Distributed demand control in power grids
and ODEs for Markov decision processes

PDE and Probability Methods for Interactions
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Joint work with Sean Meyn
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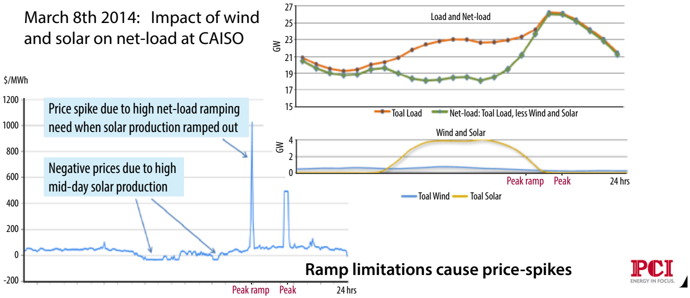
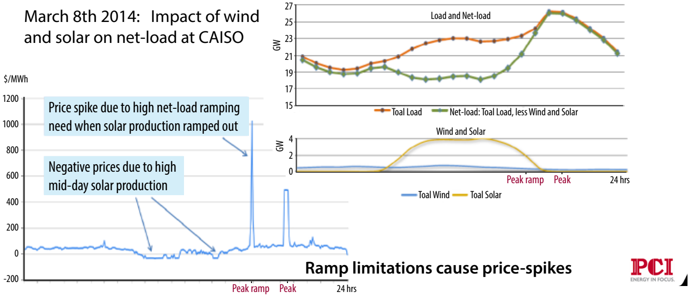
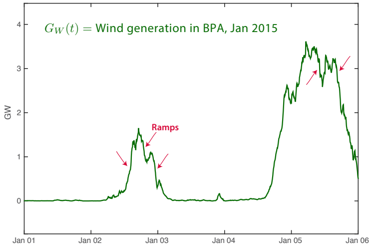
Challenges of renewable power generation



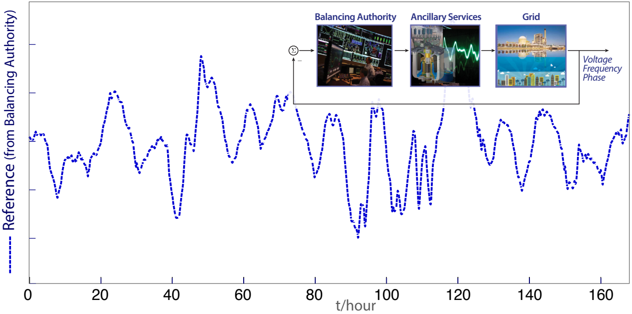
Challenges of renewable power generation





Challenges of renewable power generation

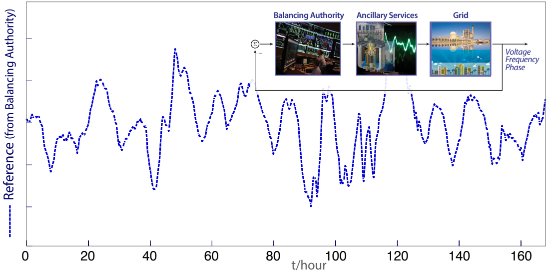
Increasing needs for ancillary services



In the past, provided by the generators - high costs!

Tracking Grid Signal with Residential Loads

Tracking objective:



Prior work

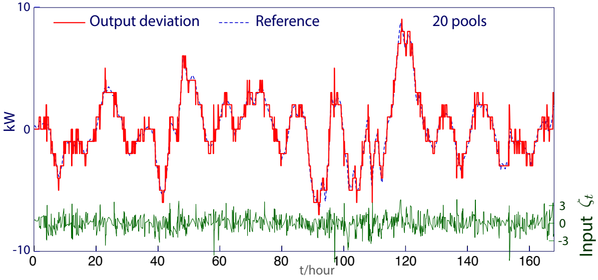
- Deterministic centralized control:
  Sanandaji et al. 2014 [HICSS], Biegel et al. 2013 [IEEE TSG]
- Randomized control:
  Mathieu, Koch, Callaway 2013 [IEEE TPS] (decisions at the BA)
  Meyn, Barooah, B., Chen, Ehren 2015 [IEEE TAC]
  (local decisions, restricted load models)

