Workspace and joint space analysis of the 3-RPS parallel robot

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Problem statement

- There are parallel robots with several operation modes [Zlatanov]
- *Can we extend the definitions of the aspects for this kind of robots?*
- *Can we define the uniqueness domains?*
- *Can we change assembly mode?*
- Application: 3-R<u>P</u>S [Husty]

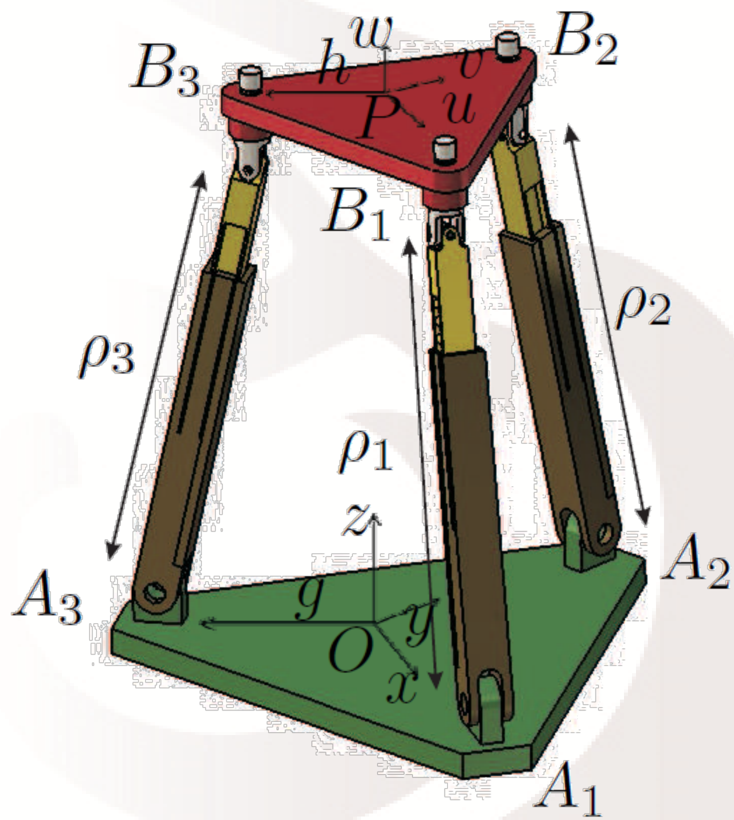
Outline of the presentation

- Mechanism under study
- Inverse and direct kinematic problem / Operation mode
- Singularities and aspects / Operation mode
- Characteristic surfaces / Operation mode
- Uniqueness domains / Operation mode
- Non-singular assembly mode changing trajectory
- Conclusions

The 3-RPS parallel robot

- Actuators ρ_1, ρ_2, ρ_3
- End-effector $x, y, z, q_1, q_2, q_3, q_4$
- Parameters $g = h = 1$
- With

$$\| A_i - B_i \| = \rho_i \quad \text{with} \quad i = 1, 2, 3$$



Kinematics

- Coordinates of the joints in local and global reference frame

$$A_1 = \begin{bmatrix} g \\ 0 \\ 0 \end{bmatrix} \quad A_2 = \begin{bmatrix} -g/2 \\ g\sqrt{3}/2 \\ 0 \end{bmatrix} \quad A_3 = \begin{bmatrix} -g/2 \\ -g\sqrt{3}/2 \\ 0 \end{bmatrix}$$

$$b_1 = \begin{bmatrix} h \\ 0 \\ 0 \end{bmatrix} \quad b_2 = \begin{bmatrix} -h/2 \\ h\sqrt{3}/2 \\ 0 \end{bmatrix} \quad b_3 = \begin{bmatrix} -h/2 \\ -h\sqrt{3}/2 \\ 0 \end{bmatrix}$$

$$B_i = P + R b_i \text{ for } i = 1,2,3 \text{ and } R = \begin{bmatrix} u_x & v_x & w_x \\ u_y & v_y & w_y \\ u_x & v_z & w_z \end{bmatrix}$$

Kinematics

- Six constraint equations

$$\| \mathrm{A}_i - \mathrm{B}_i \| = \rho_i \quad \text{with} \quad i = 1, 2, 3$$

$$u_y h + y = 0$$

$$y - u_y h / 2 + \sqrt{3} v_y h / 2 + \sqrt{3} x - \sqrt{3} u_x h / 2 + 3 v_x h / 2 = 0$$

$$y - u_y h / 2 - \sqrt{3} v_y h / 2 - \sqrt{3} x + \sqrt{3} u_x h / 2 + 3 v_x h / 2 = 0$$

- Simplification from 6 to 4 parameters

$$y = -h u_y$$

$$x = h \left(\sqrt{3} u_x - \sqrt{3} v_y - 3 u_y + 3 v_x \right) \sqrt{3} / 6$$

Operation modes

- Definition of the orientation matrix by the unit quaternions

$$
\mathrm{R} = \begin{bmatrix} 2q_1^{\,2} + 2q_2^{\,2} - 1 & -2q_1q_4 + 2q_2q_3 & 2q_1q_3 + 2q_2q_4 \\ 2q_1q_4 + 2q_2q_3 & 2q_1^{\,2} + 2q_3^{\,2} - 1 & -2q_1q_2 + 2q_3q_4 \\ -2q_1q_3 + 2q_2q_4 & 2q_1q_2 + 2q_3q_4 & 2q_1^{\,2} + 2q_4^{\,2} - 1 \end{bmatrix}
$$

with $q_1^2 + q_2^2 + q_3^2 + q_4^2 = 1$

- Two operation modes with

$$
q_1 = 0 \quad \text{or} \quad q_4 = 0
$$

Operation modes

- In the workspace W, for each operation mode, the W_{OMj} is defined such that

$$W_{OM_j} \subset W$$
$$\forall X \in W_{OM_j}, OM \text{ is constant}$$

- Inverse and direct kinematic problem

$$g_j(\mathrm{X}) = \mathrm{q}$$
$$g_j^{-1}(\mathrm{q}) = \mathrm{X} \mid (\mathrm{X},\mathrm{q}) \in OM^j$$

- In our case: $OM^1 : q_1 = 0 \quad \text{or} \quad OM^2 : q_4 = 0$

Algebraic tools

- SIROPA library (ANR SIROPA, J-P. Merlet, P. Wenger)
- Groëbner basis elimination
  - Computation of the singularities and characteristic surfaces
- Cylindrical Algebraic Decomposition (CAD)
  - Representation of the aspects and basic regions

Singularities

- Differentiating with respect to time the constraint equations leads to the velocity model:

