Supply and Shorting in Speculative Markets

Marcel Nutz
Columbia University

with Johannes Muhle-Karbe (Part I)
and José Scheinkman (Parts II, III)

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Outline

Static Model

Consider

- Agents $i \in \{1, 2, \ldots, n\}$
- Using distributions Q_i for the state $X(t)$
- Trading an asset with a single payoff $f(X(T))$ at time T
- The asset cannot be shorted and is in supply $s > 0$

Static case: Suppose the agents trade only once, at time $t = 0$

Equilibrium:

- Determine an equilibrium price p for f and portfolios $q_i \in \mathbb{R}_+$
- such that q_i maximizes $q(E_i[f(X(T))] - p)$ over $q \geq 0$, for all i
- and the market clears: $\sum_i q_i = s$

Static Equilibrium

Solution: The most optimistic agent determines the price (Miller '77),

$$p = \max_i E_i[f(X(T))]$$

- Let $i_* \in \{1, 2, \ldots, n\}$ be the maximizer
- With portfolios $q_{i_*} = s$ and $q_i = 0$ for $i \neq i_*$, this in an equilibrium
- It is unique (modulo having several maximizers).

Note:

- At price p, the optimist is invariant and will accept any portfolio
- All other agents want to have $q_i = 0$
- Price not affected by supply

Preview: The Resale Option (Harrison and Kreps '78)

- When there are several trading dates, the relatively most optimist agent depends on date and state
- Option to resell the derivative to another agent at a later time
- Adds to the static price: speculative bubble

Scheinkman and Xiong '03, '04

Continuous-Time Model

Asset can be traded on $[0, T]$.

Agents:

- Risk-neutral agents $i \in \{1, \ldots, n\}$ using models Q_i
- Here: agent i uses a local vol model Q_i for X,

$$dX(t) = \sigma_i(t, X(t))\, dW_i(t), \quad X(0) = x$$

Equilibrium:

- Find a price process $P(t)$ with $P(T) = f(X(T))$ Q_i-a.s.
- Agents choose portfolio processes Φ
- such as to optimize expected P&L: $E_i[\int_0^T \Phi(t)\, dP(t)]$
- Market clearing $\sum_i \Phi_i(t) = s$

Existence

Theorem: There exists a unique equilibrium price $P(t) = v(t, X(t))$, and v is the solution of

$$v_t(t,x) + \sup_{i \in \{1,\dots,n\}} \frac{1}{2}\sigma_i^2(t,x) v_{xx}(t,x) = 0, \quad v(T,\cdot) = f.$$

The optimal portfolios $\Phi_i(t) = \phi_i(t, X(t))$ are given by

$$\phi_i(t,x) = \begin{cases} s, & \text{if } i \text{ is the maximizer at } (t,x) \\ 0, & \text{else} \end{cases}$$

- Derivative held by the locally most optimistic agent at any time
- Agents trade as this role changes

Speculative Bubble

- The control representation (or comparison) shows that

$$P(0) \geq \max_i E_i[f(X(T))]$$

- Thus, the dynamic price exceeds the static equilibrium
- This "speculative bubble" can be attributed to the resale option

Shortcomings:

- Supply should diminish price, not reflected in this model
- Need (risk) aversion against large positions

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Model

Cost-of-Carry: For holding a position $y = \Phi(t)$ at time t, agents must pay an instantaneous cost

$$c(y) = \begin{cases} \frac{1}{2\alpha_+} y^2, & y \geq 0, \\ \infty, & y < 0. \end{cases}$$

Equilibrium: Agents optimize expected P&L − cost:

$$E_i \left[\int_0^T \Phi(t)\, dZ(t) - \int_0^T c(\Phi(t))\, dt \right].$$

Existence

Theorem: • There exists a unique equilibrium price $P(t) = v(t, X(t))$, and v is the solution of

$$v_t + \sup_{\emptyset \neq J \subseteq \{1,\dots,n\}} \left\{ \tfrac{1}{2} \tfrac{1}{|J|} \sum_{i \in J} \sigma_i^2 v_{xx} - \tfrac{s}{|J| \alpha_+} \right\} = 0.$$

• The optimal portfolios $\Phi_i(t) = \phi_i(t, X(t))$ are unique and given by

$$\phi_i(t,x) = \left\{ \alpha_+ \mathcal{L}^i v(t,x) \right\}^+$$

where $\mathcal{L}^i v(t,x) = \partial_t v(t,x) + \frac{1}{2} \sigma_i^2 \partial_{xx} v(t,x)$

Delay Effect

- The resale option is still present. Nevertheless...
- the dynamic equilibrium price can be below the static one
- "Delay effect"

Outline

Cost-of-Carry:

- For holding a position $y = \Phi(t)$ at time t, agents must pay an instantaneous cost

$$c(y) = \begin{cases} \frac{1}{2\alpha_+} y^2, & y \geq 0, \\ \frac{1}{2\alpha_-} y^2, & y < 0. \end{cases}$$

- Short is more costly than long: $\alpha_- \leq \alpha_+$

Existence

Theorem: • There exists a unique equilibrium price $Z(t) = v(t, X(t))$, and v is the solution of

$$v_t(t,x) + \sup_{I \subseteq \{1,\dots,n\}} \Big\{ \tfrac{1}{2}\Sigma_I^2(t,x) v_{xx}(t,x) - \kappa_I(t,x) \Big\} = 0, \quad v(T,\cdot) = f,$$

where the coefficients are defined as

$$\kappa_I(t,x) = \frac{s(t,x)}{|I|\alpha_- + |I^c|\alpha_+},$$

$$\Sigma_I^2(t,x) = \tfrac{\alpha_-}{|I|\alpha_- + |I^c|\alpha_+} \sum_{i \in I} \sigma_i^2(t,x) + \tfrac{\alpha_+}{|I|\alpha_- + |I^c|\alpha_+} \sum_{i \in I^c} \sigma_i^2(t,x).$$

• The optimal portfolios $\Phi_i(t) = \phi_i(t, X(t))$ are unique and given by

$$\phi_i(t,x) = \alpha_{\mathrm{sign}(\mathcal{L}^i v(t,x))} \mathcal{L}^i v(t,x), \quad \mathcal{L}^i v(t,x) = \partial_t v(t,x) + \tfrac{1}{2}\sigma_i^2 \partial_{xx} v(t,x).$$

Conclusion

Part I:

- Resale option leads to UVM price in a tractable model

Parts II–III: A tractable model where

- Supply affects the price as a running cost
- Delay effect can depress the dynamic equilibrium price
- Short-selling is possible and may further depress the price

Thank you

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Thank you