Tracking Grid Signal with Residential Loads

Example: 20 pools, 20 kW max load

Each pool consumes 1kW when operating
12 hour cleaning cycle each 24 hours

Power Deviation:



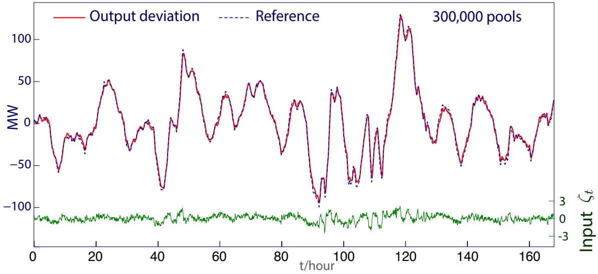
Nearly Perfect Service from Pools
Meyn, Barooah, B., Chen, Ehren 2015 [IEEE TAC]
using an extension/reinterpretation of Todorov 2007 [NIPS] (linearly solvable MDPs)

Tracking Grid Signal with Residential Loads

Example: 300,000 pools, 300 MW max load

Each pool consumes 1kW when operating
12 hour cleaning cycle each 24 hours

Power Deviation:



Nearly Perfect Service from Pools

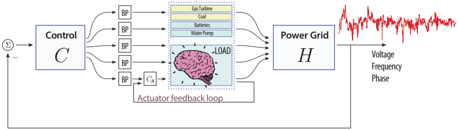
What About Other Loads?

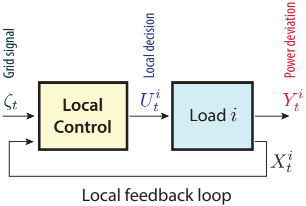
Meyn, Barooah, B., Chen, Ehren 2015 [IEEE TAC]
using an extension/reinterpretation of Todorov 2007 [NIPS] (linearly solvable MDPs)

Control Goals and Architecture

Local Control: decision rules designed to respect needs of load and grid

Demand Dispatch: Power consumption from loads varies automatically to provide *service to the grid*, *without impacting QoS* to the consumer

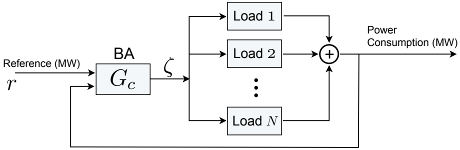




- Min. communication: each load monitors its state and a regulation signal from the grid.
- Aggregate must be controllable: **randomized policies** for finite-state loads.

Load Model

Controlled Markovian Dynamics



- Discrete time: ith load $X^i(t)$ evolves on finite state space X
- Each load is subject to *common* controlled Markovian dynamics.

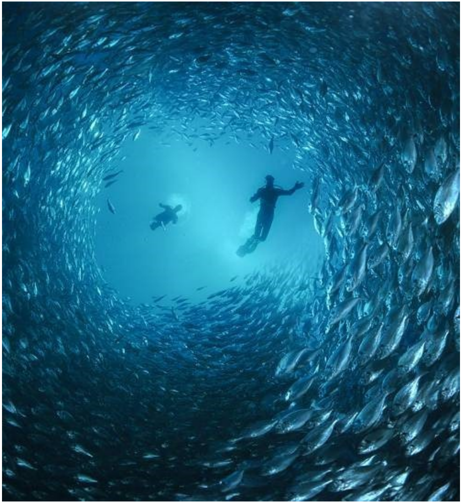
Signal $\boldsymbol{\zeta} = \{\zeta_t\}$ is broadcast to all loads

- Controlled transition matrix $\{P_\zeta : \zeta \in \mathbb{R}\}$:

$$\mathsf{P}\{X^i_{t+1} = x' \mid X^i_t = x,\, \zeta_t = \zeta\} = P_\zeta(x, x')$$

Questions

- How to analyze aggregate of similar loads?
- Local control design?