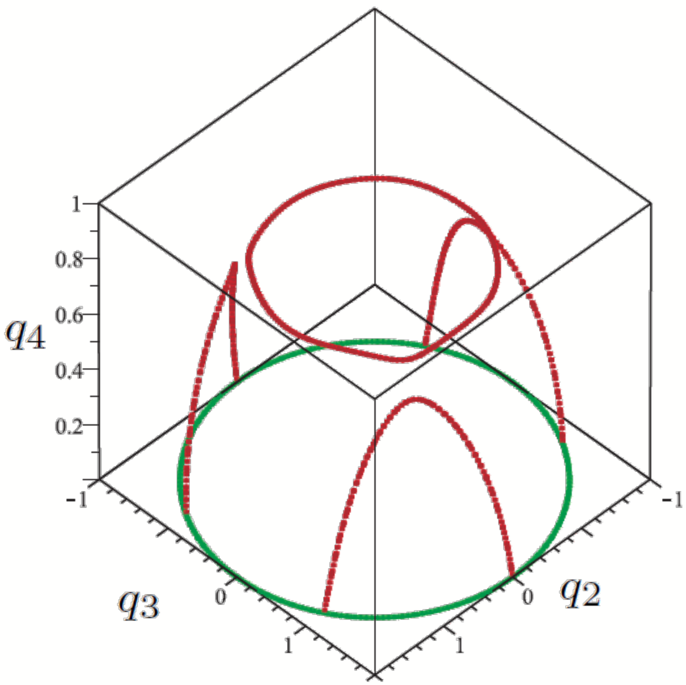
$$\mathrm{A}\,\dot{\mathrm{t}} + \mathrm{B}\,\dot{\mathrm{q}} = 0$$

$$\begin{aligned}S_{OM^1} :\ & q_4(8q_2q_3^2q_4^6 + 2q_2q_4^8 - 64zq_3^6q_4 - 96zq_3^4q_4^3 - 36zq_3^2q_4^5 - 6zq_4^7\\ & -24z^2q_2q_3^2q_4^2 - 6z^2q_2q_4^4 - 32q_2q_3^2q_4^4 - 10q_2q_4^6 + 2z^3q_4^3 + 96zq_3^4q_4\\ & +72zq_3^2q_4^3 + 23zq_4^5 + 16z^2q_2q_3^2 + 10z^2q_2q_4^2 + 8q_2q_4^4 - z^3q_4 - 36zq_3^2q_4\\ & -21zq_4^3 - 4z^2q_2 + 4zq_4) = 0\end{aligned}$$

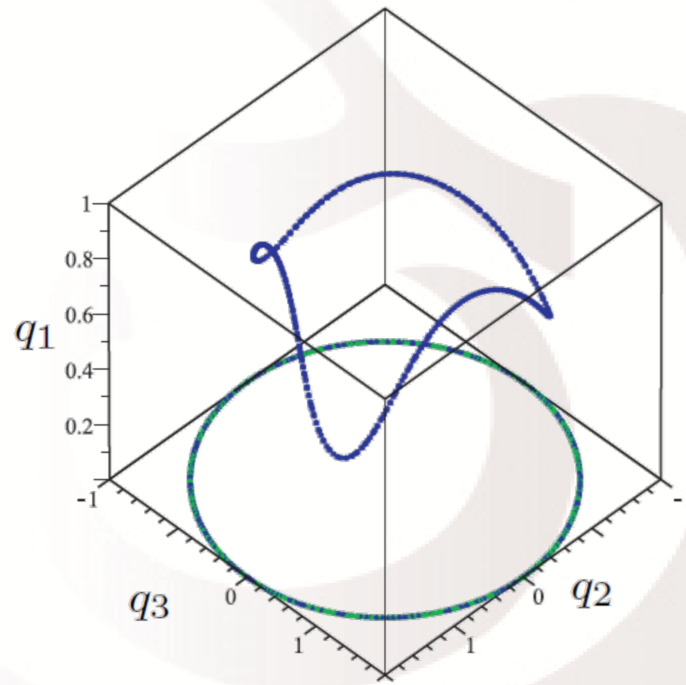
$$\begin{aligned}S_{OM^2} :\ & q_1^2(6q_1^7q_3 + 8q_1^5q_3^3 - 2zq_1^6 + 36zq_1^4q_3^2 + 96zq_1^2q_3^4 + 64zq_3^6\\ & -18z^2q_1^3q_3 - 24z^2q_1q_3^3 - 18q_1^5q_3 - 16q_1^3q_3^3 + 2z^3q_1^2 + 3zq_1{}^4 - 72zq_1^2q_3^2\\ & -96zq_3^4 + 18z^2q_1q_3 + 12q_1^3q_3 - z^3 + 3zq_1^2 + 36zq_3^2 - 4z) = 0\end{aligned}$$

Singularities

- For $z = 3$ and $q_4 > 0$ *or* $q_1 > 0$



(a)



(b)

Aspect for an operation mode

- Definition for « classical » parallel robot, an aspect WA_i is a maximal singularity free set defined such that

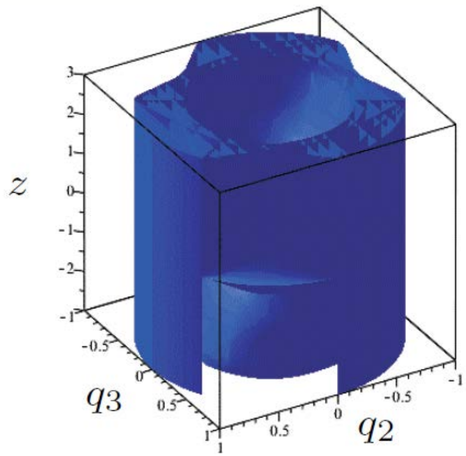
$$WA_i \subset W$$
$$WA_i \text{ is connected}$$
$$\forall X \in WA_i, \det(\mathrm{A}) \neq 0 \text{ and } \det(\mathrm{B}) \neq 0$$

- For a given operation mode *j*, an aspect WA_i^j is a maximal singularity free set defined such that

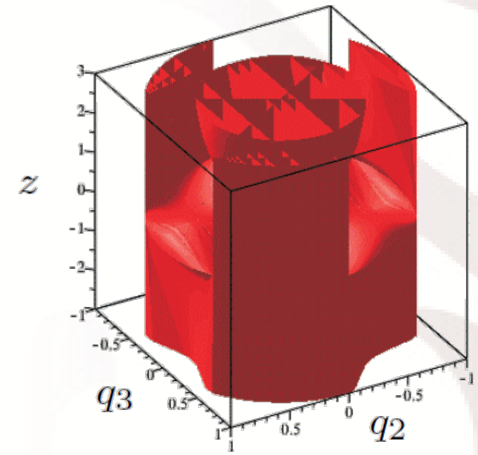
$$WA_i^j \subset W_{OM_j}$$
$$WA_i^j \text{ is connected}$$
$$\forall X \in WA_i^j, \det(\mathrm{A}) \neq 0 \text{ and } \det(\mathrm{B}) \neq 0$$

