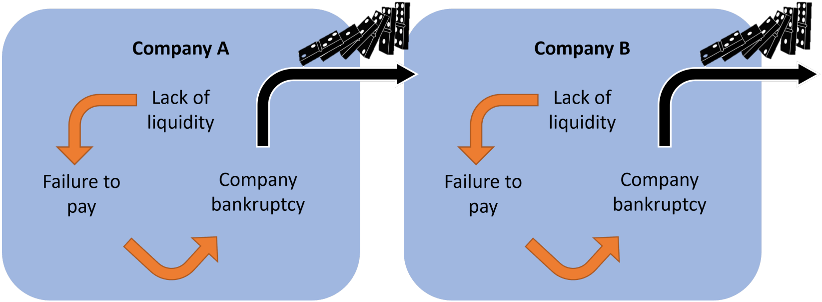

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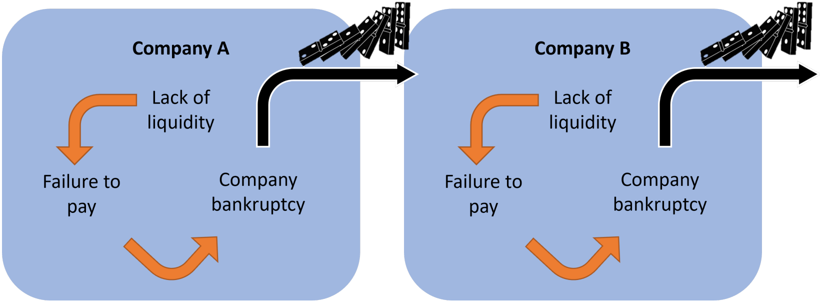

⇒ How can we lower the systemic risk of such a network ?

Debt reduction by netting inside the network

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What is netting?

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Different sorts of netting

- **Bilateral** : company A and company B remove common obligations
- **Partial** : reduction of parts of the value of invoices in cycles
- **Integral** : financing is injected at specific points inside the network to reduce chains of debt

Why integral netting?

Systemic view

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- Simpler accounting than partial netting by fully settling invoices
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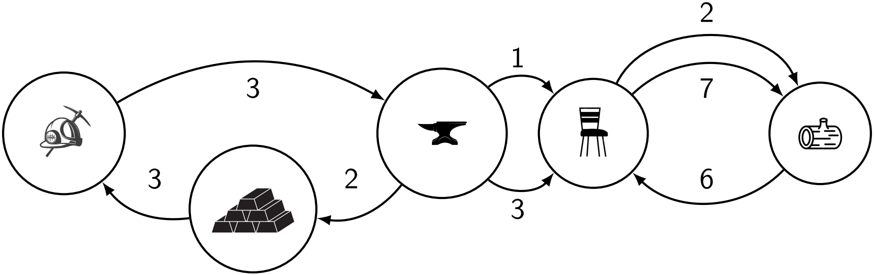
$\rightarrow$ Work on an heuristic for B2B integral netting by invoice selection

Weighted Directed Multi-Graph Representation

- **Nodes**: companies
- **Directed multi-edges**: invoices (debts) between companies
- **Weights**: amounts due

Example of a debt network as a weighted directed multi-graph:



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Debt, Financing, and Balance Computations

For a given debt graph $G = (V, E)$:

- **Total Debt** D_G:

$$D_G = \sum_{e \in E} w(e)$$

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- **External Financing** ϕ_G required to clean all debts in G:

$$\phi_G \;=\; \sum_{v \in V} \max\big(0,\; -B_G(v)\big)$$

Quality metrics

We need to assess *both* **efficiency** of financing usage and **coverage** of the graph when forming our reduction selection (sub-graph R):

