Stylized model for a grid with distributed generation and storage

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ongoing project with Clémence Alasseur and Anis Mattoussi

PDE and Probability methods for interactions March 2017

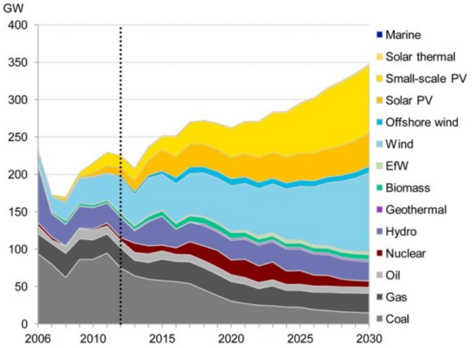

Figure: Projections for Power Generation Capacity to 2013
source: Bloomberg New Energy Finance's global forecast

Motivating Problem

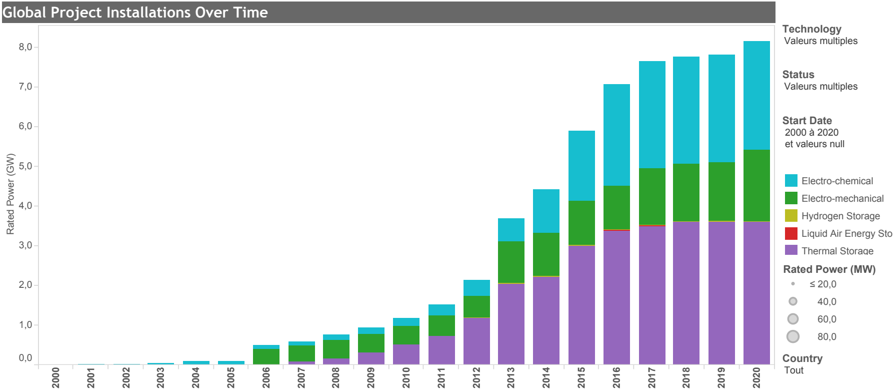

Figure: Projected evolution for Storage Installation (⊖ Pumped Hydro)
source: DOE Global Energy Storage database

Context

- Projected increase in production of renewables, the share of solar and wind will continue to increase ...
- Electricity storage triggered by the expansion of geographically (large to small scale) distributed generation

goal: impact of Decentralize Generation and Storage ?

- optimal storage management and sizing,
- impact of energy and distribution tariff structure
- competition/interaction with traditional utility industry,

Outline

- Stylized model for the power grid with N nodes
- Extended Mean Field Game approximation
- Characterization in a tractable case

Stylized model for the Power Grid

- Grid = N nodes, Γ regions ; state variables of node i in region γ:

$$
\begin{aligned}
Q_t^{i,\gamma} &= Q_0^{i,\gamma} + \int_0^t \mu^\gamma(t)dt + \int_0^t \sigma_t^\gamma dB_t^i + \int_0^t \sigma_t^{\gamma,0} dB_t^0, \quad (1)\\
S_t^{i,\gamma} &= S_0^{i,\gamma} + \int_0^t \alpha_u^i du, \quad \text{energy in storage device} \quad (2)
\end{aligned}
$$

  $\Rightarrow$ control of node i is the storage rate $\alpha^i \in \mathcal{A}$

  $\Rightarrow Q^{i,\gamma} - \alpha_t^i =$ power [injection if ≥ 0] / [consumption if ≤ 0]

- $B^0, B^1, \cdots, B^N$ independent Brownian motions on $(\Omega, \mathcal{F}, \mathbb{P}; \mathbb{F})$
- $\mathbb{F}^0 := \{\sigma(B_s^0),\ s \leq t\}_{t \in \mathbb{R}_+}$ is the filtration of common noise B^0
- Electricity price per Watt-hour, depends on all-players strategies:

$$
P_t^{N,\alpha} = p\left(-\eta N \frac{1}{N}\sum_{i=1}^N (Q_t^i - \alpha_t^i)\right) \quad (3)
$$

  η is a scaling parameter, $p(\cdot)$ inverse demand function.

Let $\alpha = (\alpha^1, \cdots, \alpha^N)$

- Cost at node i: { storage cost, network , energy}

$$J^{i,\gamma,N}(\alpha) = \mathbb{E}\int_0^T \Big[f^{\text{store}}(S_t^{i,\gamma}, \alpha_t^i) + f_\gamma^{\text{transmit}}(|Q_t^{i,\gamma} - \alpha_t^i|) - p(-N\eta\left(\bar{Q}_t - \bar{\alpha}_t\right))(Q_t^{i,\gamma} - \alpha_t^i)\Big]\, dt + \mathbb{E}[g(S_T^{i,\gamma})]$$

- Egalitarian cost function for the Social planner

$$J^{S,N}(\alpha) = \frac{1}{N}\sum_\gamma \sum_i J^{i,\gamma,N}$$

$\Rightarrow$ Nash equilibria $\alpha^\star$ for the N-players game: $\forall i, \gamma$,

$$J^{i,\gamma,N}(\alpha^\star) \leq J^{i,\gamma,N}(u, \alpha^{\star - i}), \quad \forall u \in \mathcal{A}$$

$\Rightarrow$ Pareto optima $\hat{\alpha}$ by minimizing $J^{S,N}$

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Notation: for a process $X = \{X_t\}$ denote $\bar{X}^0 = \{\bar{X}_t^0 = \mathbb{E}[X_t|\mathcal{F}_t^0]\}$.

