Lower-Mobility Parallel Manipulators: Geometrical Analysis and Singularities

Stéphane CARO

Méthodes de subdivisions pour les systèmes singuliers

December 15, 2014

Context

FIGURE: A 6-*dof* parallel manipulator

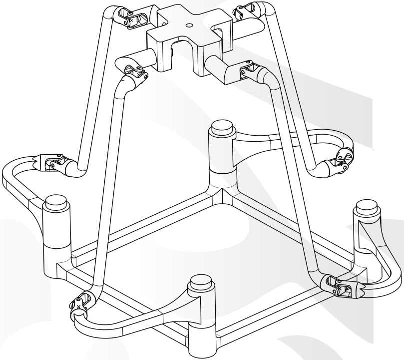
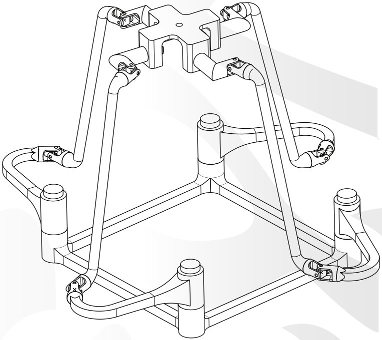


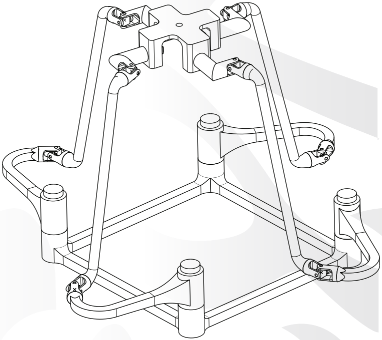
FIGURE: A lower-mobility parallel manipulator

Lower-Mobility Parallel Manipulators



Lower-Mobility Parallel Manipulators

- Great number of architectures and motion patterns



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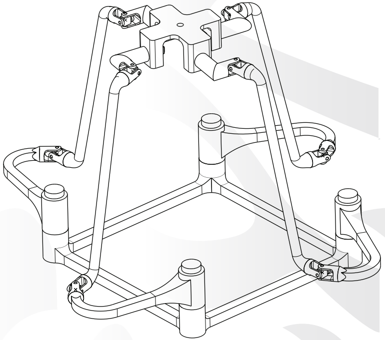

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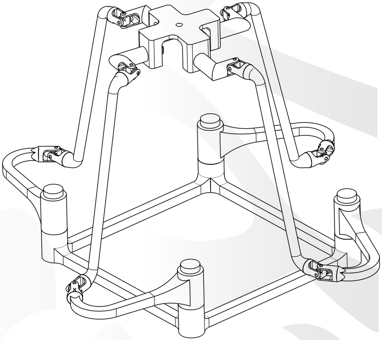
- Constraint singularities



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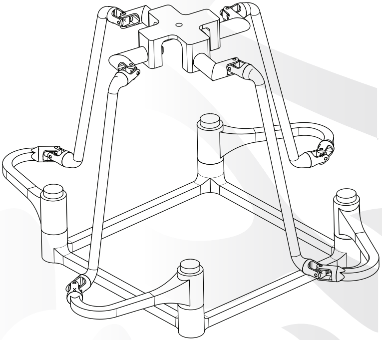

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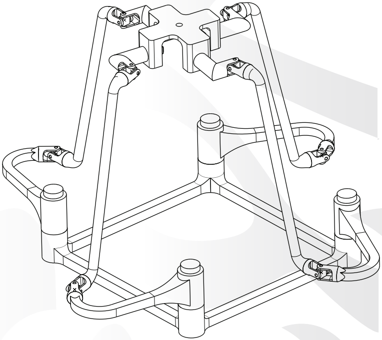

⇒ tools to analyze and compare manipulators at the conceptual design stage

Scope

Geometrical analysis

Analyze the constraints applied on the moving platform

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Parallel singularities

- Loss of control of the moving platform
- They affect the safety and the performance of the robot

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Parallel singularities

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Conceptual design

To consider the parallel singularities at the conceptual design stage

Outline

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Singularity Analysis using GCA

Objectives

- Determine the parallel singularity conditions
- Describe the motion of the moving platform in these configurations

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Tools

- The superbracket of Grassmann-Cayley algebra (GCA)

Parallel Singularities

(n<6)-*dof* parallel manipulator

- Actuators apply a n-actuation wrench system $\mathcal{W}_a$
- Limbs apply a $(6-n)$-constraint wrench system $\mathcal{W}_c$

$$\mathbf{J}_E^T = [\underbrace{\hat{\$}_a^1 \ \ldots \ \hat{\$}_a^n}_{\text{a basis of } \mathcal{W}^a} \ \underbrace{\hat{\$}_c^1 \ \ldots \ \hat{\$}_c^{6-n}}_{\text{a basis of } \mathcal{W}^c}]$$

