Dense direct solvers have long had limited application because of their relative computational cost compared to iterative solvers such as multigrid, conjugate gradient, etc. However there are many problems in which iterative solvers fail, for various reasons including typically the absence of a good and computationally cheap preconditioner. We are attempting to address these issues by developing a new class of direct solvers with a computational cost of $O(N)$ where $N$ is the size of the matrix. These solvers leverage the low-rank structure of certain off-diagonal blocks in the matrix. Recently we have applied these methods to accelerate the simulation of dislocation dynamics for modeling the strength of materials. This work involves a collaboration between INRIA (O. Coulaud, L. Giraud, J. Roman), LBL (X. Li, E. Ng) and Stanford (E. Darve).