$\rightarrow$ **Amplification factor:** $\alpha(R) = \dfrac{D_R}{\phi_R}$

- Total cleared debt over financing

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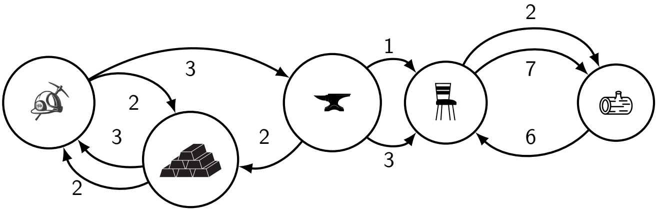
→ **Nettable gain ratio:** $\Gamma_G(R) = \dfrac{D_R - \phi_R}{D_G - \phi_G}$

- Proportion of netted debt (cleared debt - financing) between our reduction and the complete clearing of the graph

Static Reduction Algorithm



- $D_R = 0$
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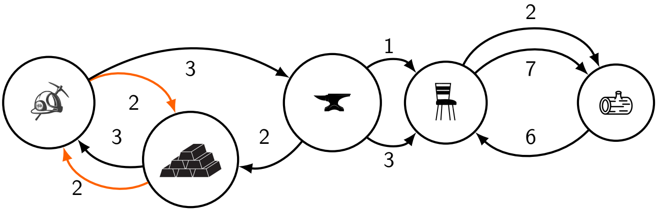

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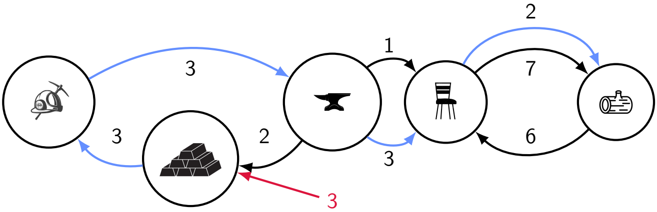

- $D_R = 11$
- $\phi_R = 3$
- $\alpha_R = 3.67$
- $\mathcal{I}_G(R) = 40.74\%$
- $\Gamma_G(R) = 34.78\%$

Reduction algorithm for a static graph :

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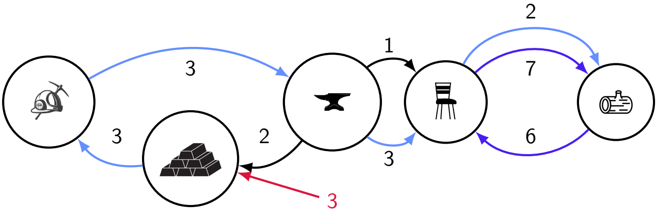

- $D_R = 24$
- $\phi_R = 3$
- $\alpha_R = 8$
- $\mathcal{I}_G(R) = 88.89\%$
- $\Gamma_G(R) = 91.30\%$

Reduction algorithm for a static graph :

- **Step 1:** Removal of perfect cycles of length 2
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- **Step 3:** Extension of the germs by adding a set of connected edges with the highest ratio of debt cleared above the needed financing

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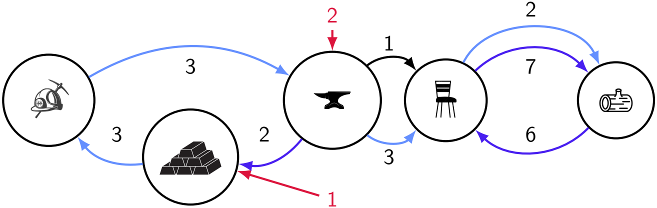

- $D_R = 26$
- $\phi_R = 3$
- $\alpha_R = 8.67$
- $\mathcal{I}_G(R) = 96.30\%$
- $\Gamma_G(R) = 100\%$

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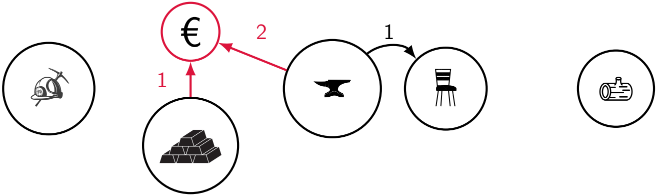