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Parallel singularities $\Leftrightarrow$ rank$(\mathbf{J}_E) < 6 \Leftrightarrow$ det$(\mathbf{J}_E) = 0$

- Constraint singularities $\rightarrow$ $\mathcal{W}^c$ degenerates $\rightarrow$ gain of some extra *dof*
- Actuation singularities $\rightarrow$ $(\mathcal{W}^a + \mathcal{W}^c)$ degenerates while $\mathcal{W}^c$ does not $\rightarrow$ actuators cannot control the motion of the moving platform

Determination of Parallel Singularities

Classical methods

- $\det(\mathbf{J}_E)$ is a large expression
- cannot provide insight into geometric conditions for parallel singularities

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Alternative methods

- Grassmann-Cayley algebra (GCA)
- Grassmann geometry (GG)

Grassmann-Cayley Algebra

- The GCA was developed by H. Grassmann (1809-1877) as a calculus for linear varieties operating on extensors with the *join* $\vee$ and *meet* $\wedge$ operators.
- GCA makes it possible to work at the symbolic level, to produce coordinate-free algebraic expressions for the singularity conditions of spatial PMs.
- The 4-dimensional vector space $\mathbb{V}$ associated with $\mathbb{P}_3$
- Extensors of step 2 $\leftrightarrow$ Plücker lines (screws of zero/infinite-pitch)

Outline

Superbracket Decomposition

$$\mathbf{J}_E^T = [\underbrace{\hat{\$}_a^1 \ \ldots \ \hat{\$}_a^n}_{\text{a basis of } \mathcal{W}^a} \ \underbrace{\hat{\$}_c^1 \ \ldots \ \hat{\$}_c^{6-n}}_{\text{a basis of } \mathcal{W}^c}]$$

$$\downarrow$$

Superbracket $S = [\mathtt{ab\ cd\ ef\ gh\ ij\ kl}] = \sum_{i=1}^{24} y_i$

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$$\downarrow$$

$y_1 = [\mathtt{abcd}][\mathtt{efgi}][\mathtt{hjkl}]$ $\qquad$ $y_2 = -[\mathtt{abcd}][\mathtt{efhi}][\mathtt{gjkl}]$

$\vdots$

$y_{23} = [\mathtt{abch}][\mathtt{defj}][\mathtt{gikl}]$ $\qquad$ $y_{24} = -[\mathtt{abdh}][\mathtt{cefj}][\mathtt{gikl}]$

Superbracket Decomposition

$$\mathbf{J}_E^T = [\underbrace{\hat{\$}_a^1 \ \dots \ \hat{\$}_a^n}_{\text{a basis of } \mathcal{W}^a} \ \underbrace{\hat{\$}_c^1 \ \dots \ \hat{\$}_c^{6-n}}_{\text{a basis of } \mathcal{W}^c}]$$

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- A bracket, [a b c d] vanishes whenever :
  1. two points are repeated
  2. three points are collinear
  3. the four points are coplanar

Formulation of the Superbracket

$$S = [\mathtt{ab\ ad\ ef\ gh\ ij\ kl}] = \sum_{i=1}^{24} y_i$$

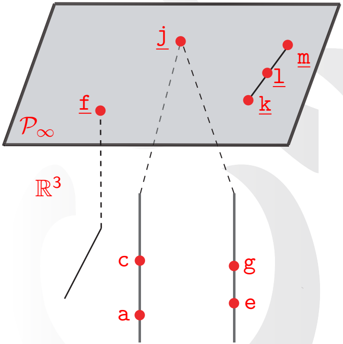
$$y_1 = [\mathtt{abcd}][\mathtt{efgi}][\mathtt{hjkl}] \qquad y_2 = -[\mathtt{abcd}][\mathtt{efhi}][\mathtt{gjkl}]$$

$$\vdots$$

$$y_{23} = [\mathtt{abch}][\mathtt{defj}][\mathtt{gikl}] \qquad y_{24} = -[\mathtt{abdh}][\mathtt{cefj}][\mathtt{gikl}]$$

Simplifications

- Reduce the number of distinct points
- Use collinear points
- Use coplanar points, points at infinity

