An adverse selection approach to power tarification

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Some elements of electricity retail

- Competition on electricity retail is a reality!
- New actors are present
- New tariff's structures proposed
  - fixed/variable price
  - duration of engagement
  - source of energy (green...)
  - dual gas/electricity offers
  - standing charges
- Competition on electricity retail is developing differently against
  - consumer's segment
  - countries

Example of providers to residential consumers in Bristol, source https://www.ukpower.co.uk

- Which tariff's structure should propose electricity providers?
  - Depending on level of competition against electricity providers?
  - Depending on power plants used to generate electricity?
  - To enable risk sharing between providers and consumers? ...
- Considering
  - Electricity is a staple good (difficult to substitute)
  - Tariff's structure should remain simple

In this work, not every questions are solved! We propose a mathematical framework simple enough so that we can explicitly solve the problem.

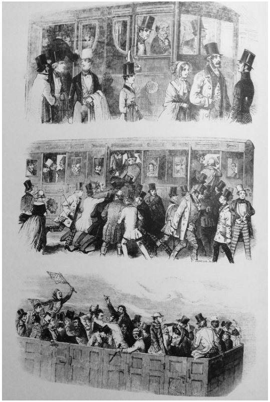
Principal-Agent model

2 types of actors :

- The Principal: proposes the contract
- The Agent: accepts or rejects the contract

Information can be incomplete for one of these two parties.

- Moral hazard : the Principal only observes the outcome and not directly the action of the agent
- **Adverse selection**: Agent's characteristic is imperfectly observed by the Principal.The Principal offers the agent a menu of contracts designed such that the Agent reveals its characteristic.
  Some examples:
  - Trains: different classes to distinguish passengers'willingness to pay
  - Insurance: deductible to distinguish drivers's nature



Objective of the Agent of type x: to select the consumption level which maximize its utility u minus the tariff p he needs to pay for the consumed electricity c.

$$U_A(p, x) := \sup_{c \in \mathbb{R}^+} \int_0^T \left(u(t, x, c(t)) - p(t, c(t)) \right) dt = p^\star(x)$$

Objective of the Principal: to propose a tariff which maximize its own profits = payments received by consumers who take its contract minus costs for producing/providing the electricity consumption of ALL its clients. Agent's type x is unknown to the Principal but the repartition f is known.

$$U_P := \sup_{p \in P} \int_0^T \left[\int_{X^\star(p^\star)} p(t, c^\star(t, x)) f(x) dx - K\left(t, \int_{X^\star(p^\star)} c^\star(t, x) f(x) dx \right) \right] dt.$$

2 conditions

- Individual Rationality: one Agent accepts the contract only if its benefit to accept it is higher than its reservation utility $\Rightarrow p^\star(x) \geq H(x)$.
- Incentive Compatibility : each Agent prefers the contract that was designed for his particular type.

Originality: electricity marginal price increases with the total consumption!

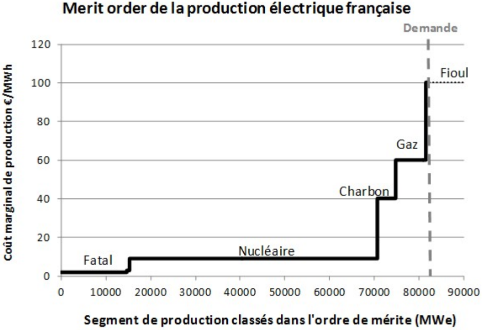


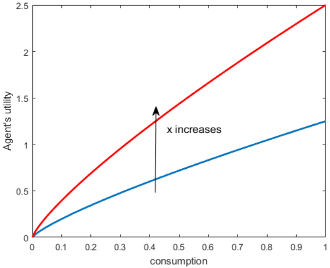
Figure: example of merit order of French electricity production, source http://conseils.xpair.com/actualite_experts/valeur-contenu-co2-electricite.htm

