Computational Methods for Blood flow

Associated Team “Cardio”

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Associated Team “Cardio”

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Coordinator
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Members
• Stanford (CVBRL, Dept Bioengineering & Mech Engng)
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• INRIA:
  - Project-team REO: J-F. Gerbeau, M. Fernández, I. Vignon-Clementel, M. Astorino, C. Bertoglio, G. Troianowski
  - Project-team MACS: D. Chapelle, P. Moireau
Associated Team “Cardio”

Similar goals and complementary approaches
- Modeling the blood flow in large arteries
- Interaction simulation / medical data

Collaboration themes
- Boundary conditions (fluid & fluid-structure)
- Advance post-processing techniques
- Image-based fluid-structure interaction
Outline

- Surgical planning
- Fluid-Structure Interaction in blood flows
- Medical Data assimilation / Inverse problems
- Viscoelasticity (Rashmi Raghu, Stanford)
Surgical planning

Glenn-Fontan surgery
- congenital heart disease
- multi-step complex procedure

Numerical simulations
- Patient-specific geometries
- Forecast pressure drop, flow split, wall-shear stress

Troianowski, Taylor, Feinstein, Vignon-Clementel
Stanford/INRIA

http://www.americanheart.org
Geometry from 5 patients MRI data

Internships of A. Birolleau, G. Trojanowski
Surgical planning

Y-graft in general improves:
- energy efficiency
- hepatic flow distribution
- SVC pressure under rest & exercise

Troianowski, Taylor, Feinstein, Vignon-Clementel
Fluid-Structure Interaction

\[ \rho^f \left( \frac{\partial \mathbf{u}}{\partial t} \bigg|_{\mathbf{x}} + (\mathbf{u} - \mathbf{w}) \cdot \nabla \mathbf{u} \right) - 2\mu \text{div} \mathbf{\epsilon}(\mathbf{u}) + \nabla p = 0, \quad \text{in} \quad \Omega^f(t) \]

\[ \text{div} \mathbf{u} = 0, \quad \text{in} \quad \Omega^f(t) \]

\[ \rho^s \frac{\partial^2 \mathbf{d}}{\partial t^2} - \text{div} \left( \mathbf{F}(\mathbf{d}) \mathbf{S}(\mathbf{d}) \right) = 0, \quad \text{in} \quad \hat{\Omega}^s \]
Blood flow in aorta

- Many works about fluid boundary conditions...
- but not that many about wall!

Typical b.c. on the external part of the vessel:

\[ \sigma_s \mathbf{n} = p_0 \mathbf{n} \]

A simple and affordable way to model the external tissues:

\[ \sigma_s \mathbf{n} = -k_s \mathbf{d} - c_s \frac{\partial \mathbf{d}}{\partial t} \]

Moireau, Xiao, Astorino, Figueroa, Chapelle, Taylor, JFG, (Biomech Model Mech 2011)
Blood flow in aorta
Comparison simulation / images

Legend:
- Solid: Contours without support
- Dashed: Contour with support
- Blue: Pressure with support (mmHg)
- Light blue: Pressure without support (mmHg)
- Red: Flow with support (cc/s)
- Brown: Flow without support (cc/s)

Moireau, Xiao, Astorino, Figueroa, Chapelle, Taylor, JFG, (Biomech Model Mech 2011)
Medical Data Assimilation

Data assimilation

- Reduce model uncertainties using observations
- Access to “hidden” quantities
- Smooth the data
Data assimilation in FSI

- FSI dynamical system: 
  \[
  \begin{align*}
  B\dot{X} &= A(X, \theta) + R \\
  X(0) &= X_0
  \end{align*}
  \]

- Time discretization: 
  \[X^{n+1} = F^{n+1}(X^n, \theta)\]

- State variable: 
  \[X = [u, p, d^f, d, \nu]\]

- Parameters: 
  \[\theta = [\text{Young modulus, viscosity, boundary conditions, ...}]\]

- Uncertainties on the initial condition \(X_0\) and the parameters \(\theta\)

- Partial observations of \(X\): 
  \[Z = H(X)\]
Data assimilation

- **State estimation** through “simple” feedback terms. For example, for velocity measurements:

\[
M \ddot{X} + KX = f - \gamma H^T (H \dot{X} - Y_{obs})
\]

*Moireau, Chapelle, Le Tallec, 2008*

- **Parameter estimation**: reduced Unscented Kalman filter (SEIK)

*Dinh Tuan Pham, 2001
Moireau, Chapelle, 2009*

- With respect to a variational approach: no tangent, no adjoint
- Counterpart: as many resolutions as parameters (“particles”)...
- ... but easy to run them in parallel
**Data assimilation**

### Summary

- **Prediction:**
  \[
  \begin{align*}
  \hat{X}^{n+1}_- &= \hat{F}^{n+1}(\hat{X}^n, \theta^n, Z^{n+1}), \\
  \hat{\theta}^{n+1}_- &= \hat{\theta}^n
  \end{align*}
  \]

- **Correction:**
  \[
  \begin{align*}
  \hat{X}^{n+1}_+ &= \hat{X}^{n+1}_- + \hat{K}^{n+1}_X (Z^{n+1} - H(\hat{X}^{n+1}_-)), \\
  \hat{\theta}^{n+1}_+ &= \hat{\theta}^{n+1}_- + \hat{K}^{n+1}_\theta (Z^{n+1} - H(\hat{X}^{n+1}_-))
  \end{align*}
  \]

**Luenberger filter** for the state

**Kalman-like filter** for the parameters
Implementation

Data assimilation: parametric estimation

Correction step of the identification process

FSI coupler

Observations:
Full displacements, normal displacements or surfaces

Structure
3D hyperelasticity

Stress on the interface

solid disp and vel
fluid disp and vel
observations
parameters

interface velocity and displ.

Fluid

Correction step of the identification process

Data assimilation: parametric estimation

Observations:
Full displacements, normal displacements or surfaces

Structure
3D hyperelasticity

Stress on the interface

solid disp and vel
fluid disp and vel
observations
parameters

interface velocity and displ.

Fluid
Stiffness estimation
Parameter estimation

- Parameter estimation: Young modulus $E$ in 3 regions
- Synthetic data with $E_1 = 0.5$, $E_2 = 2$, $E_3 = 4 MPa$
- Initial guess: $E = 2 MPa$ in the three regions
- Observations: wall velocity
• Similar experiment with 5 regions
• With noise (10%) and resampling:

Simulation: C. Bertoglio (INRIA)
Example 2
State estimation

Data assimilation
(state only)

Direct simulation

Simulation: N. Xiao (Stanford)
Future

• Charley Taylor & col. are about to leave Stanford
• ...but not the end of the collaboration!
  ★ With A. Figueroa & N. Xiao:
    - inverse problems in full-body-scale 3D vasculature
  ★ With A. Marsden (UC San Diego):
    - Pulmonary arteries in the Transatlantic Network (Leducq foundation)
    - Respiration modeling