Aspects for an operation mode

- Four aspects, two for each operation mode



(a)



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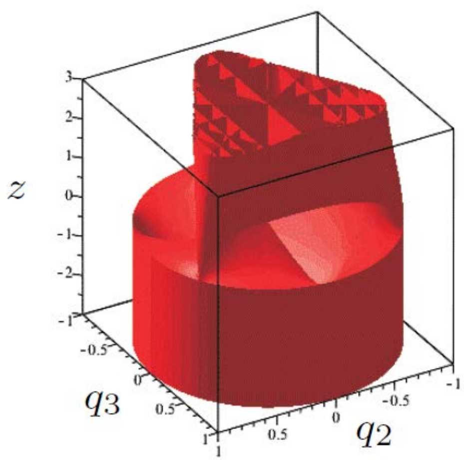

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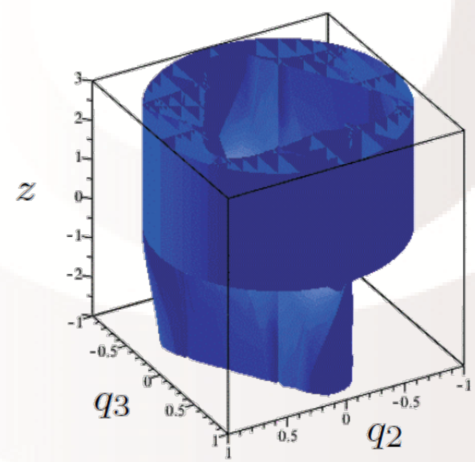
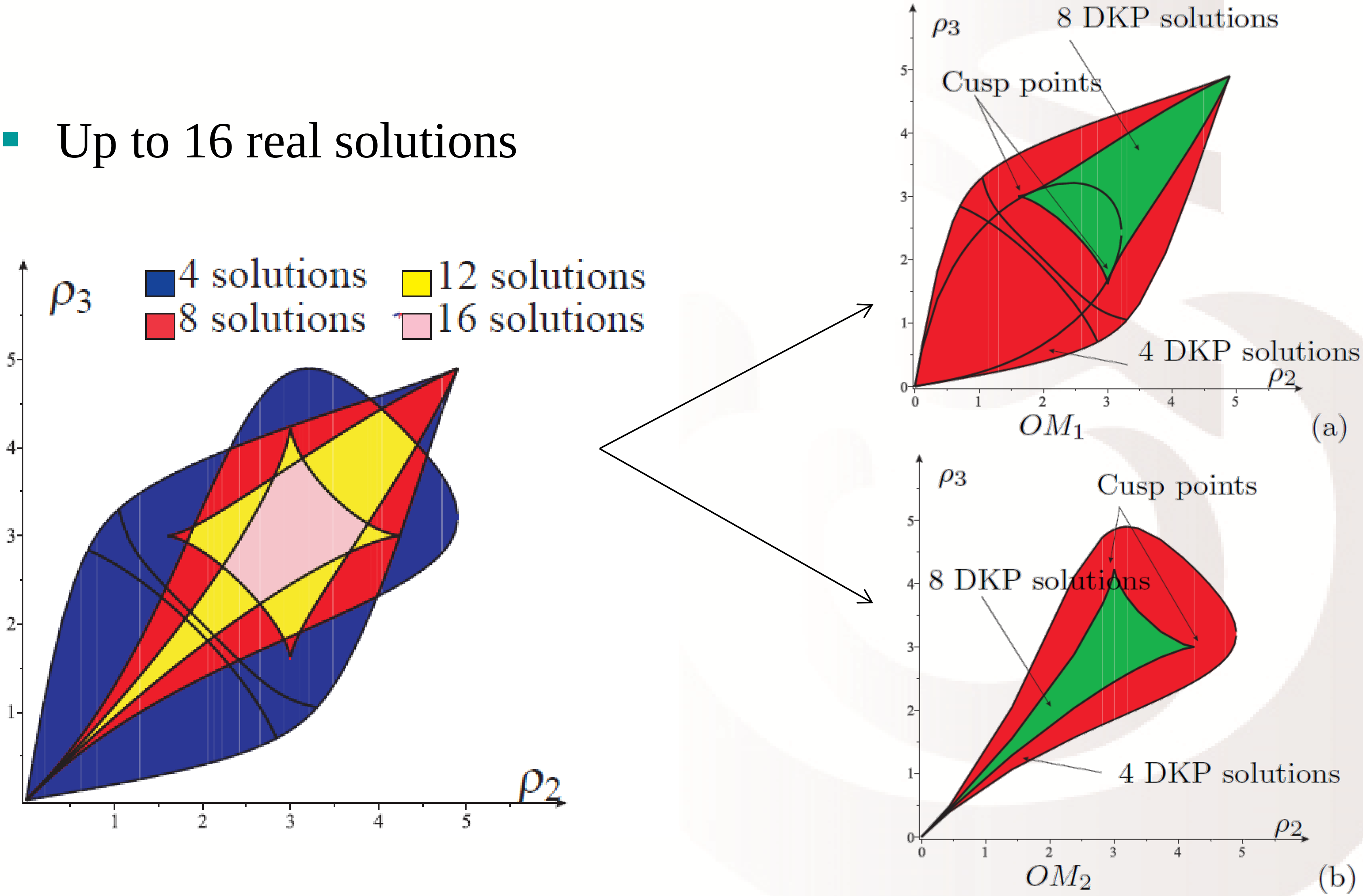


(d)