Aggregate model

How to analyze aggregate?

Mean field model

N loads running independently, each under the command ζ.

Empirical Distributions:

$$\mu_t^N(x) = \frac{1}{N}\sum_{i=1}^N \mathbb{I}\{X^i(t) = x\}, \qquad x \in \mathsf{X}$$

$\mathcal{U}(x)$ power consumption in state x,

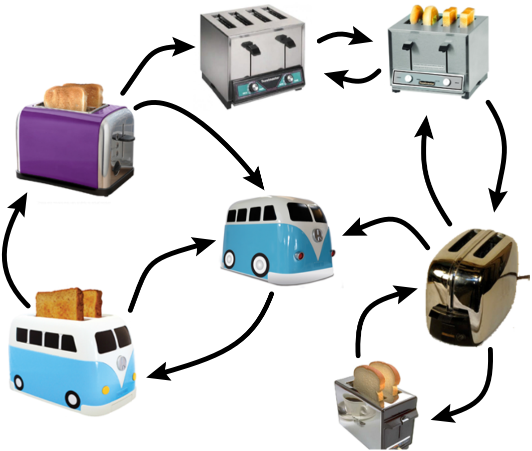
$$y_t^N = \frac{1}{N}\sum_{i=1}^N \mathcal{U}(X_t^i) = \sum_x \mu_t^N(x)\mathcal{U}(x)$$

Mean-field model:

via Law of Large Numbers for martingales

$$\mu_{t+1} = \mu_t P_{\zeta_t}, \qquad y_t = \langle \mu_t, \mathcal{U} \rangle$$

$$\zeta_t = f_t(y_0, \dots, y_t) \quad \textit{by design}$$



Local Control Design

Local Design

Goal: Construct a family of transition matrices $\{P_\zeta : \zeta \in \mathbb{R}\}$

Nominal model

A Markovian model for an individual load, based on its typical behavior.

- Finite state space $\mathsf{X} = \{x^1, \dots, x^d\}$;
- Transition matrix P_0, with unique invariant pmf π_0.

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Common structure for design

The family of transition matrices used for distributed control is of the form:

$$P_\zeta(x, x') := P_0(x, x') \exp\bigl(h_\zeta(x, x') - \Lambda_{h_\zeta}(x)\bigr)$$

with h_ζ continuously differentiable in ζ, and the normalizing constant

$$\Lambda_{h_\zeta}(x) := \log\Bigl(\sum_{x'} P_0(x, x') \exp\bigl(h_\zeta(x, x')\bigr)\Bigr)$$

Local Design

Goal: Construct a family of transition matrices $\{P_\zeta : \zeta \in \mathbb{R}\}$

Construction of the family of functions $\{h_\zeta : \zeta \in \mathbb{R}\}$

Step 1: The specification of a function $\mathcal{H}$ that takes as input a transition matrix. $H = \mathcal{H}(P)$ is a real-valued function on $\mathsf{X} \times \mathsf{X}$.

Step 2: The families $\{P_\zeta\}$ and $\{h_\zeta\}$ are defined by the solution to the ODE:

$$\tfrac{d}{d\zeta} h_\zeta = \mathcal{H}(P_\zeta), \qquad \zeta \in \mathbb{R},$$

in which P_ζ is determined by h_ζ through:

$$P_\zeta(x, x') := P_0(x, x') \exp\bigl(h_\zeta(x, x') - \Lambda_{h_\zeta}(x)\bigr)$$

The boundary condition: $h_0 \equiv 0$.

Local Design

Extending local control design to include exogenous disturbances

State space for a load model: $X = X_u \times X_n$.

Components X_n are not subject to direct control
(e.g. impact of the weather on the climate of a building).