Notations and assumptions

- $p : [0, T] \times \mathcal{C} \longrightarrow \mathbb{R}_+$ the tariff proposed by the Principal, $p(t, c)$ the price of an amount c of electricity at time t (we restrict to only one contract and not a menu).
- $K : [0, T] \times \mathcal{C} \longrightarrow \mathbb{R}_+$ is the cost of production of electricity for the Principal:
$$K(t, c) = \frac{k(t)}{n} c^n$$
with $k(t) \in \mathbb{R}_+^*$ and $n > 1$
- Electricity consumption $c \in \mathcal{C} = \mathbb{R}_+$ or $\mathbb{R}_+^*$ and $x \in [0, 1]$ the type of the agent.
- Uniform repartition of agents' type $f(x) = 1$
- CRRA utility (constant relative risk aversion) of the agent for electricity:
$$u(t, c, x) = g_\gamma(x)\phi(t)\frac{c^\gamma}{\gamma}$$
  - $\phi(t)$ eagerness to consume depending on time
  - $g_\gamma(x)$ eagerness of the Agents to pay for c and we study $g_\gamma(x) := x\mathbf{1}_{\gamma\in(0,1)} + (1 - x)\mathbf{1}_{\gamma<0}$

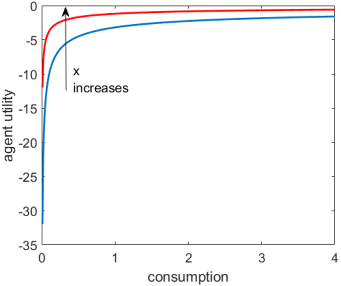
Industrial player: electricity is a market product and people can substitute it

$$u(t,c,x) = x\phi(t)\frac{c^\gamma}{\gamma}, \quad \gamma \in [0,1]$$



Residential player: electricity is a staple product and it is unconceivable not to consume it.

$$u(t,c,x) = (1-x)\phi(t)\frac{c^\gamma}{\gamma}, \quad \gamma < 0$$



Important remarks:

- NO a priori on the tariff's structure!
- the cost K is function of consumption of every consumers.

Let's consider space C^+ of maps g such that for every t, $x \longmapsto g(t,x)$ is continuous and non-decreasing.

1 - For a given tariff p, the optimal consumption $c^\star(x)$ is determined as a function of $\frac{\partial p^\star}{\partial x} \Rightarrow U_P$ is expressed in terms of $p^\star$ only.

2 - Consider the alternative problem $\tilde{U}_P = \sup_{p^* \in C^+} \dots \geq U_P = \sup_{p \in P} \dots$

3 - Compute $\tilde{U}_P$

a) Prove X^* is the union of one or two intervals $X^\star = [a_0, 1]$ or $X^\star = [0, b_0] \cup [a_0, 1]$

b) Using calculus of variations, get $\frac{\partial p^\star}{\partial x}(t,x) = f(t, x_0)$

c) Inject $\frac{\partial p^\star}{\partial x}$ into $\tilde{U}_P$ enables to find x_0

4 - Prove the two problems are equal: p^* is $u-$convex and then p is solution of initial problem

The optimal tariff has a simple structure and is the combination of three terms

$$p(t,c) = p_1(t)c^\gamma + p_2(t)c + p_3(t)$$

Classical result of informational rent: the most efficient Agents are selected. But when the reservation utility is concave, **less efficient consumers can also be selected**!

H **constant**

- Selected Agents: $X^\star = [a_0, 1]$
- Optimal tariff

$$\begin{aligned} p_{1,\gamma}(t)c^\gamma + p_{2,\gamma}(t)c + p_{3,\gamma}(t) & \qquad \gamma \in [0,1] \\ p_{2,\gamma}(t)c + p_{3,\gamma}(t) & \qquad \gamma < 0 \end{aligned}$$

H **concave**

- Selected Agents: $X^\star = [0, b_0] \cup [a_0, 1]$
- Optimal tariff

$$\begin{aligned} p_2^1(t)c + p_3^1(t) & \qquad 0 < c < \hat{c}_1^\gamma(t) \\ p_1^2 c(t)^\gamma + p_2^2(t)c + p_3^2(t) & \qquad \hat{c}_1^\gamma(t) < c < \hat{c}_2^\gamma(t) \\ p_1^3(t)c^\gamma + p_2^3(t)c + p_3^3(t) & \qquad \hat{c}_2^\gamma(t) < c \end{aligned}$$

Optimal tariff interpretation

$$p(t,c) = p_3(t) + p_2(t).c + p_1(t).c^{\gamma}.$$

- The time structure ($p_i(t)$ and power limit $\tilde{c}_i(t)$ of the contract deals typically with the peak/off peak period (ie help to limit power during peak).
- p_3 represents the standing charge of the contract
- $p_2.c$ represents the proportional part to the consumed energy
- $p_1.c^{\gamma}$ contributes to make high energy consumers to pay more.
  p_1 only depends on Agent's utility and not on Principal characteristics.