Direct kinematics

- Up to 16 real solutions

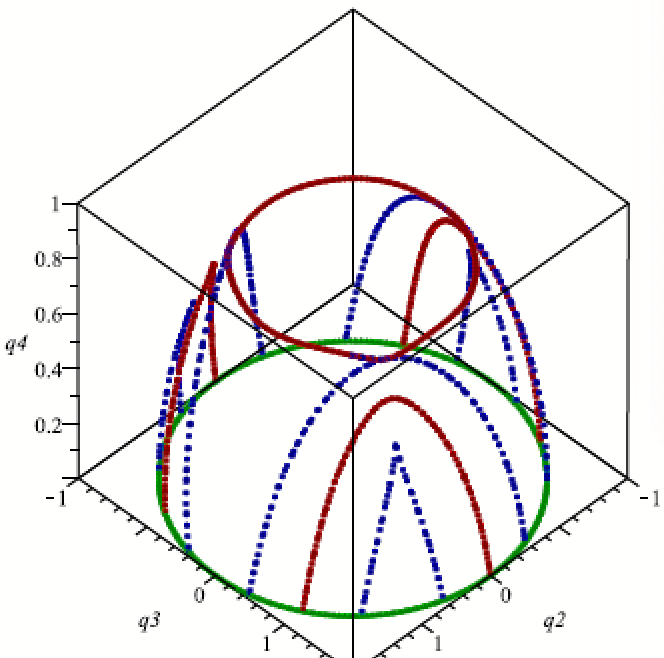


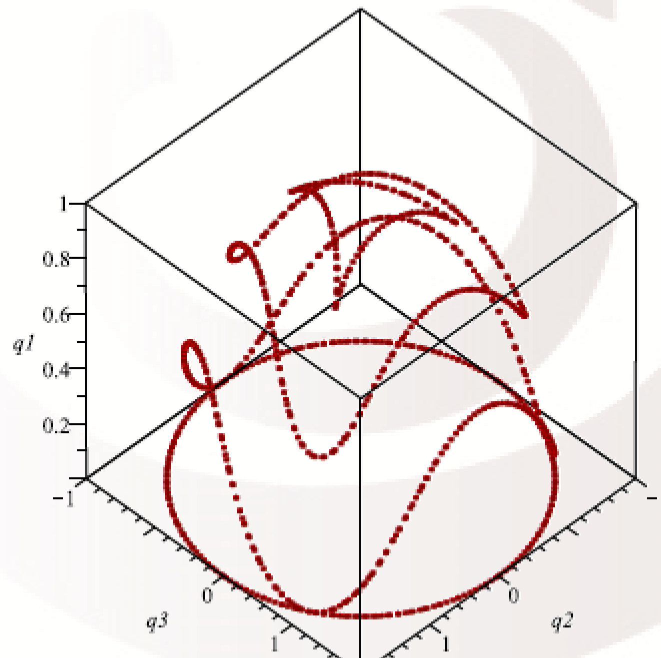
Characteristic surfaces for an operation mode

- Let WA_i^j be one aspect for the operation mode j.

- The characteristic surfaces, denoted by $S_C^j(WA_i^j)$

$$S_C^{\,j}(WA_i^{\,j}) = g_j^{-1}\left(\overline{g(WA_i^{\,j})}\right) \cap WA_i^{\,j}$$





Basic regions/Basic components for an operation mode

- Basic regions for an operation mode

$$\cup WAb_i^j = WA_i^j - S_C^j$$

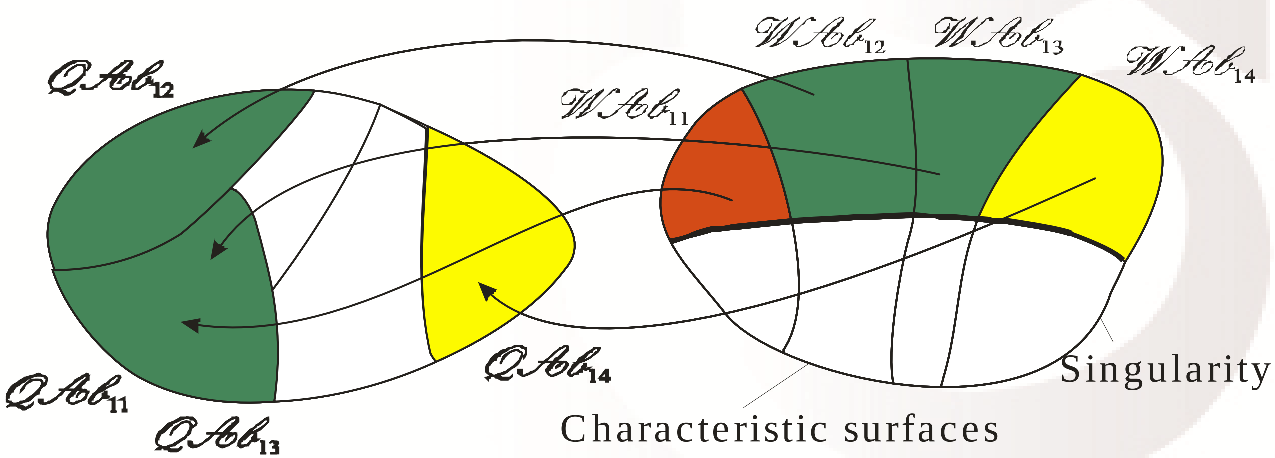
- Basic components for an operation mode

