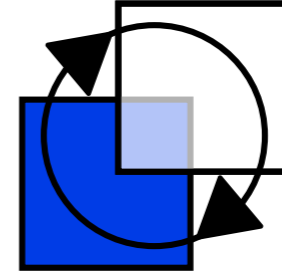




UNIVERSITÄT  
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**Reactive  
Systems  
Group**

# Causality and Hyperproperties

Norine Coenen

Shonan Meeting No. 139  
Causal Reasoning in Systems

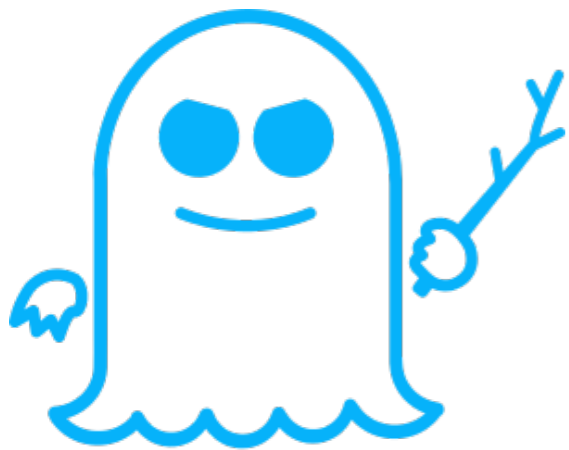
# Meltdown and Spectre



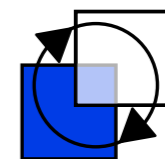
Many processors (Intel, Arm, ...) are **vulnerable** although **proven correct**

Trace properties

Side channel attacks



Attacks **compare multiple executions**



# Hyperproperties and Properties

Hyperproperty (Clarkson and Schneider)

**Set of sets of traces**  $H \subseteq 2^{\Sigma^\omega}$

$$T \models H \Leftrightarrow T \in H$$

**Compare multiple traces**

Ex: Require at least two different traces:

$$\{T \subseteq \Sigma^\omega \mid \exists t, t' \in T. t \neq t'\}$$

Trace property

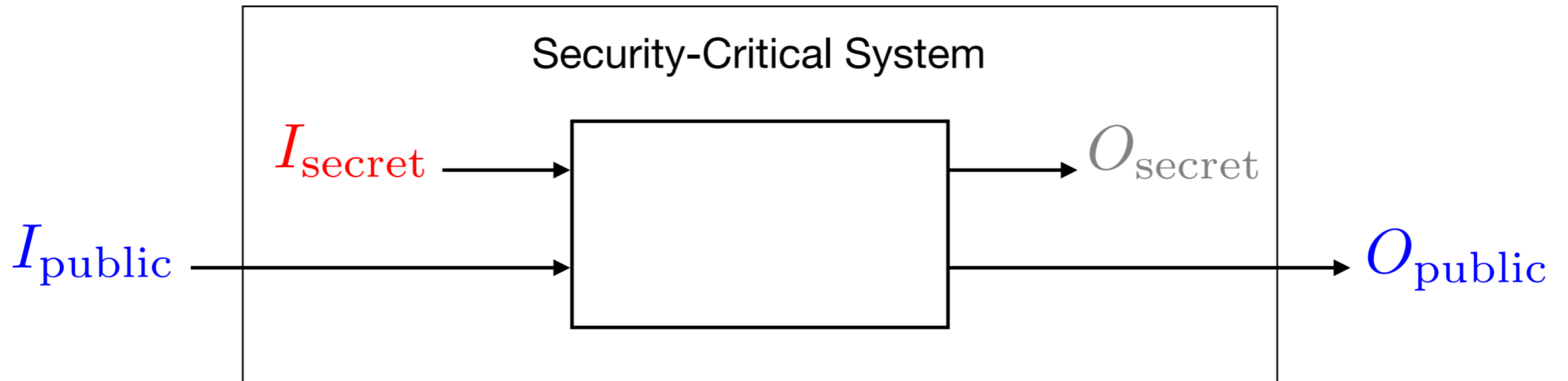
**Set of traces**  $P \subseteq \Sigma^\omega$

$$T \models P \Leftrightarrow T \subseteq P$$

No trace comparison possible

Ex: Cannot require two different traces

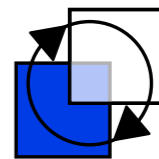
# Information-Flow Control



Public output should only depend on public input

Typical information-flow property: Noninterference

$$\{T \subseteq \Sigma^\omega \mid \forall t, t' \in T : t =_{I_{\text{public}}} t' \Rightarrow t =_{O_{\text{public}}} t'\}$$



# Other Hyperproperties

## Cleanliness (software doping)

Do traces with similar inputs also have similar outputs?

