A parametric Kantorovich theorem with application to tolerance synthesis

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1 Problem and contribution

We consider a system $f(x, q, p) = 0$ of n equations and n unknowns, denoted by $x \in \mathbb{R}^n$, with two kinds of parameters: $q \in \mathbb{R}^m$, interpreted as controlled parameters (called commands for short), and $p \in \mathbb{R}^q$, interpreted as design environment parameters with some uncertainties. For a fixed command value q, we call the solutions x_0 of $f(x_0, q, 0) = 0$ the *nominal solutions* corresponding to this command, while x satisfying $f(x, q, p) = 0$ for $p \neq 0$ is called a p-perturbed solution. The nominal solutions of interest are furthermore subject to given constraints $g(x, q) \leq 0$, possibly encoding domains for x and q. We aim at bounding rigorously the worst case distance from any nominal solution satisfying $g(x_0, q) \leq 0$ to its corresponding perturbed solution. This is done in two steps:

- Determining a uncertainty domain for which the correlation between nominal and perturbed solutions is non-ambiguous (a uniqueness condition on the existence of the perturbed solution inside a neighborhood of the nominal solution will be involved).

- Computing an upper-bound $\epsilon(e)$ on the distance between the nominal solution and its p-perturbed solution. Both a crude constant upper-bound and a sharp upper-bound depending on $\|e\|$ will be provided.

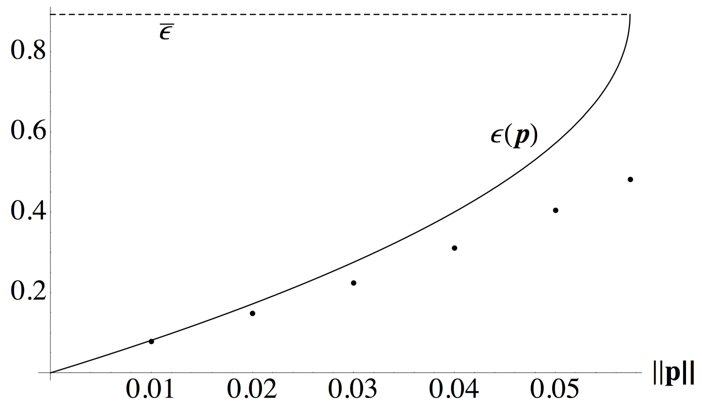

Figure 1: The upper bound $\epsilon(p)$ (dots represent exact maximal error, showing the overestimation of $\epsilon(p)$).

We propose a parametric Kantorovich theorem, which will achieve these two tasks. The idea is to compute worst case Kantorovich constants with respect to parameters q using nonlinear nonsmooth global optimization (a branch and bound algorithm and numerical constraint programming), and to use a rigorous first order model of the dependence with respect to parameters p.

2 Case study

The $\underline{\text{P}}$RRP manipulator is modeled by the following equation:

$$(x - a + p_1)^2 + (q - b + p_2)^2 = (l + p_3)^2, \tag{1}$$

where x is the pose, q is the command, parameters values are $a = 1$, $b = 1$ and $l = 3$, and p_i are uncertainties acting on them. The constraints g are $2 \leq x \leq 3 \wedge 3 \leq q \leq 4$. The proposed parametric Kantorovich theorem proves that every nominal pose has a unique corresponding perturbed pose for uncertainties satisfying $\|p\| \leq 0.057$, and that for these uncertainties the distance between the nominal pose and the perturbed pose is smaller than $\epsilon(p)$, with $\epsilon(p)$ shown on Figure 1. Experiments on parallel manipulators up to 3 degrees of freedom, i.e., $n = 3$ and $m = 3$, have been successfully conducted so far.