Local Design

Extending local control design to include exogenous disturbances

State space for a load model: $\mathsf{X} = \mathsf{X}_\mathsf{u} \times \mathsf{X}_\mathsf{n}$.

Components X_n are not subject to direct control
(e.g. impact of the weather on the climate of a building).

Conditional-independence structure of the local transition matrix

$$P(x, x') = R(x, x'_u) Q_0(x, x'_n), \quad x' = (x'_u, x'_n)$$

Q_0 models uncontroled load dynamics and exogenous disturbances.

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Q_0 models uncontroled load dynamics and exogenous disturbances.

Assumption: for all $x \in \mathsf{X}$, $x' = (x'_u, x'_n) \in \mathsf{X}$, $h_\zeta(x, x') = h_\zeta(x, x'_u)$.

Local Design

Extending local control design to include exogenous disturbances

For any function $H^\circ \colon \mathsf{X} \to \mathbb{R}$, one can define

$$H(x, x'_u) = \sum_{x'_n} Q_0(x, x'_n) H^\circ(x'_u, x'_n) \tag{1}$$

Then functions $\{h_\zeta\}$ satisfy

$$h_\zeta(x, x'_u) = \sum_{x'_n} Q_0(x, x'_n) h^\circ_\zeta(x'_u, x'_n),$$

for some $h^\circ_\zeta \colon \mathsf{X} \to \mathbb{R}$. Moreover, these functions solve the d-dimensional ODE,

$$\tfrac{d}{d\zeta} h^\circ_\zeta = \mathcal{H}^\circ(P_\zeta), \qquad \zeta \in \mathbb{R},$$

with boundary condition $h^\circ_0 \equiv 0$.

Individual Perspective Design

From the point of view of a single load

Solves an optimization problem from the point of view of a single load

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From the point of view of a single load

Solves an optimization problem from the point of view of a single load

- Local welfare function: $\mathcal{W}_\zeta(x, P) = \zeta \mathcal{U}(x) - D(P \| P_0)$,
  where D denotes relative entropy: $D(P \| P_0) = \sum_{x'} P(x, x') \log\left(\frac{P(x,x')}{P_0(x,x')}\right)$.

Individual Perspective Design

From the point of view of a single load

Solves an optimization problem from the point of view of a single load

- Local welfare function: $\mathcal{W}_\zeta(x,P) = \zeta\mathcal{U}(x) - D(P\|P_0)$,
  where D denotes relative entropy: $D(P\|P_0) = \sum_{x'} P(x,x')\log\big(\frac{P(x,x')}{P_0(x,x')}\big)$.
- Markov Decision Process: $\limsup_{T\to\infty} \frac{1}{T}\sum_{t=1}^{T} E[\mathcal{W}_\zeta(X_t,P)]$
  Average reward optimization equation (AROE):
  $$\max_P\Big\{\mathcal{W}_\zeta(x,P) + \sum_{x'} P(x,x')h^*_\zeta(x')\Big\} = h^*_\zeta(x) + \eta^*_\zeta$$
- For a fixed ζ and fully controllable dynamics, solution via an eigenvector problem using a reinterpretation of Todorov 2007 [NIPS] (linearly solvable MDPs)
  $P_\zeta(x,y) = \frac{1}{\lambda}\frac{v(y)}{v(x)}\hat{P}_\zeta(x,y)\,,\ x,y \in \mathsf{X}$,
  where $\hat{P}_\zeta v = \lambda v$, $\qquad \hat{P}_\zeta(x,y) = \exp(\zeta\mathcal{U}(x))P_0(x,y)$

Individual Perspective Design

From the point of view of a single load

- Markov Decision Process: $\limsup_{T\to\infty} \frac{1}{T}\sum_{t=1}^{T} E[\mathcal{W}_\zeta(X_t, P)]$

