

# LiDAR based relative pose and covariance estimation for communicating vehicles exchanging a polygonal model of their shape

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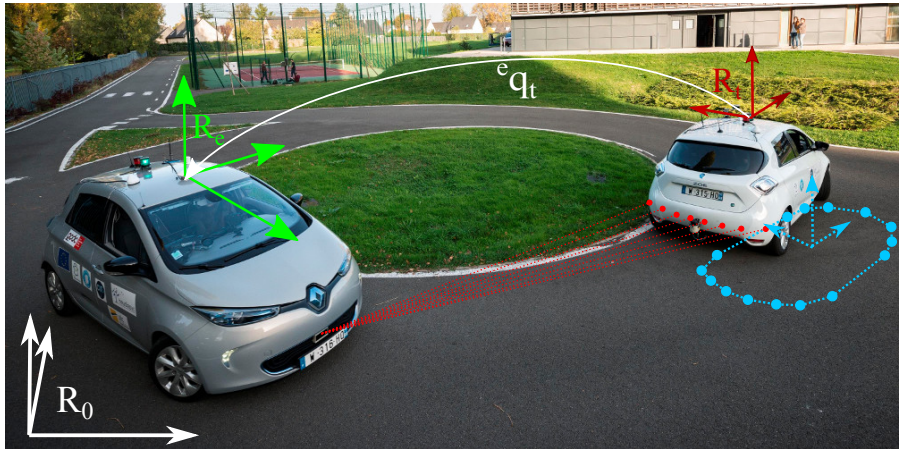
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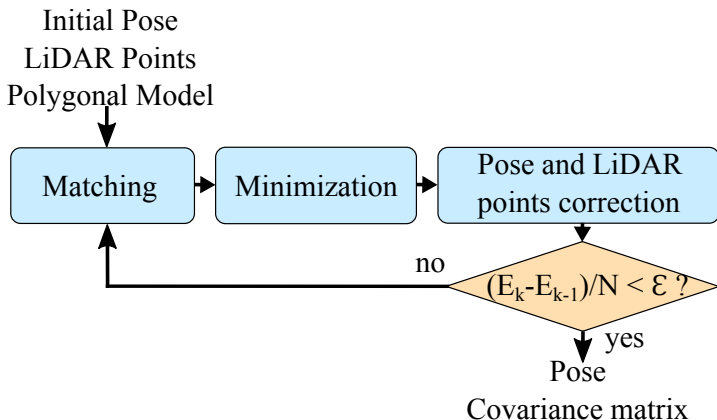


# Problem statement



- Iterative optimization
  - Matching
  - Minimization
    - Minimization using polynomial roots
    - Minimization using pseudo-inverse matrix
    - Covariance matrix approximation
  