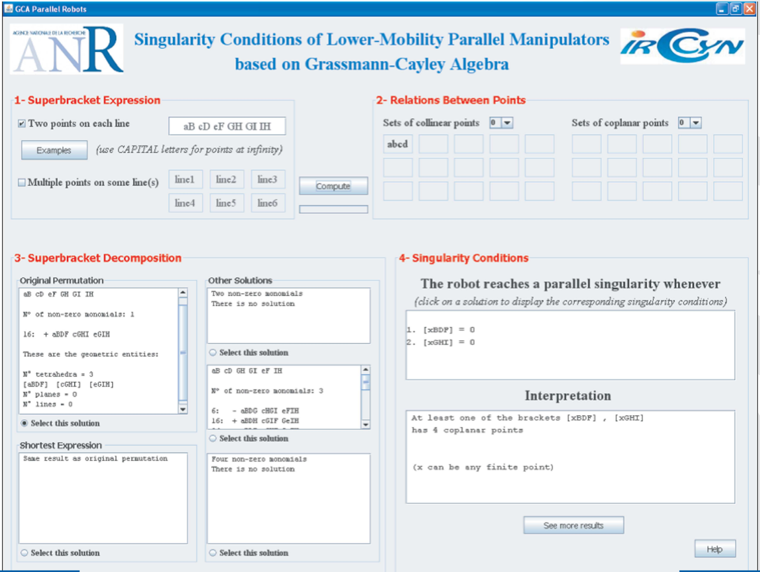


Difficulties

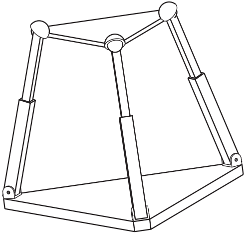
- Several possible formulations
- Effect of the permutation of lines

⇒ Long hand calculations

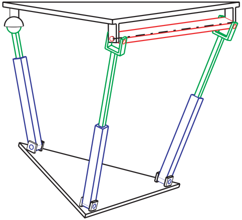
Graphical User Interface



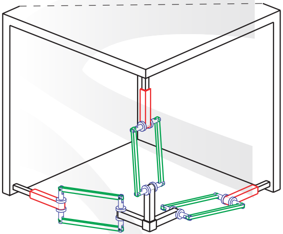
Applications



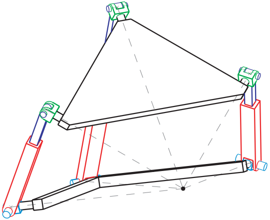
3-RPS



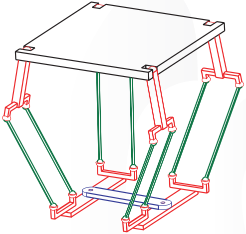
Exechon



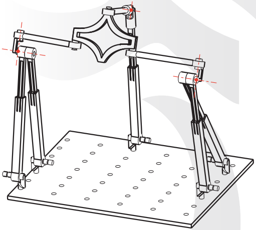
Orthoglide



3-UPU wrist



H4



5-RPUR

Outline

Case Study

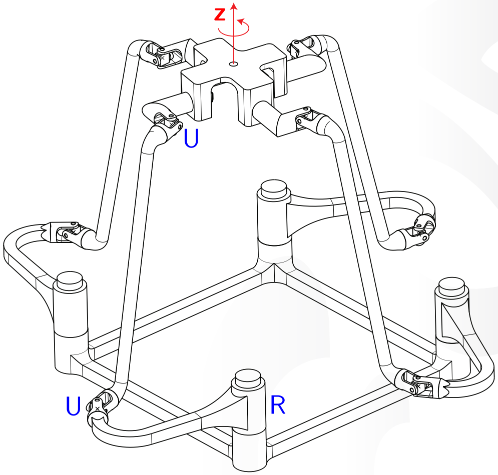


FIGURE: The 4-RUU parallel manipulator

Constraint Analysis

Limb wrench systems

- A pure constraint moment $M_i = (\mathbf{0}; \mathbf{m}_i \times \mathbf{z})$
- A pure actuation force $F_i = (\mathbf{f}_i; \mathbf{r}_{C_i} \times \mathbf{f}_i)$

Platform wrench systems

$$\underbrace{F_1,\ F_2,\ F_3,\ F_4}_{\text{4-system}},\ \underbrace{M_1,\ M_2,\ M_3,\ M_4}_{\text{2-system}}$$

$\Rightarrow$ the PM is over-constrained

Superbracket

Superbracket $\leftrightarrow [F_1,\ F_2,\ F_3,\ F_4,\ M_I,\ M_{II}]$

$\Rightarrow$ six possible superbrackets

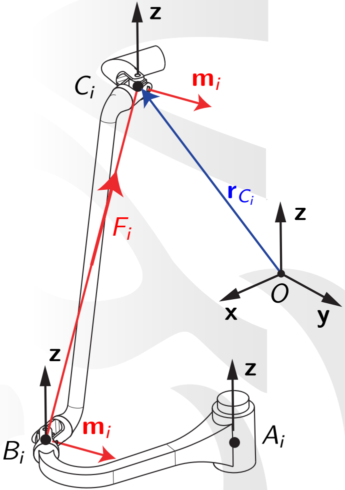


FIGURE: A RUU limb

Superbracket Points

- $S_1 = [\mathtt{ab}, \mathtt{ef}, \mathtt{cd}, \mathtt{gh}, \mathtt{ij}, \mathtt{kj}]$
- $S_2 = [\mathtt{ab}, \mathtt{ef}, \mathtt{cd}, \mathtt{gh}, \mathtt{ij}, \mathtt{mj}]$
- $S_3 = [\mathtt{ab}, \mathtt{ef}, \mathtt{cd}, \mathtt{gh}, \mathtt{ij}, \mathtt{lj}]$
- $S_4 = [\mathtt{ab}, \mathtt{ef}, \mathtt{cd}, \mathtt{gh}, \mathtt{lj}, \mathtt{mj}]$
- $S_5 = [\mathtt{ab}, \mathtt{ef}, \mathtt{cd}, \mathtt{gh}, \mathtt{lj}, \mathtt{kj}]$
- $S_6 = [\mathtt{ab}, \mathtt{ef}, \mathtt{cd}, \mathtt{gh}, \mathtt{mj}, \mathtt{kj}]$