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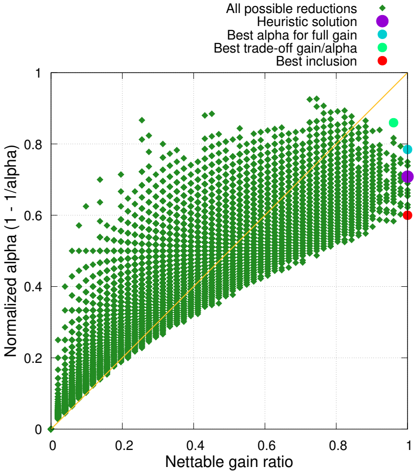
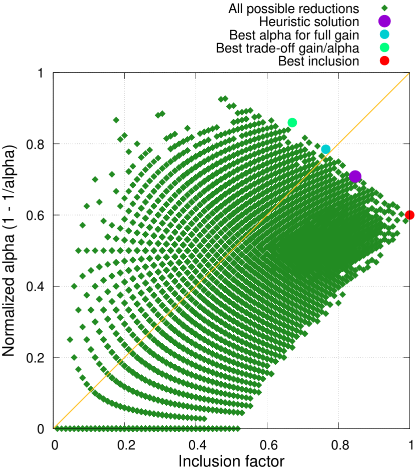
Comparison with Exact Solutions

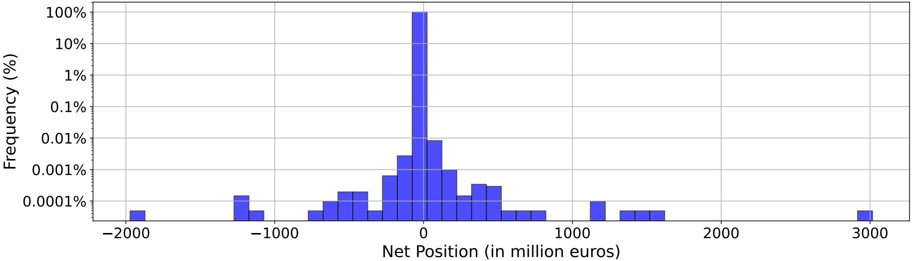
- Small random graphs of 20 edges
- Exhaustive enumeration of all possible reductions





Overview of our real invoices dataset

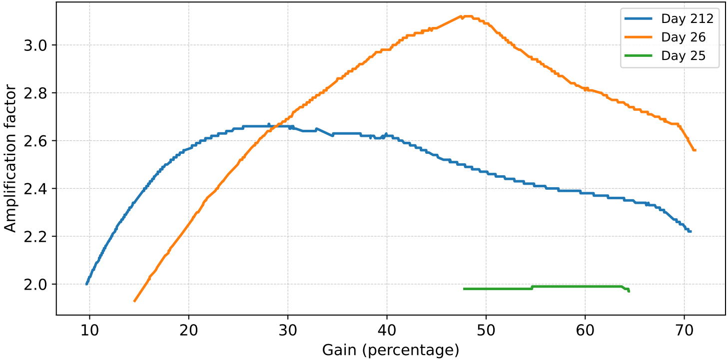
- Electronic invoices from Italy provided by the e-invoicer InfoCert
- 22 million invoices, 2 million companies, €89 billion total over the span year 2019
- Companies are mostly at equilibrium at the end of the year



Repartition of the net position on the overall year for companies in our dataset

Results on different graph sizes

- Partitioning of the year by days
- Coverage of nettable debt $> 50\%$ for an amplification factor of 1.95
- High computation time for large graphs $\Rightarrow$ need for speedup



Amplification factor as a function of the gain for three specific days

Conclusion

Contributions:

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Thank you for your attention!

Any questions or comments?

Results with a simple division strategy

- Split the graph into two sub-graphs, maximising nb of intra-edges
- Reduce each part separately, then merge with a simple greedy algorithm to re-extend
- Mean acceleration factor of 50 and 12% debt clearance loss