Impact of production competition: when H increases

- The Principal has less clients (x_0 increases)
- Remaining clients consume more

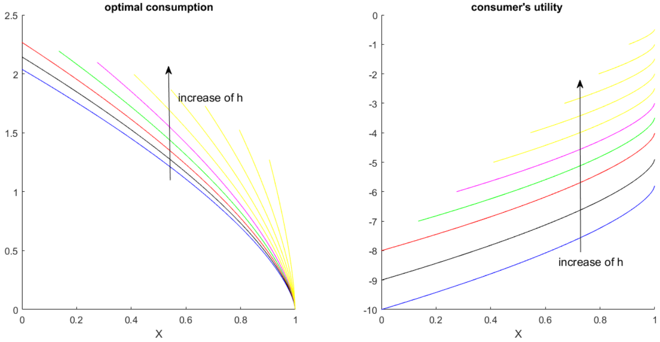


Figure: utility and consumption evolution when H increases

Impact of production competition: when H increases

- The Principal decreases $p_3(t)$ in priority

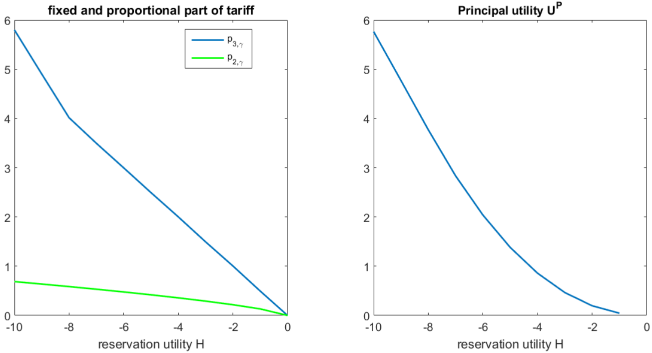


Figure: Evolution of tariff's components (left) and Principal utility U^P (right) against increase of H and $\gamma < 0$

In this example, the standing charge p_3 represents a big amount of the payment compared to sum of charges per unit consumed.

Conclusions and future extensions

In the proposed framework: **explicit tariffs** for electricity are provided and those tariffs are **simple functions of consumption**.

$$p(t, c) = p_3(t) + p_2(t).c + p_1(t).c^{\gamma}$$

Original result: **less efficient agents can also be selected**.

Future extensions we plan to study:

- To include additional Agents'characteristics such as their taste for green contracts, how fast they are changing retailers with respect to price difference...
- To introduce uncertainty on consumption and production
- To consider a mixed population of rational and non-rational consumers

Some references:

- P. Joskow, J. Tirole, Retail electricity competition, RAND Journal of Economics, 2006
- B. Salanie, The Economics of Contracts, MIT Press
- M. Boyer, M. Moreau, M. Truchon, Partage des coûts et tarification des infrastructures
- M. Rasanen, J. Ruusunen b, R.P. Hamalainen, Optimal tariff design under consumer self-selection, Energy Economics, 1997

only the Agents of type $x \geq \widehat{x}_0^\star$ will accept the contract: $X^\star(p^\star) = [x_0, 1]$

Theorem

(i) If $\gamma \in (0,1)$, then, the optimal tariff $p \in \mathcal{P}$

$$p(t,c) = \begin{cases} \phi(t)\dfrac{c^\gamma}{\gamma} + M(t)\left((2x_0^\star - 1)^{\frac{1}{1-\gamma}} - 1\right) - h(t), \ \textit{if}\ c > \left(\dfrac{2\gamma M(t)}{(1-\gamma)\phi(t)}\right)^{\frac{1}{\gamma}}, \\ \phi(t)\dfrac{c^\gamma}{2\gamma} + \left(\left(\dfrac{\phi(t)}{2}\right)^{\frac{1}{1-\gamma}} \dfrac{1-\gamma}{\gamma M(t)}\right)^{\frac{1-\gamma}{\gamma}} c - h(t) + M(t)(2x_0^\star - 1)^{\frac{1}{1-\gamma}}, \ \textit{other} \end{cases}$$

where

$$M(t) = \frac{1-\gamma}{2\gamma}\left(\frac{2(2-\gamma)}{1-\gamma}\right)^{\frac{\gamma(n-1)}{n-\gamma}} \left(\frac{\phi^n(t)}{k^\gamma(t)}\right)^{\frac{1}{n-\gamma}} \left(1 - (2x_0^\star - 1)^{\frac{2-\gamma}{1-\gamma}}\right)^{-\frac{\gamma(n-1)}{n-\gamma}},$$