$$QA_i^j = g_j\left(WAb_i^j\right)$$

Uniqueness domains for an operation mode

- For a given operation mode *j*, a uniqueness domain Wu_k^i

$$Wu_k^j = \left(\cup_{i \in I'} WAb_i^j \right) \cup S_C^j(I')$$

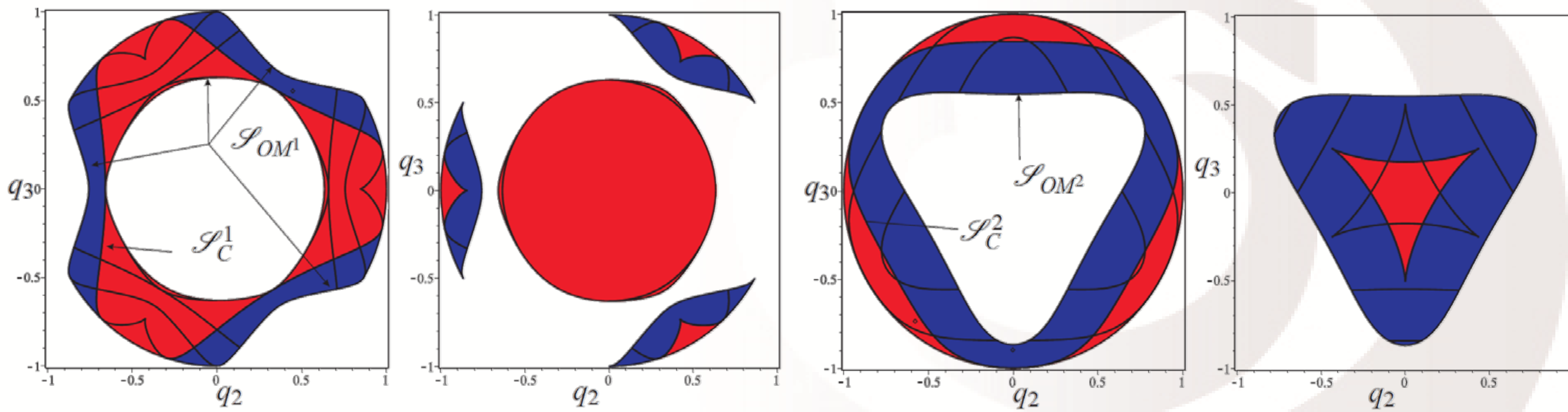


Joint space

Workspace

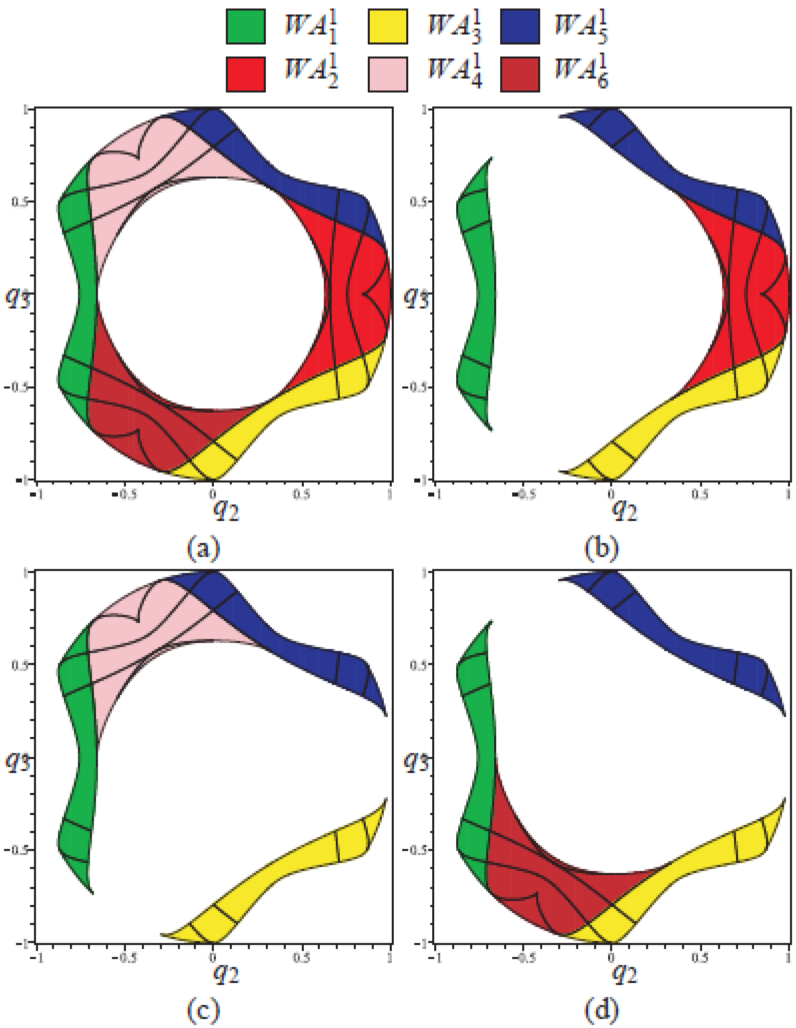
Uniqueness domains

- Slice of the workspace for z =3



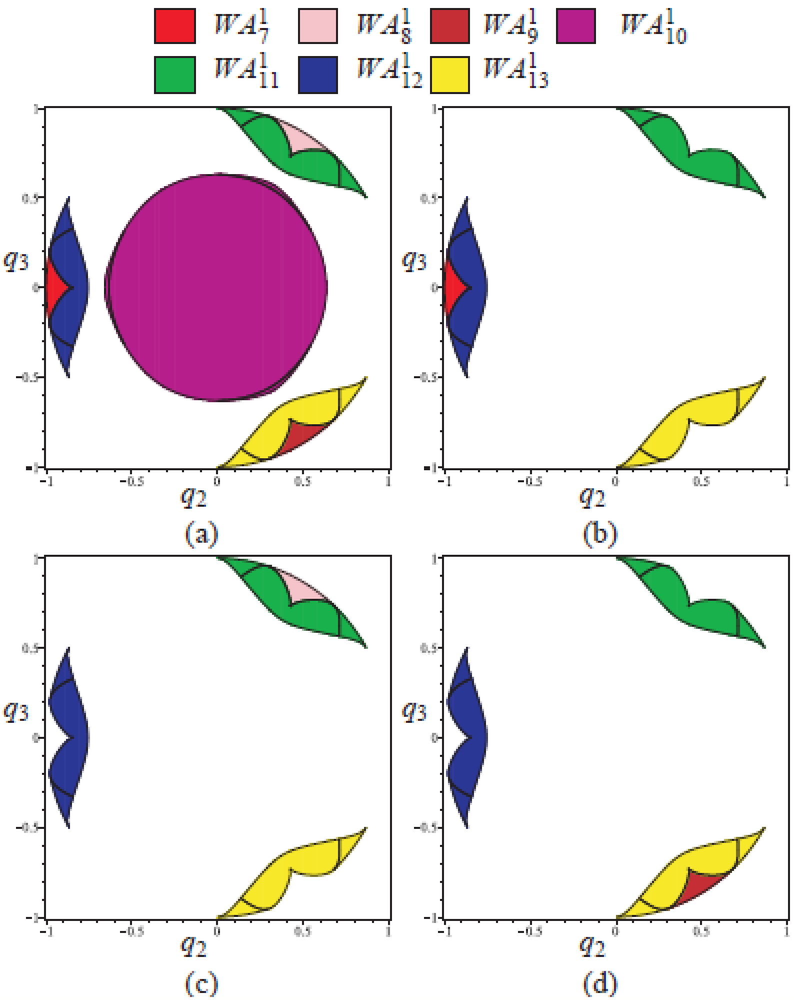
OM^1 OM^2

Uniqueness domains for OM^1 $\det(\mathbf{A}) > 0$

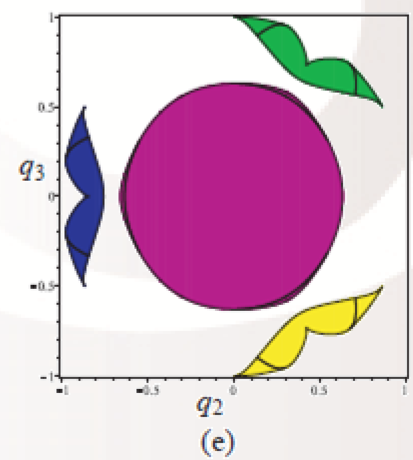


$$Wu_1^1 = WA_1^1 \cup WA_2^1 \cup WA_3^1 \cup WA_4^1$$
$$Wu_2^1 = WA_1^1 \cup WA_2^1 \cup WA_3^1 \cup WA_5^1$$
$$Wu_3^1 = WA_1^1 \cup WA_2^1 \cup WA_3^1 \cup WA_6^1$$