## Symmetry in protocols

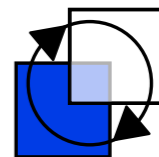
Are clients treated symmetrically?

## Error-resistant codes

Do codes for distinct inputs have at least Hamming distance  $d$ ?

## Promptness

Is there a common bound on the number of steps until a requirement is satisfied?



# Causality

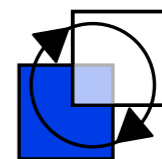
## Counterfactual dependence

**Hume:** “We may define a cause to be an object followed by another ...  
where, if the first object had not been, the second never had existed.”

Compare **actual world** with a **different possible world**

Cause and effect both occur

Cause and effect both do **not** occur



# Causality

## Contingencies

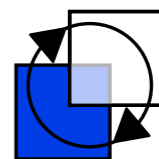
**Halpern and Pearl:**

Restrict possible alternate worlds to those **satisfying the contingencies**

## Transition systems

**Leitner-Fischer and Leue:**

Restrict possible alternate worlds to the **traces of a transition system**



# Causality Definition

Let  $\psi$  range over  $Z$  and  $\mathcal{A} \setminus Z = W$ .

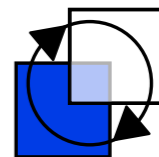
$\psi$  causes  $\varphi$  iff

$$\text{AC1: } \exists t. t \models \psi \wedge t \models \varphi$$

$$\text{AC2(1): } \exists t'. t' \not\models \psi \wedge t' \not\models \varphi \wedge \\ (\text{val}_Z(t) \neq \text{val}_Z(t') \vee \\ \text{val}_W(t) \neq \text{val}_W(t'))$$

$$\text{AC2(2): } \forall t''. t'' \models \psi \wedge \text{val}_Z(t) = \text{val}_Z(t'') \\ \wedge \text{val}_W(t) \neq \text{val}_W(t'') \rightarrow t'' \models \varphi$$

AC3: Minimality of  $\psi$





# Temporal Logics for Hyperproperties

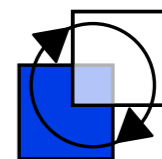
Noninterference

$$\{T \subseteq \Sigma^\omega \mid \forall t, t' \in T : t =_{I_{\text{public}}} t' \Rightarrow t =_{O_{\text{public}}} t'\}$$

Is there an **appropriate logic** for the expression of hyperproperties?



“All executions have the light on at the same time.”



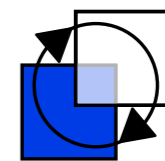
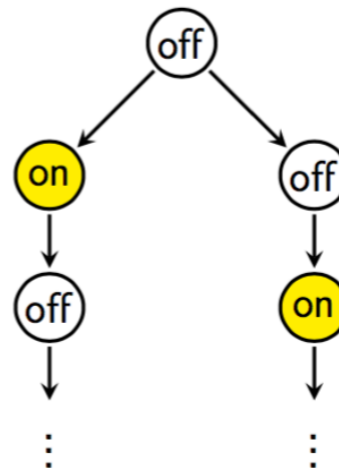
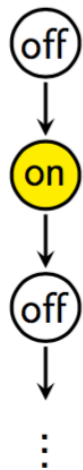
# LTL



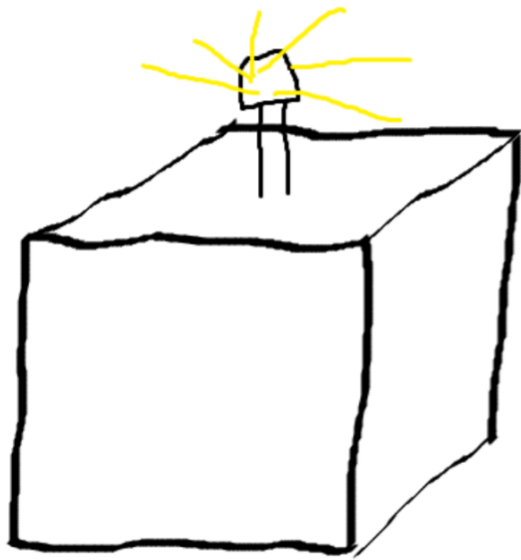
“All executions have the light on at the same time.”