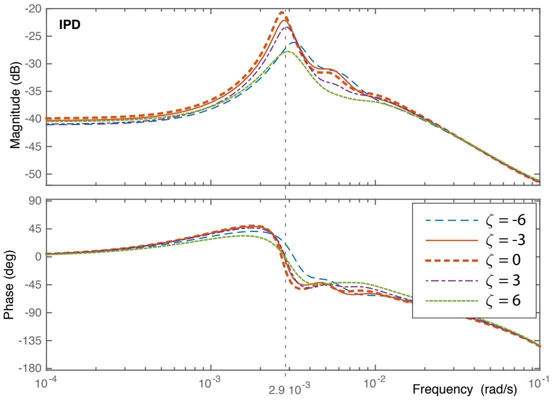
  AROE:
$$\max_R\Big\{\mathcal{W}_\zeta(x,P) + \sum_{x'} P(x,x')h^*_\zeta(x')\Big\} = h^*_\zeta(x) + \eta^*_\zeta$$
  where $P(x,x') = R(x,x'_u)Q_0(x,x'_n), \quad x' = (x'_u, x'_n)$

- ODE method for IPD design:

  Family $\{P_\zeta\}$: $P_\zeta(x,x') := P_0(x,x')\exp\big(h_\zeta(x,x') - \Lambda_{h_\zeta}(x)\big)$
  Functions $\{h_\zeta\}$: $h_\zeta(x,x'_u) = \sum_{x'_n} Q_0(x,x'_n)h^\circ_\zeta(x'_u,x'_n)$,
  for $h^\circ_\zeta \colon \mathsf{X} \to \mathbb{R}$ solutions of the d-dimensional ODE,
$$\frac{d}{d\zeta}h^\circ_\zeta = \mathcal{H}^\circ(P_\zeta), \qquad \zeta \in \mathbb{R},$$
  with boundary condition $h^\circ_0 \equiv 0$.
$$H^\circ_\zeta(x) = \frac{d}{d\zeta}h^\circ_\zeta(x) = \sum_{x'}[Z_\zeta(x,x') - Z_\zeta(x^\circ,x')]\mathcal{U}(x'), \quad x \in \mathsf{X},$$
  where $Z = [I - P + 1\otimes\pi]^{-1} = \sum_{n=0}^\infty [P_\zeta - 1\otimes\pi]^n$ is the fundamental matrix.

Individual Perspective Design

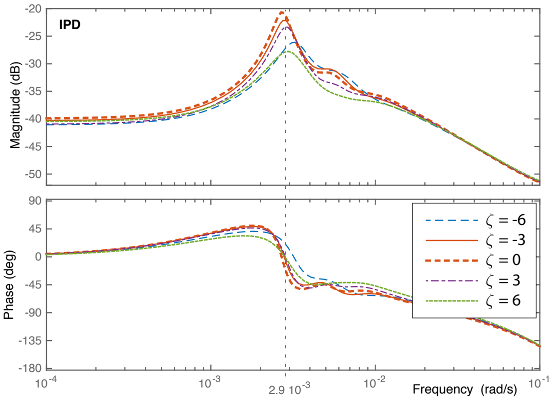
Linearized dynamics



Bode plots for IPD: Linearizations at five values of ζ

Individual Perspective Design

Linearized dynamics



Bode plots for IPD: Linearizations at five values of ζ

Proof of positive real condition for reversible load dynamics.

System Perspective Design

Strictly positive real by design

Goal: The transfer function of the linearized aggregate model is positive real.

SPD design:

- $P^{\nabla} = P^{\dagger} P$, with $P^{\dagger}$ adjoint of P in $L_2(\pi)$:
  $P^{\dagger}(x, x') = \frac{\pi(x')}{\pi(x)} P(x', x),\ x, x' \in \mathsf{X}$.
- $H^{\circ}(x) = \sum_{x'} [Z^{\nabla}(x, x') - Z^{\nabla}(x^{\circ}, x')]\mathcal{U}(x')\ x \in \mathsf{X}$
  where $Z^{\nabla} = [I - P^{\nabla} + 1 \otimes \pi]^{-1}$ the fundamental matrix for P^{∇}

Thm. (SPD design) If $P_0^{\nabla} = P_0^{\dagger} P_0$ is irreducible, and $P_0 = R_0$, then the linearized state-space model at any constant value ζ satisfies

$$G_{\zeta}^{+}(e^{j\theta}) + G_{\zeta}^{+}(e^{-j\theta}) \geq \sigma_{\zeta}^2, \qquad \theta \in \mathbb{R}$$

where σ_{ζ}^2 is the variance of $\mathcal{U}$ under π_{ζ} and $G^{+}(z) := zG(z)$.

