Obliquely Reflected Backward Stochastic Differential Equations

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Outline

BSDEs

- Backward Stochastic Differential Equation (Y, Z)

$$Y_t = \xi + \int_t^T F(t, Y_t, Z_t)\mathrm{d}t - \int_t^T (Z_t)'\mathrm{d}W_t$$

with $\xi \in L^2$ and F possibly random function.

- Markovian Setting: Forward-Backward SDEs for (b, σ, f, g) Lipschitz (say):

$$\begin{cases} X_t &= X_0 + \int_0^t b(X_u)\mathrm{d}u + \int_0^t \sigma(X_u)\mathrm{d}W_u \\ Y_t &= g(X_T) + \int_t^T f(X_t, Y_t, Z_t)\mathrm{d}t - \int_t^T (Z_t)'\mathrm{d}W_t \end{cases}$$

- Representation result: $Y_t = u(t, X_t)$ and $Z_t = \sigma(X_t)\partial_x u(t, X_t)$ with u solution to

$$\partial_t u + b(x)\partial_x u + \frac{1}{2}\mathrm{Tr}[\partial^2_{xx} u\sigma\sigma'(x)] + f(x, u, \partial_x u\sigma) = 0 \text{ and } u(T, x) = g(x) \,.$$

- Motivation: Control Problem, Pricing formula in non linear markets, Numerical probabilistic methods for PDEs, etc.

Reflected BSDEs

- One dimensional, 'Simply' reflected BSDEs on the boundary $l(X)$: (Y, Z, K)

$$Y_t = g(X_T) + \int_t^T f(X_t, Y_t, Z_t)\mathrm{d}t - \int_t^T (Z_t)'\mathrm{d}W_t + \int_t^T \mathrm{d}K_t$$

$$(\mathbf{C1})\, Y_t \geq l(X_t) \text{ (constrained value process)}$$

$$(\mathbf{C2}) \int_0^T \Big(Y_t - l(X_t)\Big)\mathrm{d}K_t = 0 \text{ ("optimality" of K)}$$

K increasing & continuous.

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- Linked to optimal stopping (pricing of US options)

$$Y_t = \operatorname{ess}\sup_{\tau \in \mathcal{T}_{[0,T]}} \mathbb{E}\Big[g(X_\tau) + \int_t^\tau f(X_t, Y_t, Z_t)\mathrm{d}t | \mathcal{F}_t\Big]$$

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- Doubly reflected BSDEs: upper boundary (Dynkin games)

Multidimensional case

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- Direction of Reflection? $n(y)$ denotes the (set of) outward normal for $y \in \partial\mathcal{D}$
  1. Normal reflection: $\mathrm{d}K_t = -\Phi_t \mathrm{d}t$, $\Phi_t \in n(Y_t)$ increasing
  2. Oblique reflection: $\mathrm{d}K_t = -H(X_t, Y_t, Z_t)\Phi_t \mathrm{d}t$, H matrix valued s.t. $\nu(X_t, Y_t, Z_t) = H_t n(Y_t)$ is the oblique outward direction.

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- Note: we require $\langle \frac{\nu}{\|\nu\|}, \frac{n}{\|n\|}\rangle \geq \epsilon > 0$.

Starting and Stopping problem (1)

Hamadene and Jeanblanc (01) - Carmona and Ludkovski (10):

- Consider e.g. a power station producing electricity whose price is given by a diffusion process X: $\mathrm{d}X_t = b(X_t)\mathrm{d}t + \sigma(X_t)\mathrm{d}W_t$
- Two modes for the power station:
  mode **1**: operating, profit is then $f^1(X_t)\mathrm{d}t$
  mode **2**: closed, profit is then $f^2(X_t)\mathrm{d}t$
  $\hookrightarrow$ switching from one mode to another has a cost: $c > 0$
- Management decide to produce electricity only when it is profitable enough.
- The management strategy is (θ_j, α_j) : θ_j is a sequence of stopping times representing switching times from mode α_{j-1} to α_j.

  $(a_t)_{0\le t\le T}$ is the state process (the management strategy).

Starting and Stopping problem (2)

- Following a strategy a from t up to T, gives

$$J(a,t) = \int_t^T f^{a_s}(X_s)\mathrm{d}s - \sum_{j\geq 0} c\mathbf{1}_{\{t\leq\theta_j\leq T\}}$$

- The optimisation problem is then (at $t=0$, for $\alpha_0 = 1$)

$$Y_0^1 := \sup_a \mathbb{E}[J(a,0)]$$

At any date $t \in [0,T]$ in state $i \in \{1,2\}$, the value function is Y_t^i.

- Y is solution of a coupled optimal stopping problem

$$Y_t^1 = \mathrm{ess}\sup_{t\leq\tau\leq T} \mathbb{E}\left[\int_t^\tau f(1,X_s)\mathrm{d}s + (Y_\tau^2 - c)\mathbf{1}_{\{\tau<T\}} \mid \mathcal{F}_t\right]$$

$$Y_t^2 = \mathrm{ess}\sup_{t\leq\tau\leq T} \mathbb{E}\left[\int_t^\tau f(2,X_s)\mathrm{d}s + (Y_\tau^1 - c)\mathbf{1}_{\{\tau<T\}} \mid \mathcal{F}_t\right]$$