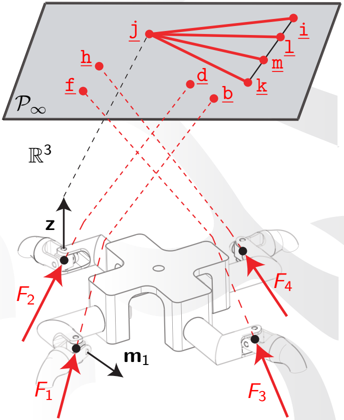
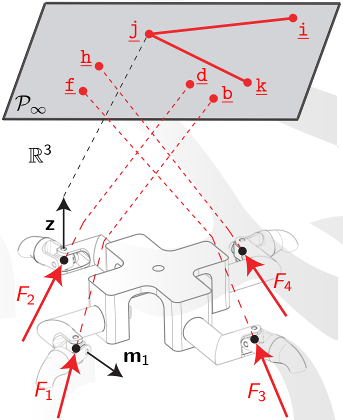


FIGURE: Wrench graph of the 4-RUU PM

Superbracket Points

- $S_1 = [a\underline{b}, e\underline{f}, c\underline{d}, g\underline{h}, \underline{i}\underline{j}, \underline{k}\underline{j}]$



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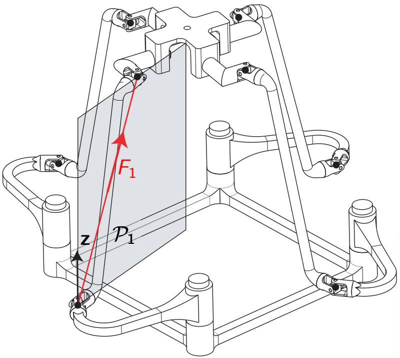
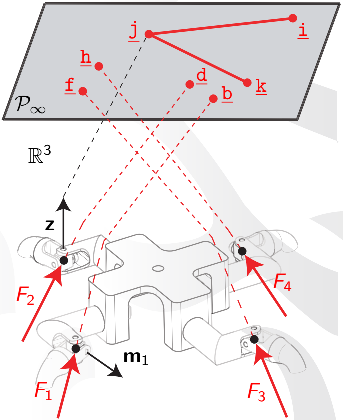


FIGURE: The 4-RUU PM



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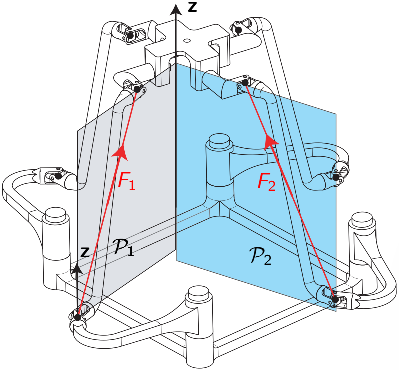
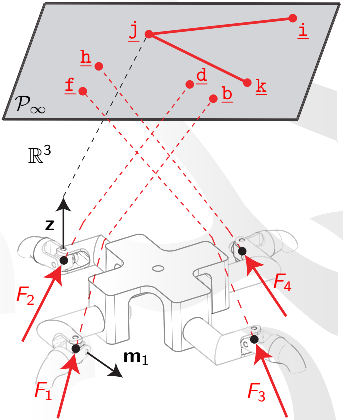


FIGURE: The 4-$\underline{R}$UU PM



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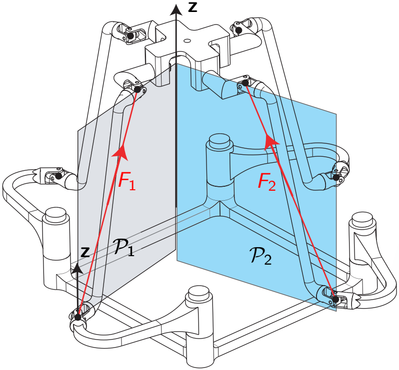

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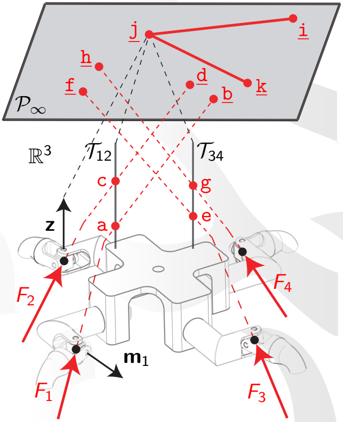