The linearized aggregate model is passive: $\sum_{t=0}^{\infty} u_t y_{t+1} \geq 0,\ \forall \{u_t\}$.

Exponential family

Alternative to solving an ODE

For a function $H^\circ_{\mathrm{e}} \colon \mathsf{X} \to \mathbb{R}$, define for each x, x'_u and ζ,

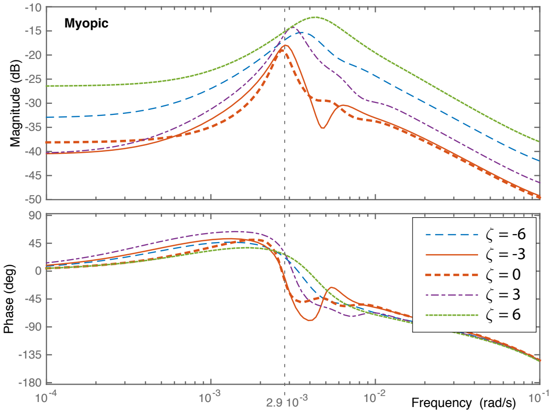
$$h_\zeta(x, x'_u) = \zeta H_{\mathrm{e}}(x'_u \mid x)$$

$$\text{with} \quad H_{\mathrm{e}}(x'_u \mid x) := \sum_{x'_n} Q_0(x, x'_n) H^\circ_{\mathrm{e}}(x'_u, x'_n)$$

- Myopic design: $H^\circ_{\mathrm{e}} = \mathcal{U}$.
- Linear approximations to the IPD or SPD solutions, with $H^\circ_{\mathrm{e}} = \mathcal{H}^\circ(P_0)$.

Myopic Design

Linearized dynamics



Bode plots for myopic design: Linearizations at five values of ζ

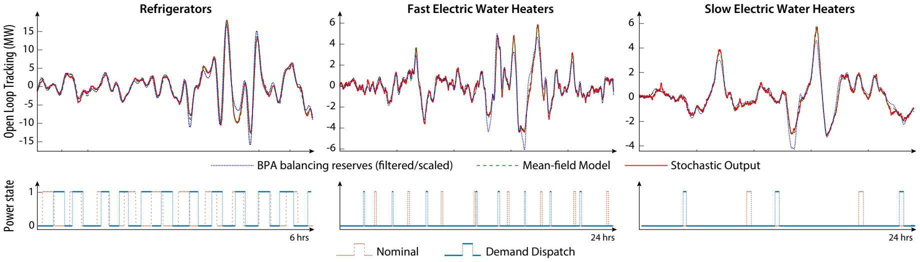
Tracking performance

and the controlled dynamics for an individual load

Heterogeneous setting:

- 40 000 loads per experiment;
- 20 different load types in each case

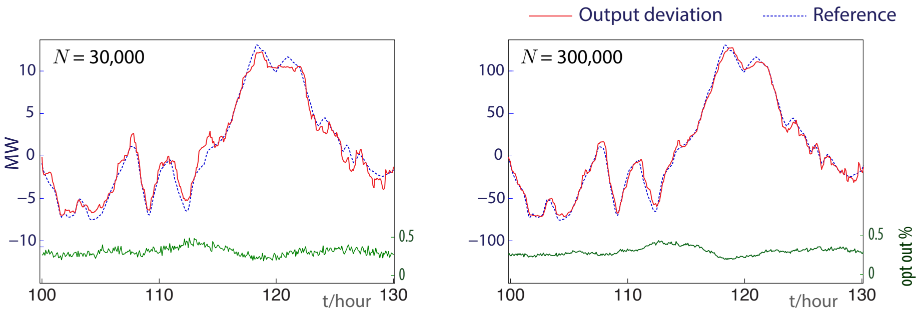
Lower plots show the on/off state for a typical load