- initial data: random vector $x_0 = \{(s_0^\gamma, q_0^\gamma)\} \perp \mathbb{F}^0$
- for $\bar{\nu}^0 = \{\bar{\nu}^{\gamma,0}\}$ $\mathbb{F}^0$-adapted process associate the cost functions

$$\alpha^\gamma \in \mathcal{A} \quad \mapsto \quad J_{x_0}^{\gamma,\mathrm{MFG}}(\alpha^\gamma, \bar{\nu}^0) \; = \; \mathbb{E}\int_0^T \Big[f^{\mathrm{store}}(S_t^\gamma, \alpha_t^\gamma) + f_\gamma^{\mathrm{transmit}}(|Q_t^\gamma - \alpha_t^\gamma|)$$
$$-p(-\sum_\gamma \Big(\bar{Q}_t^{\gamma,0} - \bar{\nu}_t^0\Big))(Q_t^\gamma - \alpha_t^\gamma)\Big] \, dt + \mathbb{E}[g(S_T^\gamma)]$$

Definition (Mean field Nash equilibrium)

$\alpha^\star = \{\alpha^{\gamma,\star}\}$ is a mean field Nash equilibrium if for each $\gamma \in \Gamma$, $\alpha^{\gamma,\star}$ minimizes $\alpha \in \mathcal{A} \mapsto J_{x_0}^{\gamma,\mathrm{MFG}}(\alpha, \{\mathbb{E}[\alpha_t^\star|\mathcal{F}_t^0]\})$.

A tractable setting: Quadratic cost functions

- $\mathcal{A}$ $\mathbb{F}$-adapted processes $a = \{a_t \in \mathbb{R}\}$ s.t: $\mathbb{E}\left[\int_0^T |a_u|^2 du\right] < \infty$.
- $f^{\text{store}}(S_t, \alpha_t) + f_\gamma^{\text{transmit}}(|Q_t - \alpha_t|) = \frac{C}{2}|\alpha_t|^2 + \frac{A}{2}|S_t^\alpha|^2 + \frac{K^\gamma}{2}|Q_t - \alpha_t|^2$
- terminal cost $g(S_T) = B(S_T - b)^2$
- $p(\cdot)$ convex and strictly increasing function.

Probabilistic approach $\rightsquigarrow$ Characterization of ext.MFG

Some References: R. Carmona and F. Delarue (2013), (2015);
R. Carmona and F. Delarue and D. Lacker (2014).
Linear Quadratic case: A. Bensoussan, K. Sung, S. Yam and Yung (2011), Yong (2013), Pham (2016).
Graber (2016).

- **Characterization of Mean field Nash equilibrium**
  Existence (and unicity) is equivalent to the existence (and unicity) of a solution to the FBSDE

$$
\begin{aligned}
d\bar{S}^0_t &= (\bar{Q}^0_t + H(\bar{Y}^0_t + \bar{Q}^0_t))dt, \quad \bar{S}^0_0 = \bar{s}^0_0 \qquad (4) \\
d\bar{Y}^0_t &= -A\bar{S}^0_t dt + \bar{Z}^0_t dB^0_t, \quad \bar{Y}^0_T = 2B(\bar{S}^0_T - b) \qquad (5)
\end{aligned}
$$

  where $G(x) = p(x) + (C + K)$, $H(y) = G^{-1}(-y)$.
  $\Rightarrow$ based on existence and uniqueness results for Linear FBSDE: existence and uniqueness holds for small time.

- **Under a linear pricing rule:** $p(d) = P_0 + \pi d$
  $\Rightarrow$ Existence and uniqueness of a Mean field Nash equilibrium: $\alpha^\star(\pi)$
  $\Rightarrow$ can be obtained as the solution of a convenient Mean Field Type (MFC) problem

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The Mean Field type Control Problem

- initial data: random vector $x_0 = \{(s_0^\gamma, q_0^\gamma)\} \perp \mathbb{F}^0$
- to an $\mathbb{F}$-adapted process $\alpha \in \mathcal{A}$ we associate the cost

$$J_{x_0}^{\mathrm{MFC}}(\alpha) = \mathbb{E}\int_0^T \Big[\frac{C}{2}|\alpha_t|^2 + \frac{A}{2}|S_t^\alpha|^2 + \frac{K}{2}|Q_t - \alpha_t|^2 \\ -p(-\sum_\gamma \Big(\bar{Q}_t^{\gamma,0} - \bar{\alpha}_t^{\gamma,0}\Big))(Q_t^\gamma - \alpha_t)\Big] dt + \mathbb{E}[g(S_T^\gamma)]$$