FIGURE: Wrench graph of the 4-$\underline{\text{R}}$UU PM

Wrench Graph

$S_1 = [\underline{ab}, \underline{ef}, \underline{cd}, \underline{gh}, \underline{ij}, \underline{kj}]$

- Points at infinity $\{\underline{b}, \underline{d}, \underline{f}, \underline{h}, \underline{i}, \underline{j}, \underline{k}\}$
- $[a, c, \underline{j}, x] = 0$
- $[e, g, \underline{j}, x] = 0$

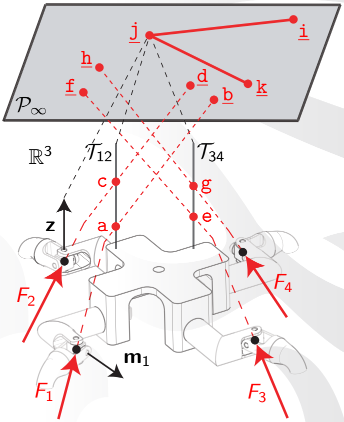

FIGURE: Wrench graph of the 4-RUU PM

Superbracket Decomposition

$$S_j = A_j\big(\underbrace{[\mathtt{a}\,\underline{\mathtt{d}}\,\underline{\mathtt{f}}\,\underline{\mathtt{h}}][\mathtt{e}\,\mathtt{c}\,\underline{\mathtt{b}}\,\underline{\mathtt{j}}] - [\mathtt{a}\,\underline{\mathtt{b}}\,\underline{\mathtt{f}}\,\underline{\mathtt{h}}][\mathtt{e}\,\mathtt{c}\,\underline{\mathtt{d}}\,\underline{\mathtt{j}}]}_{B}\big) \quad ; \quad j = 1, \ldots, 6$$

$$A_1 = [\mathtt{g}\,\underline{\mathtt{i}}\,\underline{\mathtt{k}}\,\underline{\mathtt{j}}],\ A_2 = [\mathtt{g}\,\underline{\mathtt{i}}\,\underline{\mathtt{l}}\,\underline{\mathtt{j}}],\ A_3 = [\mathtt{g}\,\underline{\mathtt{i}}\,\underline{\mathtt{m}}\,\underline{\mathtt{j}}],\ A_4 = [\mathtt{g}\,\underline{\mathtt{k}}\,\underline{\mathtt{l}}\,\underline{\mathtt{j}}],$$
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Two parallel singularity cases

1. All terms A_j vanish simultaneously $\rightarrow$ Constraint singularity

Superbracket Decomposition

$$S_j = A_j\Big(\underbrace{[\mathtt{a}\ \underline{\mathtt{d}}\ \underline{\mathtt{f}}\ \underline{\mathtt{h}}][\mathtt{e}\ \mathtt{c}\ \underline{\mathtt{b}}\ \underline{\mathtt{j}}] - [\mathtt{a}\ \underline{\mathtt{b}}\ \underline{\mathtt{f}}\ \underline{\mathtt{h}}][\mathtt{e}\ \mathtt{c}\ \underline{\mathtt{d}}\ \underline{\mathtt{j}}]}_{B}\Big) \quad ; \quad j = 1, \dots, 6$$

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Two parallel singularity cases

1. All terms A_j vanish simultaneously $\rightarrow$ Constraint singularity
2. Term B vanishes $\rightarrow$ Actuation singularity

Constraint Singularities

$\mathbf{m}_1 \parallel \mathbf{m}_2 \parallel \mathbf{m}_3 \parallel \mathbf{m}_4$

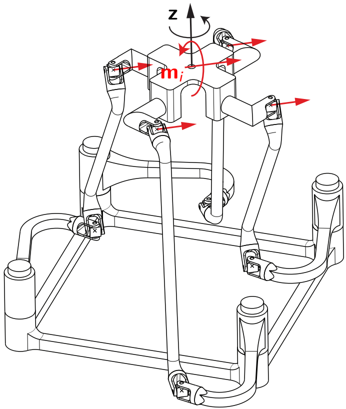


FIGURE: A constraint singular configuration

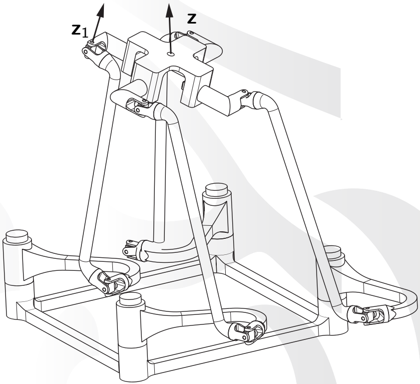
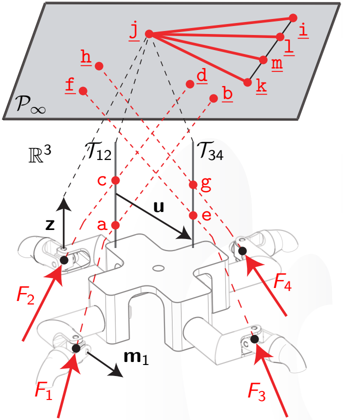