LTL: Specifies computations

Syntax:  $\varphi ::= a \mid \bigcirc \psi \mid \square \psi \mid \diamond \psi \mid \psi \mathcal{U} \psi \mid \dots$



# CTL\*

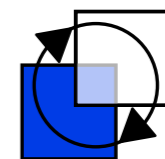
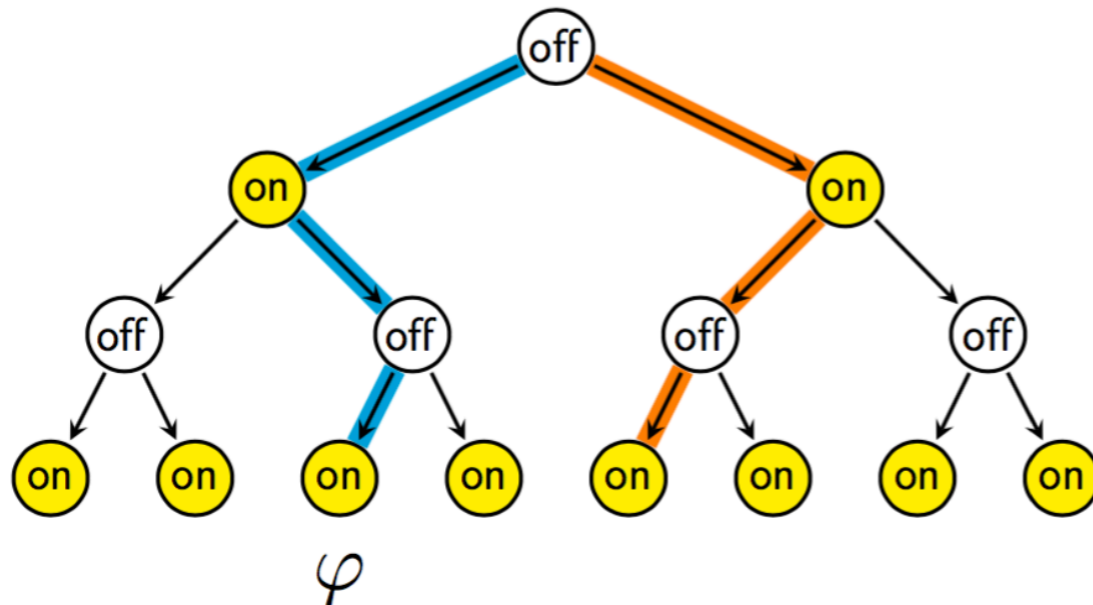


“All executions have the light on at the same time.”

$AA \varphi ?$

CTL\*: Specifies computation trees

Syntax:  $\varphi ::= a \mid A\varphi \mid E\varphi \mid O\varphi \mid \square\varphi \mid \varphi \mathcal{U} \varphi \mid \dots$



# HyperLTL

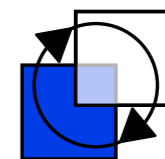
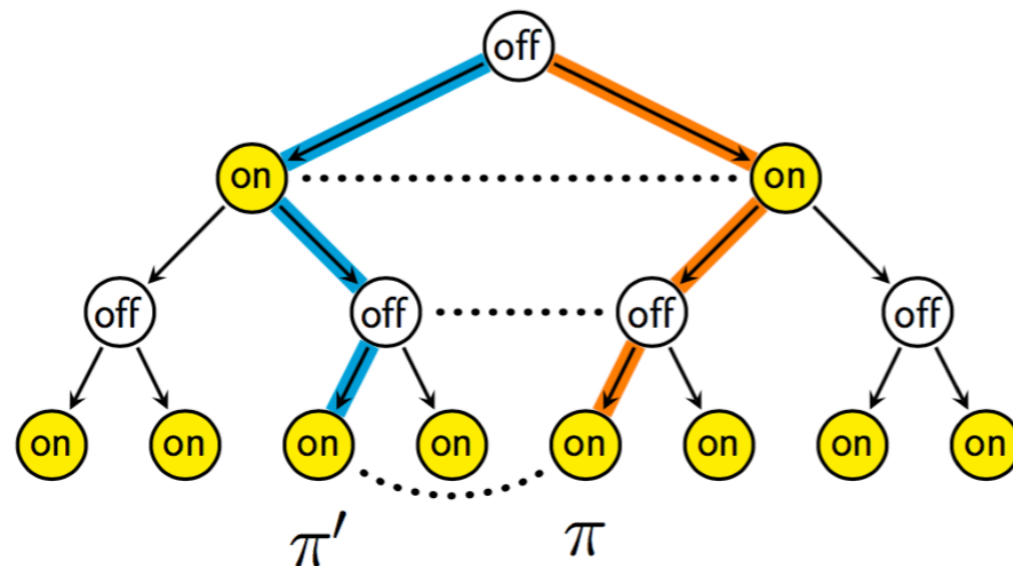
Add trace quantifiers:  $\forall \pi. \varphi$   $\exists \pi. \varphi$