- Simulation Results
  - Scenarios
  - Errors and consistencies

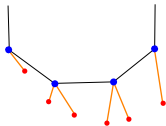
# Iterative optimization



# Matching

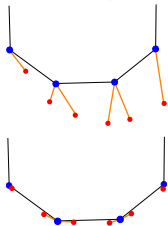
# Matching

ICP  
Point-to-point



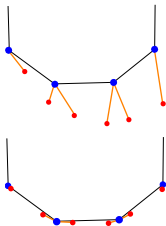
# Matching

ICP  
Point-to-point

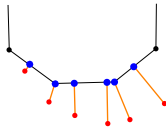


# Matching

ICP  
Point-to-point



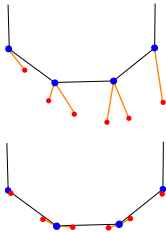
ICPP  
Point-to-projection



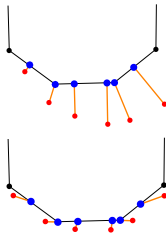


# Matching

ICP  
Point-to-point

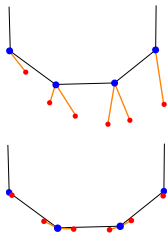


ICPP  
Point-to-projection

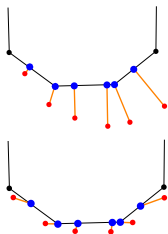


# Matching

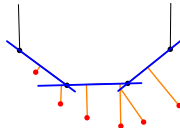
ICP  
Point-to-point



ICPP  
Point-to-projection

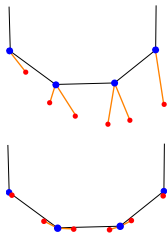


PLICP  
Point-to-line

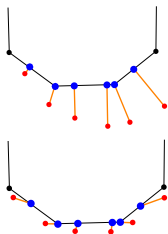


# Matching

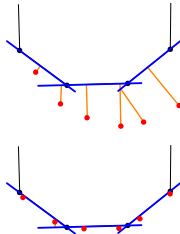
ICP  
Point-to-point



ICPP  
Point-to-projection

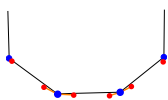
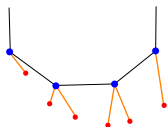


PLICP  
Point-to-line

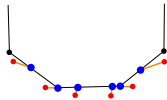
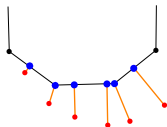


# Matching

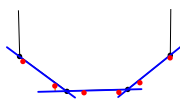
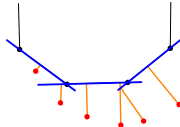
ICP  
Point-to-point



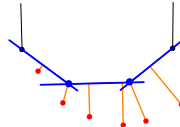
ICPP  
Point-to-projection



PLICP  
Point-to-line

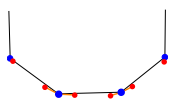
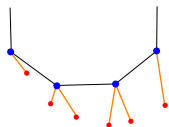


mixICP  
Mix-matching

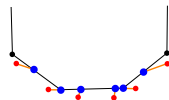
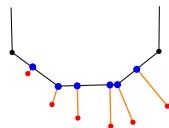


# Matching

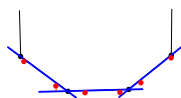
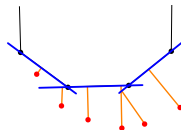
ICP  
Point-to-point



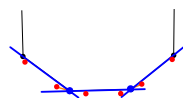
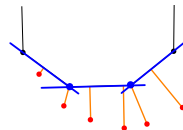
ICPP  
Point-to-projection



PLICP  
Point-to-line



mixICP  
Mix-matching



# Minimization

Minimization problem:

$$\hat{q} = \arg \min_q E(q) = \arg \min_q \sum_{i=1}^n d(q; p_i, M)$$

Euclidean distance

$$d(q, p_i, M) = \|m_j - \Delta T p_i\|^2$$

Orthogonal distance

$$d(q, p_i, M) = ((m_j - \Delta T p_i) \cdot n_j)^2$$

Transformation:

$$\Delta T = \begin{bmatrix} \cos(\Delta\theta) & -\sin(\Delta\theta) & \Delta x \\ \sin(\Delta\theta) & \cos(\Delta\theta) & \Delta y \\ 0 & 0 & 1 \end{bmatrix}$$

## Minimization using polynomial roots

With  $q_{4D} = [x \ y \ \cos(\theta) \ \sin(\theta)]^T$

$$\begin{cases} \min_{q_{4D}} & E(q_{4D}) = q_{4D}^T A q_{4D} + b^T q_{4D} + c \\ \text{subject to} & q_{4D}^T W q_{4D} = 1 \end{cases}$$



A. Censi. An ICP variant using a point-to-line metric. In IEEE International Conference on Robotics and Automation, pages 19–25, May 2008.