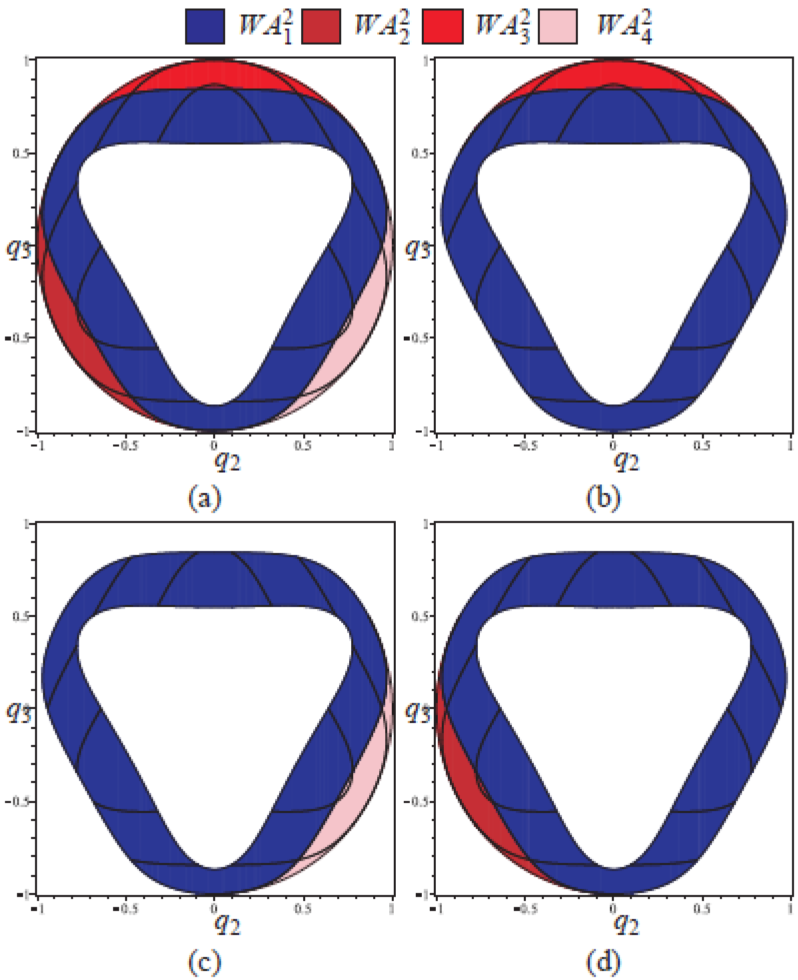
Uniqueness domains for OM^1 $\det(\mathbf{A}) < 0$



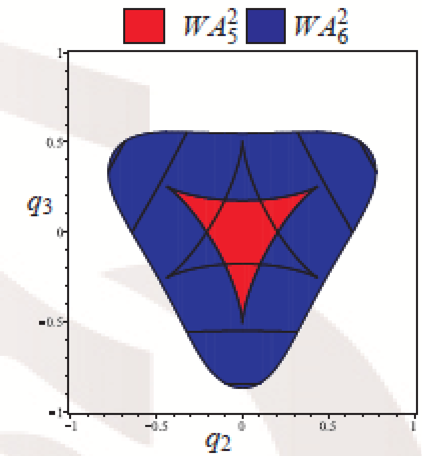
$$Wu_4^1 = WA_7^1 \cup WA_{11}^1 \cup WA_{12}^1 \cup WA_{12}^1$$
$$Wu_5^1 = WA_8^1 \cup WA_{11}^1 \cup WA_{12}^1 \cup WA_{12}^1$$
$$Wu_6^1 = WA_9^1 \cup WA_{11}^1 \cup WA_{12}^1 \cup WA_{12}^1$$
$$Wu_7^1 = WA_{10}^1 \cup WA_{11}^1 \cup WA_{12}^1 \cup WA_{12}^1$$



Uniqueness domains for OM^2



$$Wu_1^2 = WA_1^2 \cup WA_2^2$$
$$Wu_2^2 = WA_1^2 \cup WA_3^2$$
$$Wu_3^2 = WA_1^2 \cup WA_4^2$$
$$Wu_4^2 = WA_5^2 \cup WA_6^2$$



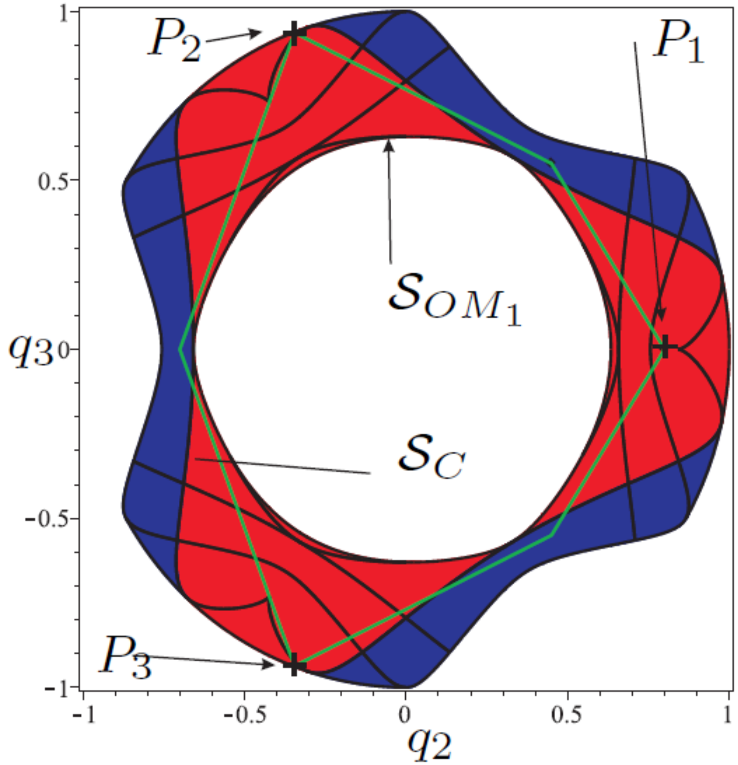
Non-singular assembly mode changing trajectories