Syntax:  $\varphi ::= \forall \pi. \varphi \mid \exists \pi. \varphi \mid \psi$

$\psi ::= a_\pi \mid \bigcirc \psi \mid \square \psi \mid \diamond \psi \mid \psi \mathcal{U} \psi \mid \dots$

“All executions have the light on at the same time.”

$$\forall \pi. \forall \pi'. \square (\text{on}_\pi \leftrightarrow \text{on}_{\pi'})$$



# HyperLTL

Require at least two different traces in a system:

$$\{T \subseteq \Sigma^\omega \mid \exists t, t' \in T. t \neq t'\}$$

In HyperLTL:  $\exists \pi. \exists \pi'. \pi \neq \pi'$

$$\pi = \pi' := \Box \left( \bigwedge_{a \in AP} a_\pi \leftrightarrow a_{\pi'} \right)$$

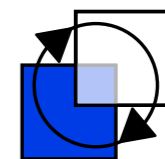
Semantics:

trace assignment  $\Pi : Vars \rightarrow T$

$\Pi \models_T a_\pi$  iff  $a \in \Pi(\pi)(0)$

$\Pi \models_T \Box \varphi$  iff  $\forall i \geq 0 : \Pi[i, \infty] \models_T \varphi$

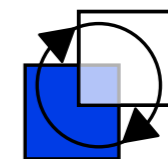
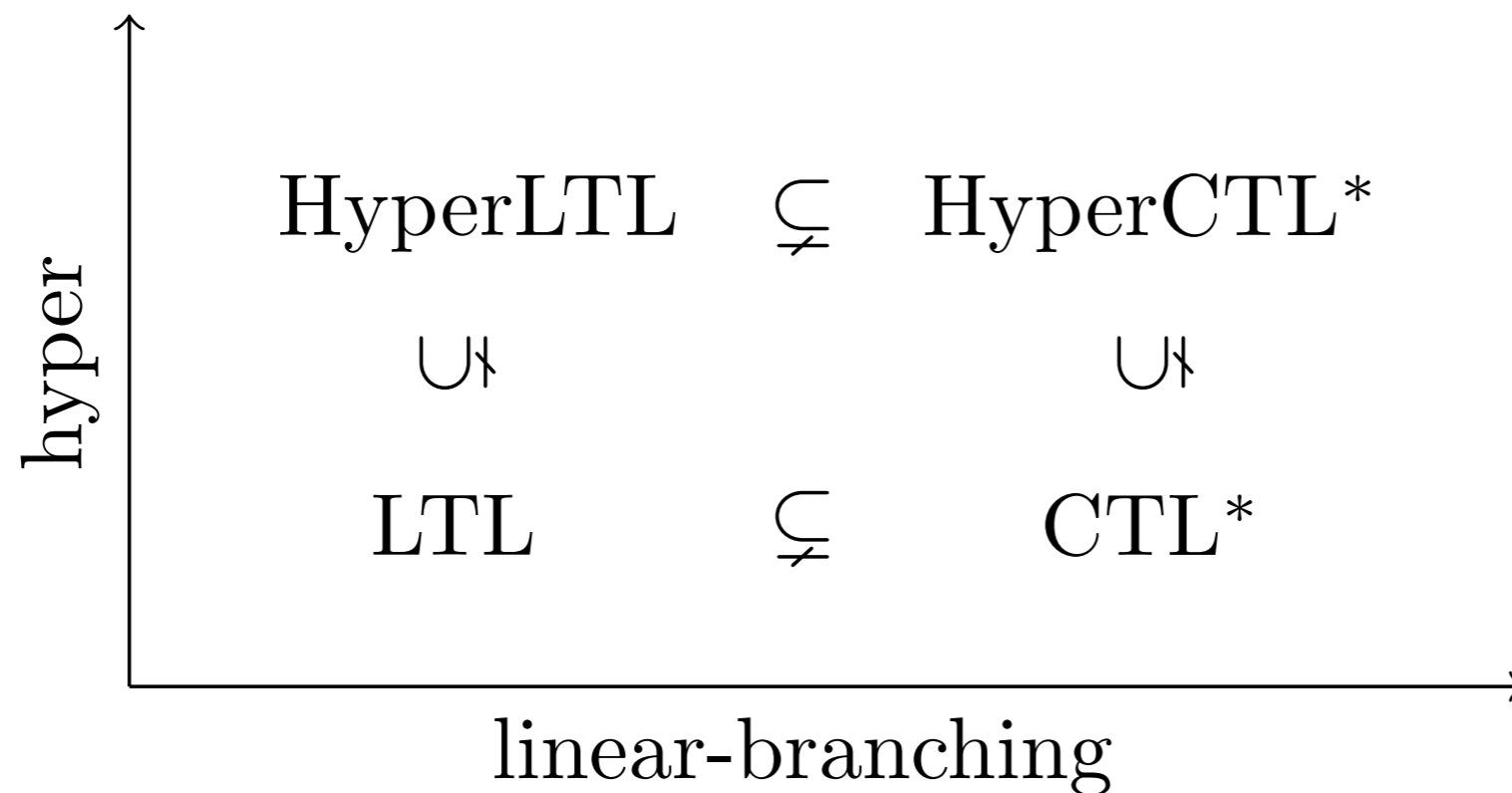
$\Pi \models_T \forall \pi. \varphi$  iff  $\forall t \in T : \Pi[\pi \mapsto t] \models_T \varphi$