and where $x_0^\star$ is the unique solution in $(1/2, 1)$ of the equation

$$\int_0^T h(t)dt = 2nA_\gamma(T)\frac{2-\gamma}{n-\gamma}(2x_0^\star - 1)^{\frac{1}{1-\gamma}}\left(1 - (2x_0^\star - 1)^{\frac{2-\gamma}{1-\gamma}}\right)^{-\frac{\gamma(n-1)}{n-\gamma}}.$$

Constant reservation utility $h(t,x) = h(t)$ and $\gamma < 0$

only the Agents of type $x \geq \widehat{x}_0^\star$ will accept the contract: $X^\star(p^\star) = [x_0, 1]$

Theorem

The optimal tariff $p \in \mathcal{P}$

$$p(t,c) = \begin{cases} \phi(t)\dfrac{c^\gamma}{\gamma} - h(t) - \widehat{M}(t)(1-\widehat{x}_0^\star)^{\frac{1}{1-\gamma}} + \widehat{M}(t), \text{ if } c > \left(-\dfrac{\gamma \widehat{M}(t)}{\phi(t)(1-\gamma)}\right)^{\frac{1}{\gamma}}, \\ -\gamma c\left(-\dfrac{\phi(t)}{\gamma}\right)^{\frac{1}{\gamma}}\left(\dfrac{1-\gamma}{\widehat{M}(t)}\right)^{\frac{1-\gamma}{\gamma}} - h(t) - \widehat{M}(t)(1-\widehat{x}_0^\star)^{\frac{1}{1-\gamma}}, \text{ otherwise.} \end{cases}$$

where

$$\widehat{M}(t) = -\frac{1-\gamma}{\gamma}\left(\frac{2-\gamma}{1-\gamma}\right)^{\frac{\gamma(n-1)}{n-\gamma}}\left(\frac{2^\gamma \phi^n(t)}{k^\gamma(t)}\right)^{\frac{1}{n-\gamma}}(1-\widehat{x}_0^\star)^{-\frac{\gamma(2-\gamma)(n-1)}{(n-\gamma)(1-\gamma)}},$$

and where

$$\widehat{x}_0^\star := \left(1 - \left(\frac{n-\gamma}{n(1-\gamma)B_\gamma(T)}\int_0^T h(t)dt\right)^{\frac{n-\gamma}{n(1-\gamma)+\gamma}}\left(\frac{2-\gamma}{1-\gamma}\right)^{\frac{-\gamma(n-1)}{n(1-\gamma)+\gamma}} 2^{\frac{-n}{n(1-\gamma)+\gamma}}\right)^+.$$

$$
\text{p(t,c)}=\begin{cases}
\phi(t)\dfrac{c^\gamma}{\gamma} - N_\gamma\left(2^{-\frac{1}{1-\gamma}} - \left(a_0^\star - \dfrac{1}{2}\right)^{\frac{1}{1-\gamma}}\right) - h(t,a_0^\star), \text{ if } c > L_\gamma(t)2^{-\frac{1}{1-\gamma}}, \\
\phi(t)\dfrac{c^\gamma}{2\gamma} + \phi(t)L_\gamma(t)^{\gamma-1}c + N_\gamma\left(a_0^\star - \dfrac{1}{2}\right)^{\frac{1}{1-\gamma}} - h(t,a_0^\star), \text{ if } L_\gamma(t)\left(a_0^\star - \dfrac{1}{2}\right)^{\frac{1}{1-}} \\
\tilde{x}^\star(c)\phi(t)\dfrac{c^\gamma}{\gamma} - \tilde{p}^\star(t,\tilde{x}(c)), \text{ if } L_\gamma(t)(b_0^\star)^{\frac{1}{1-\gamma}} < c \leq L_\gamma(t)\left(a_0^\star - \dfrac{1}{2}\right)^{\frac{1}{1-\gamma}}, \\
\phi(t)L_\gamma(t)^{\gamma-1}c - h(t,b_0^\star) + N(b_0^\star)^{\frac{1}{1-\gamma}}, \text{ if } 0 \leq c \leq L_\gamma(t)(b_0^\star)^{\frac{1}{1-\gamma}}.
\end{cases}
$$