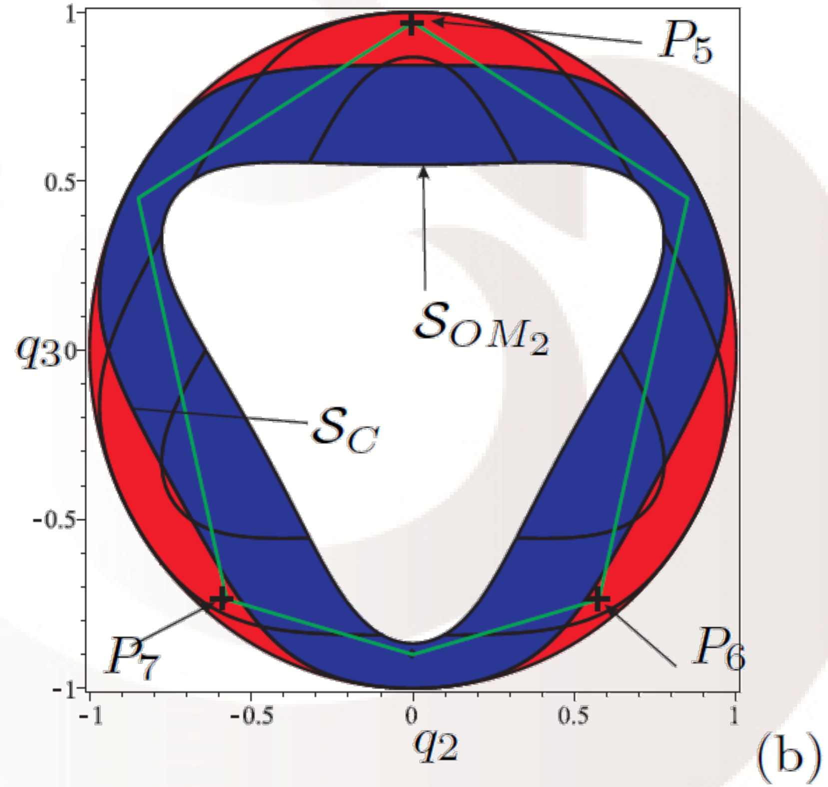
- Can I connect three configurations?
- When the robot is changing assembly mode?
- Example:

OM_1					OM_2				
$\rho_1 = 3.90,\ \rho_2 = 3.24,\ \rho_3 = 3.24$					$\rho_1 = 3.79,\ \rho_2 = 3.24,\ \rho_3 = 3.24$				
P	z	q_2	q_3	q_4	P	z	q_1	q_2	q_3
P_1	3.01	−0.34	−0.94	0.06	P_5	3.04	0.35	−0.58	−0.74
P_2	3.01	−0.34	0.94	0.06	P_6	3.04	0.35	0.586	−0.74
P_3	3	0.85	0.0	0.53	P_7	3	0.24	0.0	0.97
P_4	−2.88	−0.35	0.0	0.93	P_8	−3.42	0.98	0.0	0.19

Non-singular assembly mode changing trajectories

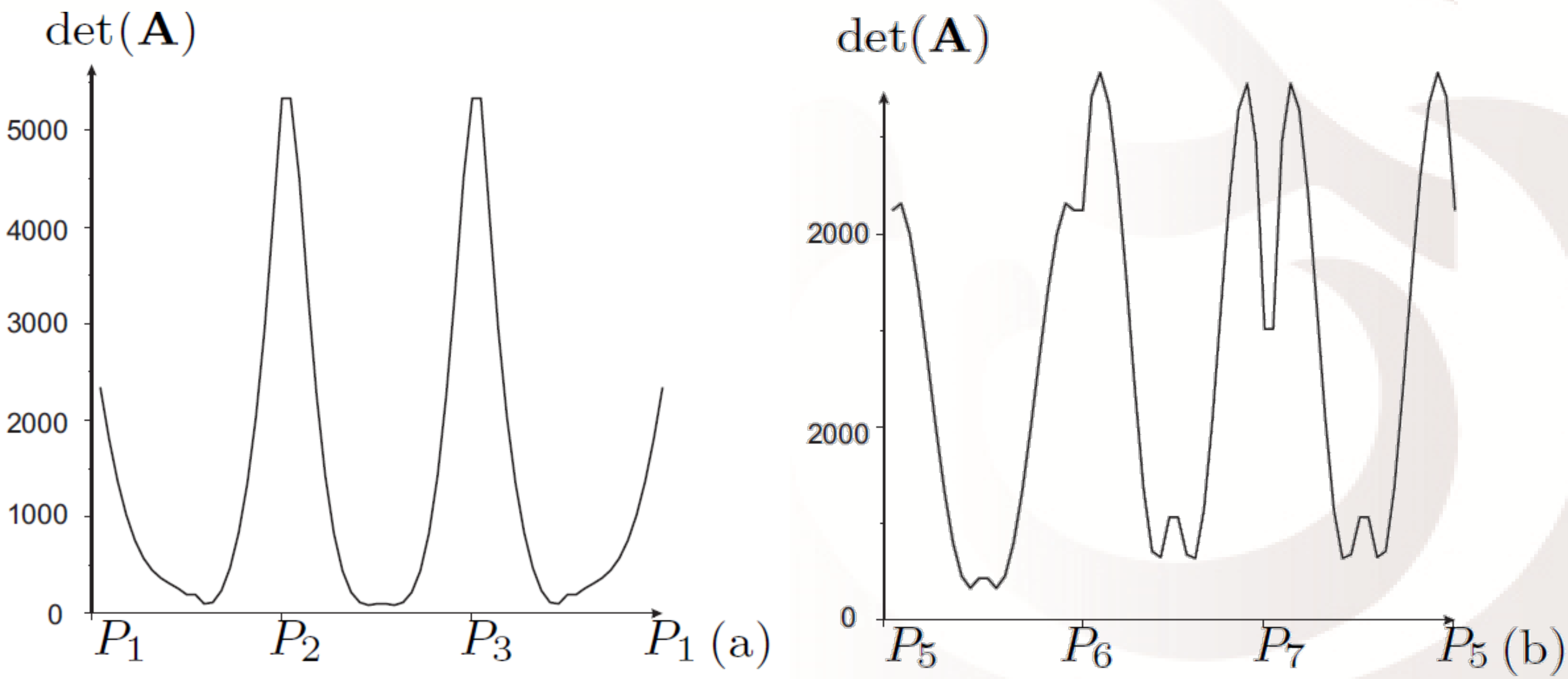
- Two trajectories $P_1P_2P_3$ and $P_5P_6P_7$ for z=3