Unmodeled dynamics

Setting: 0.1% sampling, and

1. *Heterogeneous* population of loads
2. Load i overrides when QoS is out of bounds

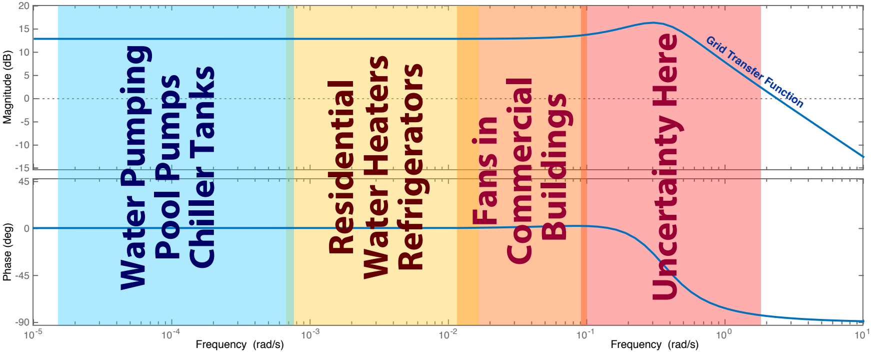


Closed-loop tracking

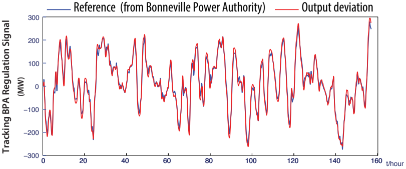
PI control: $\zeta_t = k_P e_t + k_I e^I_t, \quad e_t = r_t - y_t, \quad e^I_t = \sum_{s=0}^{t} e_s$

Control Architecture

Frequency *Allocation* for Demand Dispatch



10,000 pools



Bandwidth centered around its natural cycle

Conclusions

Virtual storage from flexible loads

Approach: creating Virtual Energy Storage through direct control of flexible loads - helping the grid while respecting user QoS

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Virtual storage from flexible loads

Approach: creating Virtual Energy Storage through direct control of flexible loads - helping the grid while respecting user QoS

Challenges:

- Stability properties for IPD and myopic design?
- Information Architecture: $\zeta_t = f(?)$
  Different needs for communication, state estimation and forecast.
- Capacity estimation (time varying)
- Network constraints
- Resource optimization & learning
  Integrating VES with traditional generation and batteries.
- Economic issues
  Contract design, aggregators, markets . . .

Conclusions

Thank You!

References: this talk

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Mean Field Model

Linearized Dynamics

Mean-field model: $\mu_{t+1} = \mu_t P_{\zeta_t}, \qquad y_t = \langle \mu_t, \mathcal{U} \rangle$

$$\zeta_t = f_t(y_0, \dots, y_t)$$

Linear state space model:

$$\Phi_{t+1} = A\Phi_t + B\zeta_t$$
$$\gamma_t = C\Phi_t$$

Interpretations: $|\zeta_t|$ is small, and π denotes invariant measure for P_0.

- $\Phi_t \in \mathbb{R}^{|\mathsf{X}|}$, a column vector with
$$\Phi_t(x) \approx \mu_t(x) - \pi(x),\ x \in \mathsf{X}$$
- $\gamma_t \approx y_t - y^0$; deviation from nominal steady-state
- $A = P_0^{\intercal}$, $C = \mathcal{U}^{\intercal}$, and input dynamics linearized:

$$B^{\intercal} = \frac{d}{d\zeta} \pi P_\zeta \Big|_{\zeta=0}$$