FIGURE: Coupled motions

Actuation Singularities

$$\Big((\mathbf{f}_1 \times \mathbf{f}_2) \times (\mathbf{f}_3 \times \mathbf{f}_4) \Big) \cdot (\mathbf{u} \times \mathbf{z}) = 0$$



Actuation Singularities (Cont'd)

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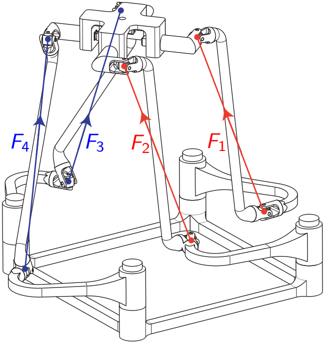

FIGURE: Two actuation forces F_1 and F_2 are parallel.

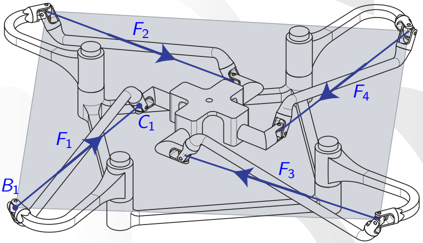

FIGURE: All actuation forces are coplanar.

Outline

Correspondence between GCA and GG

Objectives

- Use Grassmann Geometry (GG) with points and lines at infinity
- Highlight the correspondence between GCA and GG

Correspondence between GCA and GG

Objectives

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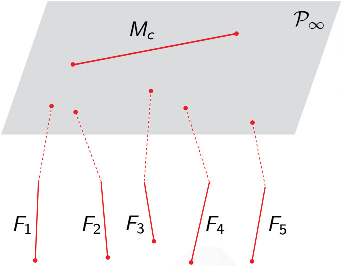
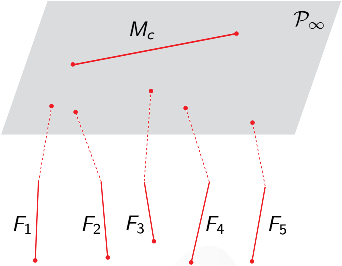
Tools

- The classification of linear varieties

Use of Grassmann Geometry

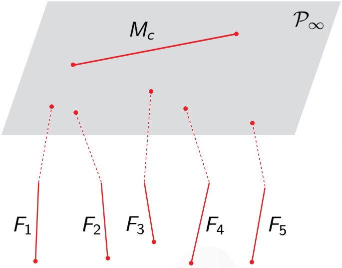


Use of Grassmann Geometry



Rank	Class	Linear line variety	
3	*planes*	(3a)	a regulus
		(3b)	the union of two flat pencils having a line in common but lying in distinct planes and with distinct centers
		(3c)	all lines through a point
		(3d)	all lines in a plane
4	*congruences*		
5	*complexes*	(5a)	non singular complex ; generated by five skew lines
		(5b)	singular complex ; all the lines meeting one given line

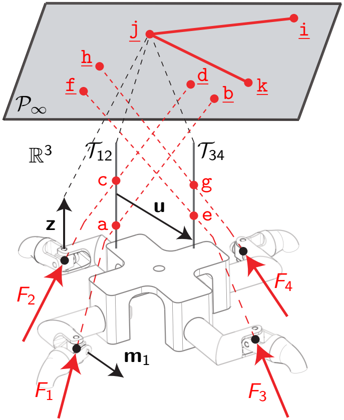
Use of Grassmann Geometry



Rank	Class	Linear line variety	
3	*planes*	(3a)	a regulus
		(3b)	the union of two flat pencils having a line in common but lying in distinct planes and with distinct centers
		(3c)	all lines through a point
		(3d)	all lines in a plane
4	*congruences*		
5	*complexes*	(5a)	non singular complex ; generated by five skew lines
		(5b)	singular complex ; all the lines meeting one given line

Rank Deficiency of the Jacobian Matrix

$$\mathbf{J}_E^T = [F_1,\ F_2,\ F_3,\ F_4,\ M_I,\ M_{II}]$$



$$\Big((\mathbf{f}_1 \times \mathbf{f}_2) \times (\mathbf{f}_3 \times \mathbf{f}_4)\Big) \cdot (\mathbf{u} \times \mathbf{z}) = 0$$

$$\downarrow$$

$$\mathbf{f}_1 \parallel \mathbf{f}_2$$

$$\downarrow$$

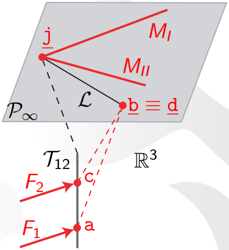


FIGURE: Wrench graph of the 4-RUU PM

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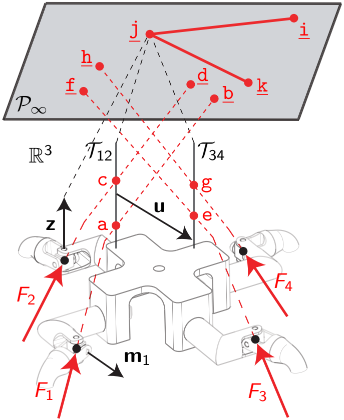