## Minimization using pseudo-inverse matrix

Using the 1st order small-angle approximation:

$$\min_q \|Aq - b\|^2$$

Then

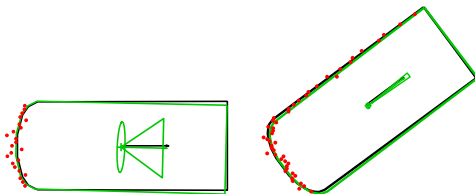
$$\hat{q} = \text{pinv}(A)b$$



K.L. Low. Linear Least-Squares Optimization for Point-to-Plane ICP Surface Registration. Technical Report TR04-004, Department of Computer Science University of North Carolina at Chapel Hill, February 2004.



# Covariance matrix approximation



$$\hat{\Sigma} = \frac{E(\hat{q})}{n-k} \left( \frac{\partial^2 E(\hat{q})}{\partial q^2} \right)^{-1}$$

$E(q)$ : cost function,

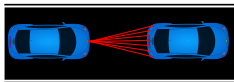
$n$ : number of LiDAR Points

$k$ : number of parameters (3 in 2D)

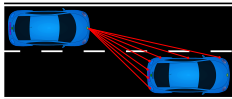


O. Bengtssons and A.J. Baerveldt. Robot localization based on scan-matching-estimating the covariance matrix for the IDC algorithm. *Robotics and Autonomous Systems*, 44(1):29–40, July 2003.

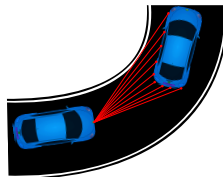
# Scenarios



Straight lane

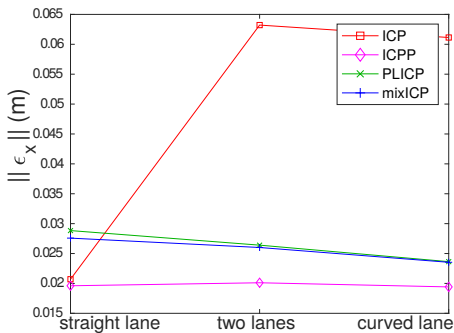


two lane



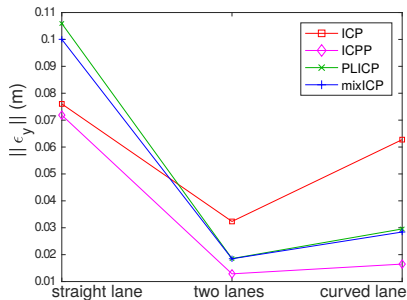
curved lane

# Results

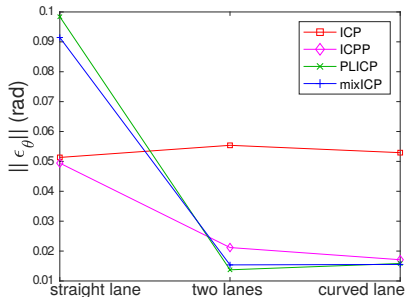


Longitudinal error

# Results



Lateral error



Heading error

## Errors and consistencies

Consistency test:

$$({}^e q_t - {}^e \hat{q}_t)^T e \hat{\Sigma}_t^{-1} ({}^e q_t - {}^e \hat{q}_t) < \chi_{3,0.05}^2$$

		ICP	ICPP	PLICP	mixICP
Polynomial minimization	$\ \bar{\epsilon}\ $ (cm)	8.2	7.8	13.7	11.0
	consistency (%)	85.5	58.8	69.8	70.0
Pseudo-inverse minimization	$\ \bar{\epsilon}\ $ (cm)	8.2	7.8	11.5	10.8
	consistency (%)	93.5	83.9	91.6	89.8

The minimization using pseudo-inverse matrix with point-to-line matchings gives the best consistencies.

## Conclusion

- The relative pose and covariance matrix estimation using an iterative minimization algorithm was tested with different matching and minimization methods.
- The geometry of the vehicle is well represented by the point-to-line matching. The approximation of the covariance matrix is then more consistent.
- The minimization using a pseudo-inverse matrix formulation is more accurate and consistent.
- When two sides of the vehicle are seen by the perception sensor, the estimated pose becomes more accurate.

**Thank you for your attention!**

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