# HyperCTL\*

Add **path variables** to path quantifiers:

Syntax:  $\varphi ::= a_\pi \mid \forall \pi. \varphi \mid \exists \pi. \varphi \mid \bigcirc \varphi \mid \square \varphi \mid \varphi \mathcal{U} \varphi \mid \dots$



# Causality in HyperLTL

Let  $\psi$  range over  $Z$  and  $\mathcal{A} \setminus Z = W$ .

$\psi$  causes  $\varphi$  iff

$$\text{AC1: } \exists t. t \models \psi \wedge t \models \varphi$$

$$\text{AC2(1): } \exists t'. t' \not\models \psi \wedge t' \not\models \varphi \wedge \\ (val_Z(t) \neq val_Z(t') \vee \\ val_W(t) \neq val_W(t'))$$

$$\text{AC2(2): } \forall t''. t'' \models \psi \wedge val_Z(t) = val_Z(t'') \\ \wedge val_W(t) \neq val_W(t'') \rightarrow t'' \models \varphi$$

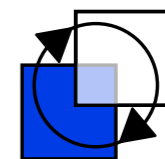
$$\exists \pi. \exists \pi'. \forall \pi''.$$

$$\psi_\pi \wedge \varphi_\pi \wedge$$

$$\neg \psi_{\pi'} \wedge \neg \varphi_{\pi'} \wedge \pi \neq \pi' \wedge$$

$$\psi_{\pi''} \wedge \pi =_Z \pi'' \rightarrow \varphi_{\pi''}$$

$$\pi =_Z \pi' := \square \left( \bigwedge_{z \in Z} z_\pi \leftrightarrow z_{\pi'} \right)$$



# Results and Tools

## Satisfiability

- Decidability results for HyperLTL, HyperCTL\*, HyperQPTL
- EAHyper
- MGHyper

## Model Checking

- Automata-based algorithm for HyperCTL\*
- MCHyper: Alternation-free fragment of HyperLTL

Recently extended to one quantifier alternation

- MCQHyper: Model checking of quantitative hyperproperties → Responsibility

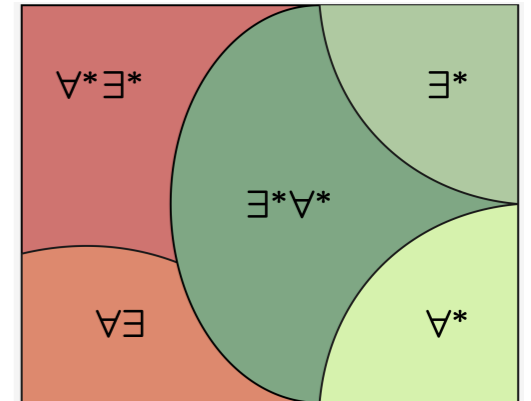
## Synthesis

- Decidability results
- BoSyHyper

## Runtime Monitoring

- RVHyper

How can we use hyperproperties for causality checking?



Further information: <https://www.react.uni-saarland.de>