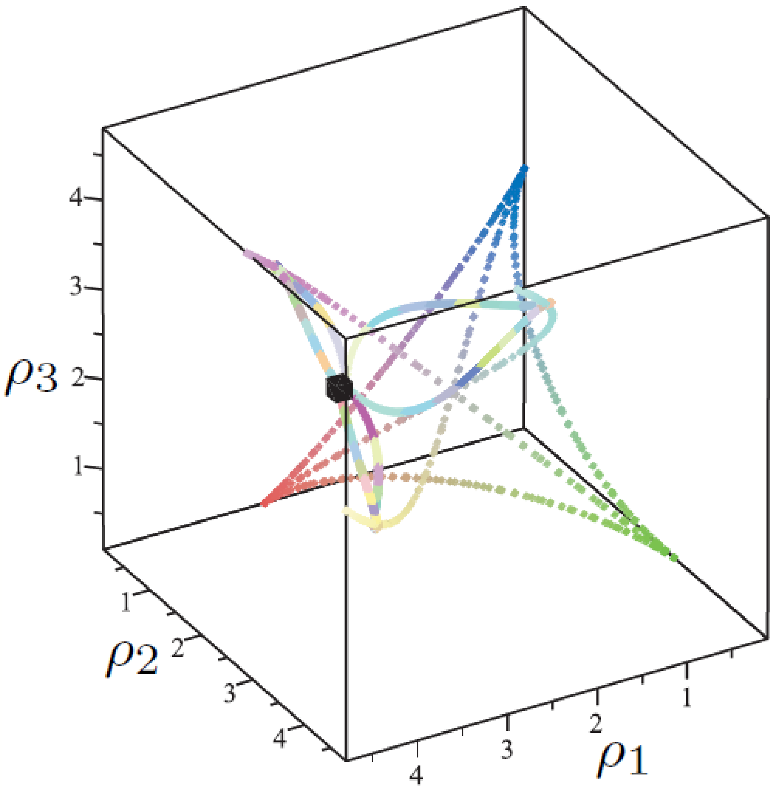
Non-singular assembly mode changing trajectories

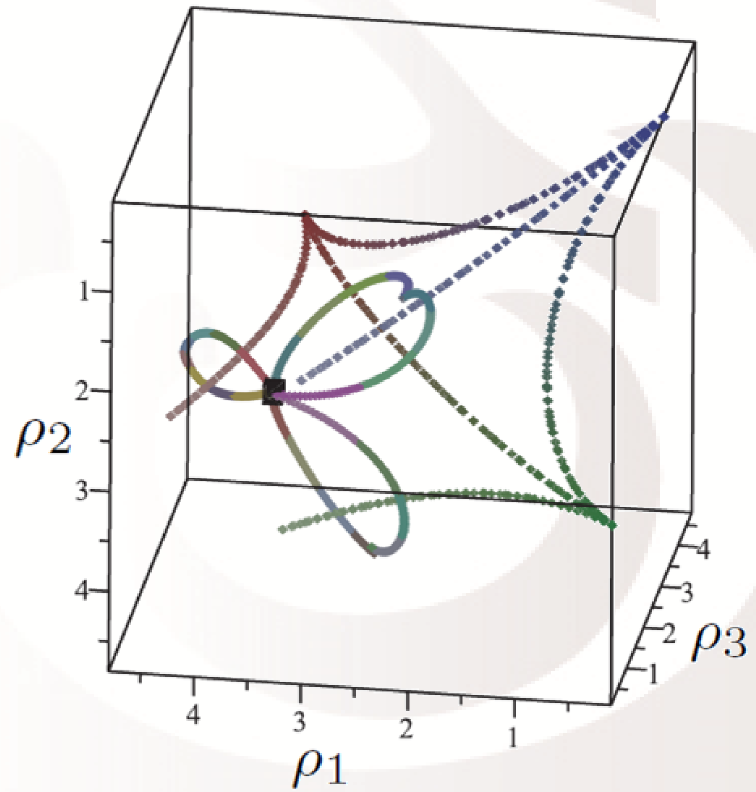
- Images in the joint space



Non-singular assembly mode changing trajectories

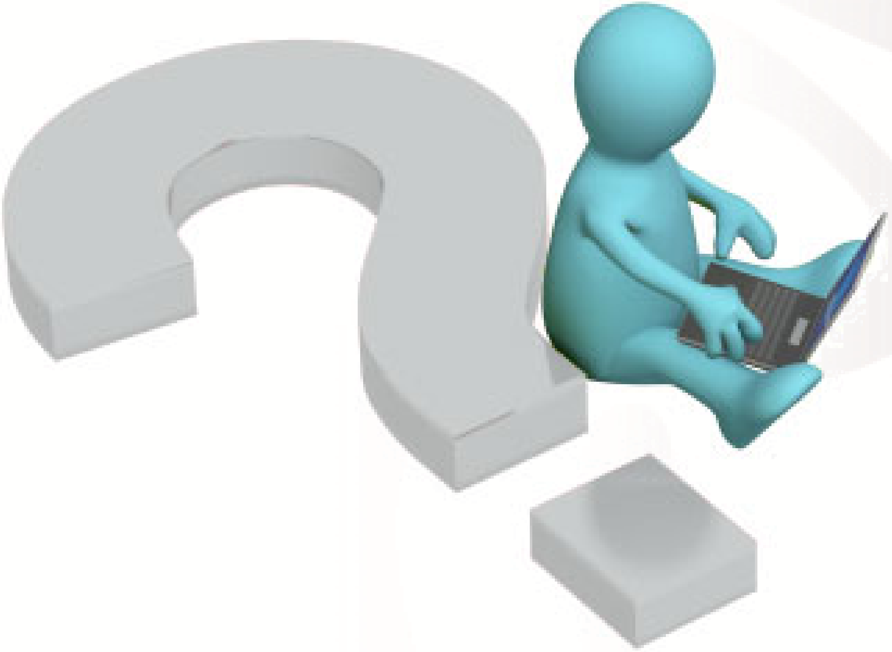
- Images in the joint space





Conclusions

- A study of the joint space and workspace of the 3-RPS parallel robot
- Two non-singular assembly mode changing trajectories are made.
- The two operation modes are divided into two aspects
- This mechanism admits a maximum of 16 real solutions to the direct kinematic problem, eight for each operation mode.
- The basic regions for each operation mode are defined.
- The uniqueness domains for each operation mode are defined.
- A path is defined through several basic regions which are images of the same basic component with 8 solutions for the DKP.
- The proof is made by the analysis of the determinant of Jacobian.



Thank you for your attention

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