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$$\downarrow$$

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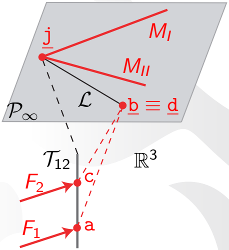


$\Rightarrow$ Condition (3b) of GG :

$$\dim\Big(\mathrm{span}(F_1,\ F_2,\ M_I,\ M_{II})\Big) \leq 3$$

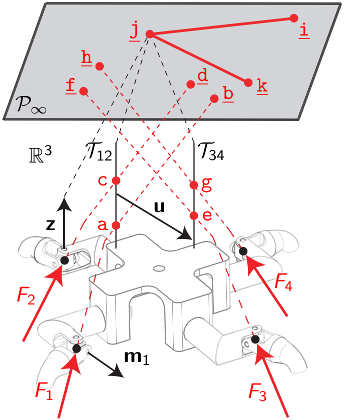
Unconrolled Motions

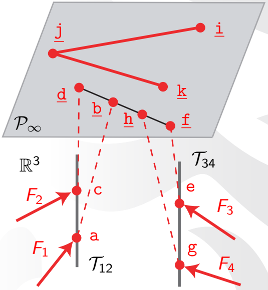
$$\mathbf{J}_E^T = [F_1,\ F_2,\ F_3,\ F_4,\ M_I,\ M_{II}]$$

$$\Big((\mathbf{f}_1 \times \mathbf{f}_2) \times (\mathbf{f}_3 \times \mathbf{f}_4)\Big) \cdot (\mathbf{u} \times \mathbf{z}) = 0$$

$$\downarrow$$

$$(\mathbf{f}_1 \times \mathbf{f}_2) \parallel (\mathbf{f}_3 \times \mathbf{f}_4)$$





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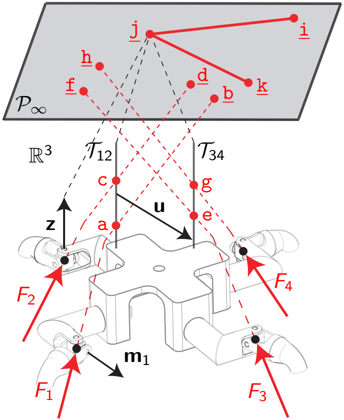

FIGURE: Wrench graph of the 4-RUU PM

$$\Big((\mathbf{f}_1 \times \mathbf{f}_2) \times (\mathbf{f}_3 \times \mathbf{f}_4)\Big) \cdot (\mathbf{u} \times \mathbf{z}) = 0$$

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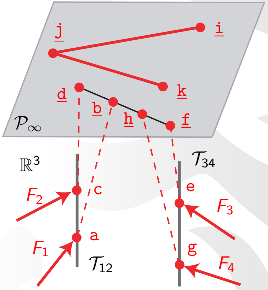


$\Rightarrow$ Condition (5b) of GG :
A line at infinity intersects all the lines of $\mathbf{J}_E^T$

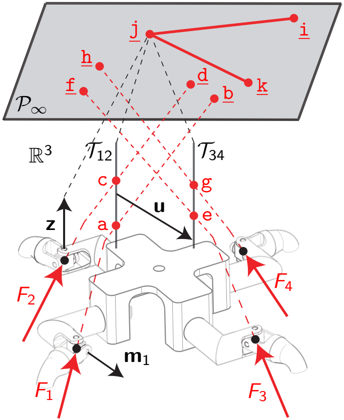
Unconrolled Motions (Cont'd)

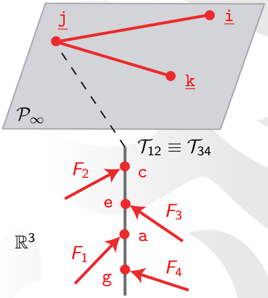
$$\mathbf{J}_E^T = [F_1,\ F_2,\ F_3,\ F_4,\ M_I,\ M_{II}]$$

$$\big((\mathbf{f}_1 \times \mathbf{f}_2) \times (\mathbf{f}_3 \times \mathbf{f}_4)\big) \cdot (\mathbf{u} \times \mathbf{z}) = 0$$

$$\downarrow$$

$$\mathbf{u} \times \mathbf{z} = \mathbf{0}$$





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Unconrolled Motions (Cont'd)

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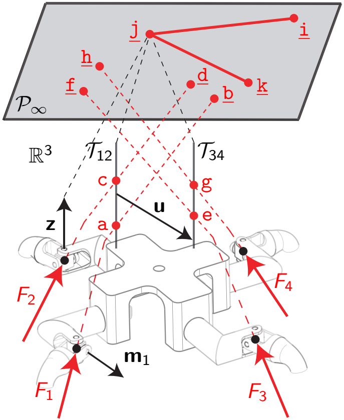