Oblique RBSDE

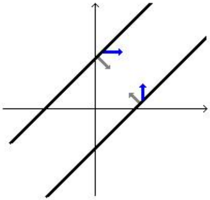
- Y is the solution of the following system of reflected BSDEs:

$$Y_t^i = \int_t^T f^i(X_s, Y_s, Z_s)\mathrm{d}s - \int_t^T (Z_s^i)'\mathrm{d}W_s + \int_t^T \mathrm{d}K_s^i \;,\; i \in \{1,2\}\;,$$

$$(\mathbf{C1})\; Y_t^1 \geq Y_t^2 - c \text{ and } Y_t^2 \geq Y_t^1 - c \quad \text{(constraint - coupling)}$$

$$(\mathbf{C2})\; \int_0^T \Big(Y_s^1 - (Y_s^2 - c)\Big)\mathrm{d}K_s^1 = 0 \text{ and } \int_0^T \Big(Y_s^2 - (Y_s^1 - c)\Big)\mathrm{d}K_s^2 = 0$$

- d=2:



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- More generally, d modes, the convex set is

$$\mathcal{D} = \{ \, y \in \mathbb{R}^d | y^i \geq \max_j (y_j - c) \ , \ 1 \leq i \leq d \, \}$$

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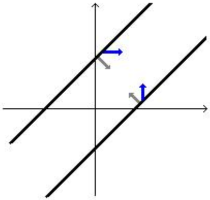

Randomised switching

- $d \geq 3$ modes, (ξ_n) homogeneous Markov Chain on $\{1, \dots, d\}$ s.t. $\mathbb{P}(\xi_n = j | \xi_{n-1} = i) = p_{ij}$ and $p_{ii} = 0$.

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- $Y_0 = \sup_a \mathbb{E}[J(a, 0)]$, J reward following strategy a.
- The solution is given by an obliquely reflected BSDEs with domain

$$\mathcal{D} := \{y \in \mathbb{R}^d \mid y^j > \sum_i p_{ji} y^i - c\}$$

The reflection is along the axis.

Known results

- Normal Reflection: Existence and uniqueness in classical L^2, Lipschitz setting for a fixed domain (Gegout-Petit & Pardoux 95),
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- Oblique Reflection:
  1. Reflection in an orthant: Ramasubramanian (02), $f := f(Y)$.
  2. Switching problem: $f^i(Y, Z) = f^i(Y, Z^i)$
     (do not cover non-zero sum game)
     contributions by several authors: Hu & Tang (08), Hamadene & Zhang (09), C., Elie & Kharroubi (11)
  3. Attempt to general case: existence of a (weak) solution when $f = f(Y)$ and $H = H(t, Y)$ is Lipschitz in the Markovian setting.
     By Gassous, Rascanu & Rotenstein (15)

Outline

Penalised BSDEs

- Let $y \mapsto \varphi_n(y) = nd^2(y, \mathcal{D})$, $\nabla \varphi_n(y) = 2n(y - \mathcal{P}(y))$, $\mathcal{P}$ normal projection:

$$Y_t^n = g(X_T) + \int_t^T f(Y_s^n, Z_s^n)\mathrm{d}s - \int_t^T Z_s^n \mathrm{d}W_s - \int_t^T H(Y_s^n, Z_s^n)\nabla\varphi_n(Y_s^n)\mathrm{d}s$$

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- A priori estimates yields $(Y, Z) \in \mathcal{S}^2 \times \mathcal{H}^2$ with norms uniform in n and

$$\sup_t \mathbb{E}\Big[nd^2(Y_t^n, \mathcal{D}) + \int_t^T |\nabla\varphi_n(Y_s^n)|^2 \mathrm{d}s\Big] \leq C$$

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- Is there a limit and does it satisfy an oblique RBSDE?
- Show that $u^n(t, x)$ where $u^n(t, X_t) = Y_t^n$ is a Cauchy sequence.
  $\hookrightarrow$ Need σ non degenerated, use weak convergence: idea from Hamadene, Lepeltier, Peng (97) in the red book...

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- Apply Ito's formula to $\Gamma_t(\alpha, \beta)(Y_t - Y'_t)^\top H(t, Y_t)^{-1}(Y_t - Y'_t)$ where

$$\Gamma_t = e^{\alpha t + \beta \int_0^t \left(|\Phi_s| + |\Phi'_s| + |Z_s|^2\right)\mathrm{d}s}.$$

Stability result

Small time result

- Main question: Is Γ integrable? $\rightarrow$ In fact the most difficult part is: $e^{\beta \int_0^t |Z_s|^2 \mathrm{d}s}$ integrable?
- We rely on BMO techniques and we are able to get that integrability holds for all $\delta \leq c(\beta, \Lambda)$ (specialy does not depend on the Lipschitz constant of g).
- This leads to

$$\sup_{t \leq \delta} \mathbb{E}\Big[\|\delta Y_t\|^2\Big] \leq C \mathbb{E}\Big[|g(X_T) - \xi'|^4\Big]^{\frac{1}{2}}$$

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Arbitrary time result

- Divide interval $[0, T]$ in small interval and apply stability result using Backward induction: Importantly $c(\beta, \Lambda)$ is fixed in this procedure.

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Non markovian case:

- Use the stability approach to obtain that Y^n is a Cauchy sequence (smooth setting for H and $\mathcal{D}$)
- Existence for bounded terminal condition, f Lipschitz and sublinear in z.
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Various application to switching problem: RBSDEs for randomised switching is well posed,

For $d = 2$ and RBSDEs for switching problem, existence and uniqueness for full dependence in Z for f.