- modified pricing rule $p(x) = P_0 + \mathbf{2}\pi x$

Numerical Example: 1 region with storage , 1 region without storage

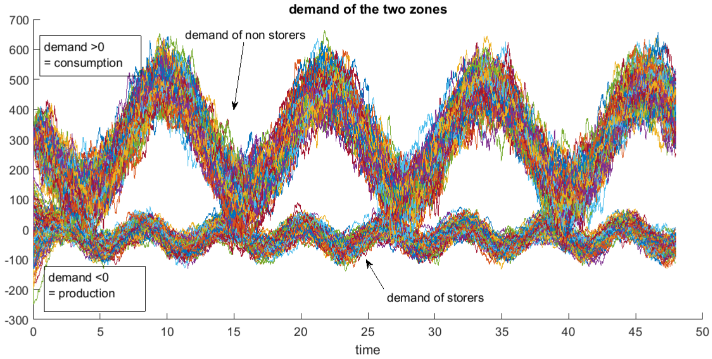

Figure: Local net power demand, 2 regions.

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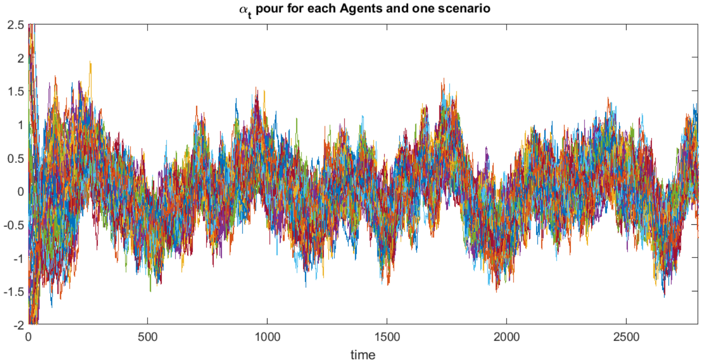

Figure: battery charge/discharge rate α, region with storage

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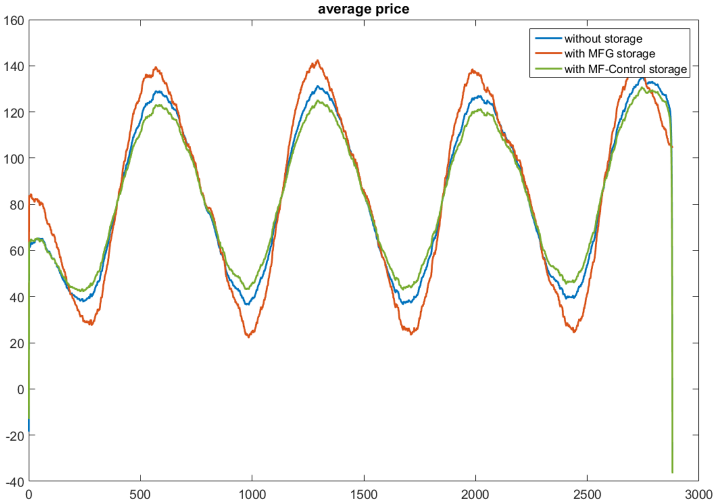

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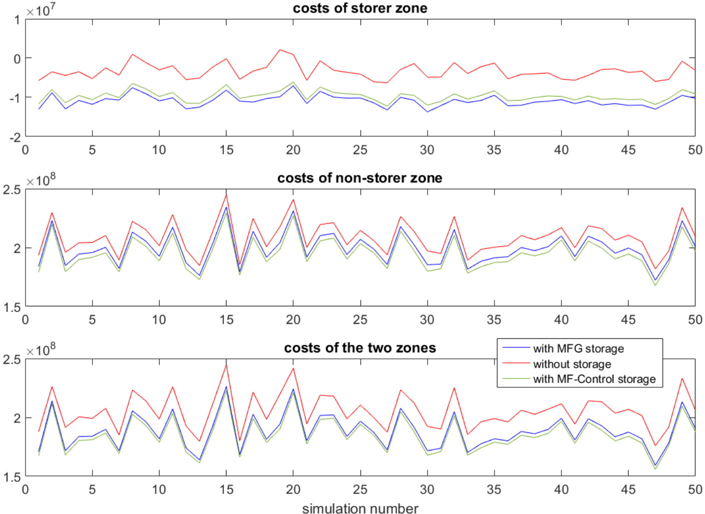

Ongoing work

- under which conditions the extended MFG $\sim$ MFC ?
- more general cost functions ?