FIGURE: Wrench graph of the 4-RUU PM

$$\Big((\mathbf{f}_1 \times \mathbf{f}_2) \times (\mathbf{f}_3 \times \mathbf{f}_4)\Big) \cdot (\mathbf{u} \times \mathbf{z}) = 0$$

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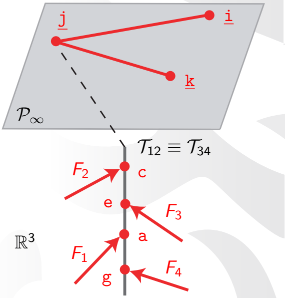

$\Rightarrow$ Condition (5b) of GG : A finite line intersects all the lines of $\mathbf{J}_E^T$

Correspondence between GCA and GG

Case	Singularity condition GCA	Corresponding case of GG
(a)	$\mathbf{f}_i \parallel \mathbf{f}_j$	condition (3b)
(b)	$\mathbf{u}_{ij}^{kl} \parallel \mathbf{z}$	condition (5b)
(c)	$(\mathbf{f}_i \times \mathbf{f}_j) \parallel (\mathbf{u}_{ij}^{kl} \times \mathbf{z})$	condition (5b)
(d)	$(\mathbf{f}_i \times \mathbf{f}_j) \parallel (\mathbf{f}_k \times \mathbf{f}_l)$	conditions (5b), (3d)
(e)	$(\mathbf{f}_i \times \mathbf{f}_j) \parallel (\mathbf{u}_{ij}^{kl} \times \mathbf{z}) \parallel (\mathbf{f}_k \times \mathbf{f}_l)$	condition (5b)
(f)	$\Big((\mathbf{f}_i \times \mathbf{f}_j) \times (\mathbf{f}_k \times \mathbf{f}_l)\Big) \perp (\mathbf{u}_{ij}^{kl} \times \mathbf{z})$	condition (5a)
(g)	$\mathbf{m}_i \parallel \mathbf{m}_j \parallel \mathbf{m}_k \parallel \mathbf{m}_l$	conditions 1, (5b)

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Case	Singularity condition GCA	Corresponding case of GG
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(d)	$(\mathbf{f}_i \times \mathbf{f}_j) \parallel (\mathbf{f}_k \times \mathbf{f}_l)$	conditions (5b), (3d)
(e)	$(\mathbf{f}_i \times \mathbf{f}_j) \parallel (\mathbf{u}_{ij}^{kl} \times \mathbf{z}) \parallel (\mathbf{f}_k \times \mathbf{f}_l)$	condition (5b)
(f)	$\Big((\mathbf{f}_i \times \mathbf{f}_j) \times (\mathbf{f}_k \times \mathbf{f}_l)\Big) \perp (\mathbf{u}_{ij}^{kl} \times \mathbf{z})$	condition (5a)
(g)	$\mathbf{m}_i \parallel \mathbf{m}_j \parallel \mathbf{m}_k \parallel \mathbf{m}_l$	conditions 1, (5b)

Correspondence between GCA and GG

- The use of GCA and GG as complementary approaches to better understand the singularities

Amine, S., Tale Masouleh, M., Caro, S., Wenger, P., and Gosselin, C., 2011 "*Singularity Conditions of 3T1R Parallel Manipulators with Identical Limb Structures*", ASME Journal of Mechanisms and Robotics, DOI : 10.1115/1.4005336

Amine, S., Tale Masouleh, M., Caro, S., Wenger, P., and Gosselin, C., "*Singularity Analysis of 3T2R Parallel Mechanisms using Grassmann-Cayley Algebra and Grassmann Line Geometry*", Mechanism and Machine Theory, 10.1016/j.mechmachtheory.2011.11.015

Correspondence between GCA and GG

- The use of GCA and GG as complementary approaches to better understand the singularities

- How to consider the parallel singularities at the conceptual design stage ?

Amine, S., Tale Masouleh, M., Caro, S., Wenger, P., and Gosselin, C., 2011 "*Singularity Conditions of 3T1R Parallel Manipulators with Identical Limb Structures*", ASME Journal of Mechanisms and Robotics, DOI : 10.1115/1.4005336

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Outline

Contributions

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   - enumerate the parallel singularity conditions
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   - enumerate the parallel singularity conditions
   - describe the behavior of the moving platform
3. A graphical user interface for the parallel singularity analysis

Future Works

- Identify the limits of Grassmann-Cayley algebra by studying other motion patterns such as 3R2T, 3R1T and 2R2T
- Singularity analysis of PMs with finite-pitch wrenches
- Analysis of the motion mode changing for lower-mobility PMs

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Conclusions & Future Works

Thank you !

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Lower-Mobility Parallel Manipulators: Geometrical Analysis and Singularities

Stéphane CARO

Méthodes de subdivisions pour les systèmes singuliers

December 15, 2014